Market Size, Firm Sourcing, and Vertical Specialization*

Tomohiro Ara†

Fukushima University

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Abstract

This paper studies how market size shapes vertical specialization across production stages. We develop a model in which heterogeneous firms jointly decide whether to export final goods and import intermediate inputs. The model shows that market size alone can generate endogenous specialization across production stages, even in the absence of technological differences. A larger country tends to specialize in downstream activities and export relatively more final goods, while a smaller country specializes in upstream stages and exports relatively more intermediate inputs. The mechanism depends on how market size affects firms' joint exporting and importing decisions, altering the composition of firms across production modes and reallocating activity across stages. When input trade costs are high, the model generates a conventional home-market effect in which larger countries sustain higher relative wages. When input trade costs become sufficiently low, however, firms substitute more intensively toward foreign inputs, reversing the relative wage response to market size, although larger countries continue to export relatively more final goods. Lower input trade costs also attenuate the welfare gains associated with larger markets and redistribute income across countries participating in global value chains.

Keywords: Market size; home-market effects; vertical specialization; input trade; heterogeneous firms
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†Faculty of Economics and Business Administration, Fukushima University, Fukushima 960-1296, Japan. *Email address*: `tomohiro.ara@gmail.com`

1 Introduction

Production is increasingly fragmented across countries, with economies specializing in different stages of global value chains. Recent research shows that larger economies tend to occupy more downstream positions along these chains. Antràs and de Gortari (2020) find that countries with greater economic size or centrality specialize in downstream stages—a pattern they term the "centrality–downstreamness nexus"—which they explain through the accumulation of iceberg trade costs along production chains and countries' positions within the network. Johnson and Moxnes (2023) likewise document systematic differences in specialization across stages driven by technology and trade costs. While this literature highlights the roles of trade costs, technology, and production structure in shaping specialization and trade patterns across stages, it does not isolate how market size operates through firm-level decisions. This raises the question of whether market size alone can generate such differences by reshaping firms' exporting and importing decisions and, in turn, aggregate outcomes.

To understand the mechanism, it is useful to consider firm-level evidence on sourcing and exporting behavior. A key feature of modern trade is that a large share of cross-border transactions consists of intermediate inputs, and trade in intermediate inputs and final goods captures distinct but closely related margins of specialization: final-good trade reflects where goods are consumed, whereas intermediate-input trade reflects how production is organized internationally. Firms that expand sourcing tend to exhibit stronger export performance (Feng, Li, and Swenson 2016), while firm heterogeneity shapes sourcing relationships (Huang et al. 2024). Participation in international trade is also highly selective, with exporters and importers systematically larger, more productive, and more input-intensive than purely domestic firms (Bernard et al. 2018). Market size affects not only demand but also competition, thereby generating heterogeneous responses across firms (Lileeva and Trefler 2010; Aghion et al. 2024). Taken together, this evidence suggests that firms jointly make exporting and importing decisions through selection, whereby market size shapes their incentives to engage in both activities and, consequently, the organization of production across stages.[1]

Despite this firm-level evidence, much of the existing theoretical literature builds on competitive frameworks (Caliendo and Parro 2015; Antràs and de Gortari 2020; Johnson and Moxnes 2023), abstracting from firms' exporting and importing decisions. As a result, this literature does not capture how firm behavior translates into country-level outcomes, in particular how selection shapes specialization across production stages. A key missing mechanism is that market size affects revenues and profitability, altering incentives to export and import and the equilibrium composition of firms across production modes, which in turn gives rise to endogenous specialization. This mechanism matters because the composition of firms shapes aggregate trade flows and firms' participation in global production networks, as emphasized by Bernard et al. (2018).

We develop a heterogeneous-firm model in which market size affects specialization through general-equilibrium adjustments driven by joint exporting and importing behavior and selection. In the model, a larger domestic market raises demand for local labor, generating a standard market-size effect that tends to increase wages. When inputs are tradable, it also alters firms' incentives to import intermediates and increases foreign labor demand. This reallocation shifts the composition of activity across stages, creating countervailing effects on local labor demand. As a result, the impact of market size on wages is ambiguous and depends on the interaction between exporting and importing. While this mechanism shares elements with economic geography models such as Krugman (1991), which emphasize agglomeration forces across locations, our framework instead highlights how joint exporting–importing behavior and selection shape specialization across production stages.

[1] While existing work establishes that market size shapes firms' export behavior, much less is known about how it affects their joint exporting and importing decisions and, in turn, specialization patterns across production stages.

We show that market size alone can generate endogenous specialization across production stages, even in the absence of technological differences. Firms jointly choose whether to export final goods and import intermediate inputs, and these choices shape the organization of production across stages. A larger country supports more firms producing final goods and tends to specialize in downstream activities, exporting relatively more final goods, whereas a smaller country specializes in upstream stages and exports relatively more intermediate inputs. Importantly, this pattern depends on the level of input trade costs, which determines the relative importance of final-good and input trade. When input trade costs are relatively high, scale effects reinforce downstream specialization in larger countries. When input trade costs become sufficiently low, however, sourcing incentives become more influential, weakening—and potentially reversing—the tendency for larger countries to specialize in downstream activities. These results arise because market size affects firms' exporting and importing incentives, reshaping the composition of activity across production stages.

These results have key implications for how market size shapes trade patterns and welfare in supply chains. In standard models, differences in specialization are typically attributed to technology, trade costs, or exogenous network structure. In contrast, our model shows that market size can itself generate endogenous specialization across production stages through firms' joint exporting and importing decisions. Because these decisions are jointly determined, the mapping from market size to specialization depends on the composition of trade between final goods and intermediate inputs. This highlights a novel channel through which firm decisions translate into aggregate outcomes. As a result, globalization not only reshapes the organization of production across countries but also alters the distribution of income by weakening the wage advantages associated with larger markets and attenuating the welfare gains from market size.

Related Literature This paper contributes to the literature on how market size shapes trade patterns, firm behavior, and specialization across production stages. In models with final-good trade, a central result is the home-market effect (hereafter, HME), i.e., larger markets tend to specialize in and export products subject to increasing returns and trade costs across countries and regions (Krugman 1980, 1991).[2] Subsequent work shows that the strength of the HME may depend on the surrounding general-equilibrium environment and market structure (Davis 1998; Behrens et al. 2009). A related line of research examines the role of intermediate inputs in economic geography, showing how production linkages affect industrial location, specialization patterns, and the strength of the HME by reshaping the organization of production across stages (Venables 1996; Amiti 1998, 2005). Our paper differs from these papers because we focus on how firm-level exporting and importing decisions interact with production linkages to shape specialization patterns under vertical specialization.

Building on the role of input-output linkages in international production, related studies explore how trade costs, technology, and production structure shape countries' positions along production chains in competitive models. Early work in this strand analyzes fragmentation and vertical production linkages across industries (Jones and Kierzkowski 1990; Yi 2003), whereas more recent studies examine global value chains across countries (Costinot, Vogel, and Wang 2013; Antràs and de Gortari 2020; Johnson and Moxnes 2023). However, since this literature is developed in perfectly competitive settings, it does not generate the HME mechanisms arising from increasing returns and demand linkages under monopolistic competition. By abstracting from the role of firms, it also cannot capture how firm-level selection shapes specialization across production stages. Our framework incorporates these firm-level aspects into a model of global production fragmentation with market-size-driven trade patterns.

[2] See Hanson and Xiang (2004) for empirical evidence on the HME in U.S. export data, showing that larger markets export disproportionately more in industries characterized by increasing returns to scale or high trade costs.

In heterogeneous-firm models, market size affects trade patterns not only through scale economies but also through export selection and productivity differences (Melitz and Ottaviano 2008). Existing studies show that firm heterogeneity can modify the HME through export selection, agglomeration, and productivity sorting (Okubo 2009; Erhardt 2017; Felbermayr and Jung 2018). Existing work typically includes an "outside" good that limits endogenous wage adjustment, so that trade affects specialization primarily through firm entry, export selection, and relocation decisions. However, once this adjustment is taken into account, competitiveness effects can substantially alter—or even overturn—standard implications (Demidova and Rodríguez-Clare 2013). Our framework extends this perspective by allowing endogenous wage adjustment to interact with firms' importing behavior under vertical specialization.

A growing literature combines firm heterogeneity with endogenous input sourcing, showing how imported inputs affect firms' productivity, export performance, and sourcing adjustments in response to trade frictions and tariff shocks (Kasahara and Lapham 2013; Halpern, Koren, and Szeidl 2015; Antràs, Fort, and Tintelnot 2017; Bernard et al. 2018; Blaum, Lelarge, and Peters 2018; Handley, Kamal, and Monarch 2025).[3] This literature highlights important complementarities between importing and exporting at the firm level, but typically does not examine how importing decisions feed back into general-equilibrium specialization patterns or modify the HME itself. Our framework instead emphasizes how firm-level importing interacts with endogenous wage adjustment and vertical specialization, thereby shaping trade patterns and production allocation across stages.

Firm-level empirical evidence suggests that market size, firm heterogeneity, and firms' global engagement are closely linked. Larger markets enable productive firms to expand their export activities, which, in turn, may strengthen incentives to source foreign inputs. Lileeva and Trefler (2010) and Aghion et al. (2024) provide evidence that larger markets created by trade integration improve firm productivity and export performance, although their focus is primarily on firms' export-side responses. Evidence from China also highlights the links between importing, exporting, and production networks (Feng et al. 2016; Huang et al. 2024; Li et al. 2024), typically abstracting from the role of market size. To the best of our knowledge, the existing empirical literature does not examine how market size shapes firms' joint importing and exporting decisions or how these decisions translate into specialization patterns under vertical specialization.

To summarize, our paper highlights a distinct mechanism that is largely absent from the existing literature: by affecting firms' exporting and importing decisions, market size endogenously generates countries' specialization patterns across production stages and yields novel welfare implications through the HME in the presence of international fragmentation of production.

Roadmap The remainder of the paper is organized as follows. Section 2 presents the model setting, describing the environment, firm decisions, and equilibrium conditions. Section 3 characterizes the general equilibrium and develops the main results on how market size endogenously shapes specialization, trade patterns, and welfare across countries. Section 4 discusses the roles of monopolistic competition and firm heterogeneity in our results relative to existing competitive frameworks and derives empirically testable implications that can be examined using firm-level and aggregate trade data. Section 5 concludes. The appendix provides proofs of the theoretical results and additional quantitative results.

[3] A separate literature focuses on the formation of firm-to-firm trade networks in input sourcing. For example, Dhyne et al. (2023) analyze how firms adjust buyer–supplier relationships in response to trade costs, while Dhyne et al. (2021) highlight how sourcing decisions shape trade flows and firm performance. Kopytov et al. (2024) and Arkolakis et al. (2025) also examine endogenous production-network formation under uncertainty or across multiple sourcing locations, respectively. In contrast, our focus is on how market size and trade costs shape specialization across production stages through firms' joint exporting and importing decisions, while abstracting from endogenous matching between individual buyers and suppliers.

2 Model Setup

2.1 Demand and Production

Environment The world consists of two asymmetric countries, indexed by $i, j = 1, 2$. Each country has a single industry that produces final goods (downstream stage) using intermediate inputs (upstream stage), and labor is the only factor of production for both types of goods. Country i is endowed with L_i units of labor, and the wage w_i is determined endogenously.

Intermediate inputs are produced under perfect competition using labor with a linear technology. Since we abstract from technological differences across countries, the unit labor requirement is common across countries, which is normalized to one for simplicity, so that the input price in country i equals the wage w_i. By contrast, final goods are produced under monopolistic competition by heterogeneous firms with productivity φ, which combine domestic and imported inputs, with importing determined endogenously.

In this framework, the only exogenous difference between countries is market size, given by L_i and L_j. Differences in market size generate differences in equilibrium wages, w_i and w_j, which in turn shape firms' exporting and importing decisions and the allocation of production across stages.

Preferences and Demand Consumers in country i have CES preferences over domestic varieties produced in country i and imported varieties from country j, denoted by $y_{Di}(v)$ and $y_{Xj}(v)$, with elasticity of substitution $\sigma = 1/(1-\rho) > 1$. Let $p_{Di}(v)$ and $p_{Xj}(v)$ denote the corresponding prices. Utility maximization yields the standard CES demand system for respective varieties, $y_{Di}(v) = p_{Di}(v)^{-\sigma} P_i^{\sigma-1} Y_i$, $y_{Xj}(v) = p_{Xj}(v)^{-\sigma} P_i^{\sigma-1} Y_i$, where P_i and Y_i denote the CES price index and aggregate expenditure in country i, respectively. Hereafter, we suppress the variety index v unless needed for clarity.

Production and Expenditure The structure of firm behavior follows that in Melitz (2003). Upon paying a fixed entry cost f_E, firms draw productivity $\varphi \sim G(\varphi)$ and choose their mode of operation. Domestic producers incur a fixed cost f_D; exporters incur f_X; importers incur f_{DM}; and firms that both export and import incur f_{XM}, which captures additional fixed costs associated with jointly managing exporting and sourcing activities (e.g., coordinating foreign sales and input procurement), with all fixed costs measured in units of labor in the country where they are incurred. In addition, trade in final goods and intermediate inputs is subject to standard iceberg costs: exporting firms incur $\tau_X \geq 1$, while importing firms incur $\tau_M \geq 1$.

A firm with productivity φ produces output according to a linear technology, $y_i = \varphi x_i$. Here x_i is a CES aggregate of input bundles, $x_i = (x_{Di}^{\rho} + \mathbb{1}_{Mi} x_{Mi}^{\rho})^{1/\rho}$, where x_{Di} and x_{Mi} denote domestic and imported inputs, respectively.[4] The indicator $\mathbb{1}_{Mi}$ equals one for importing firms and zero otherwise, so that access to imported inputs is endogenous and limited to a subset of firms. Note that although labor is immobile across countries, trade in intermediate inputs allows firms to access foreign labor indirectly.

Cost minimization implies that firms choose input demand based on relative input prices. Since inputs are produced competitively using labor in the origin country and trade costs are symmetric, the price of domestic inputs in country i is w_i, while imported inputs from country j are available at price $\tau_M w_j$. Unit costs are therefore given by the CES aggregate of input prices:

$$c_i = \left[w_i^{1-\sigma} + \mathbb{1}_{Mi} (\tau_M w_j)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} .$$

[4] We assume that the elasticity of substitution is identical across preferences and production, and that firms produce using only the input bundle; however, these assumptions do not affect the qualitative results.

Non-importing firms use only domestic inputs and therefore face unit costs $c_i = w_i$. By contrast, importing firms combine domestic and foreign inputs according to the CES technology, substituting toward the relatively cheaper source. In particular, when imported inputs are relatively cheap (i.e., when $\tau_M w_j < w_i$), importing leads to a larger reduction in unit costs, with the magnitude of this reduction governed by the elasticity of substitution σ. Even when imported inputs are more expensive, importing can still reduce unit costs through input-variety effects, although these gains are small.

Define the domestic input share as $s_i = e_{Di}/e_i$, where e_{Di} and e_i denote firm-level expenditures on domestic inputs and total inputs, respectively. Because input expenditure depends on relative input prices, we obtain

$$s_i = \frac{1}{1 + \mathbb{1}_{Mi} \left(\frac{w_i}{\tau_M w_j} \right)^{\sigma - 1}}.$$

Non-importing firms have $s_i = 1$, while importing firms have $s_i < 1$, with the domestic share decreasing as foreign inputs become relatively cheaper. Unit costs can then be expressed as

$$c_i = w_i s_i^{1/(\sigma-1)}.$$

This formulation is useful because it links (unobservable) unit costs to (observable) firm-level input expenditure shares, which summarize firms' sourcing decisions.

Firm productivity φ scales marginal costs through the production technology, so the effective marginal costs are given by c_i/φ. As a result, more productive firms face lower marginal costs and are more likely to find it profitable to incur the fixed cost of importing and benefit from access to foreign inputs.

Economic Forces of Market Size Since market size is the only exogenous difference in the present model, the two countries are symmetric when $L_1 = L_2$. In this case, wages are equalized, i.e., $w_1 = w_2$, implying that the relative FOB price of domestic and foreign inputs is identical across countries. As a result, importing firms allocate expenditures symmetrically across domestic and foreign inputs under the CES technology.

When market size in country 1 increases and $L_1 > L_2$, the relative wage w_1/w_2 is endogenously shaped by two opposing forces. First, the HME raises w_1 by increasing demand for domestic labor in country 1 through higher production of final goods. Second, greater use of imported inputs by firms in country 1 increases demand for inputs produced in country 2, thereby raising labor demand and wages in country 2. Because these forces affect the relative wage in opposite directions, the overall effect depends on their relative strength: the HME tends to increase w_1/w_2, while the input-sourcing channel tends to reduce it by transmitting labor demand from country 1 to country 2 through imported inputs.

These channels operate through distinct margins. The first reflects demand for final goods, while the second operates through firms' sourcing of imported inputs. As country 1 expands, changes in relative wages induce firms to adjust their sourcing decisions, potentially shifting labor demand toward country 2. The interaction between these margins determines whether production becomes concentrated in the larger country or fragmented across countries, in the spirit of the economic geography framework of Krugman (1991).

Changes in wages further affect unit costs. In country 1, a higher relative wage w_1/w_2 reduces the domestic input share of importing firms, s_1, increasing their reliance on foreign inputs. Thus, access to foreign inputs lowers unit costs c_1, although higher wages w_1 increase production costs. At the same time, a larger market expands exporting activity by increasing firm revenues, allowing more firms to cover the fixed cost of exporting. These forces strengthen the interaction between firms' exporting and importing decisions.

2.2 Firm Behavior

Firm Decisions Conditional on productivity, firms choose whether to serve the domestic and foreign markets and whether to source imported inputs for each activity. In the domestic market, firms choose between domestic production without importing and domestic production with imported inputs, paying the additional fixed cost f_{DM} when importing. Likewise, firms serving the foreign market choose whether to export using only domestic inputs or to combine exporting with imported-input sourcing, paying the additional fixed cost f_{XM}. Thus, exporting and importing decisions are jointly determined and market-specific.

Although labor is immobile across countries, trade in intermediate inputs allows firms to access foreign labor indirectly through imported inputs. Importing firms substitute toward the relatively cheaper input source under the CES technology. Since importing requires additional fixed costs, only productive firms choose to import.

Domestic Market Consider first firms serving the domestic market in country i. Using the demand system, the marginal cost, and the fixed operating costs, domestic profits are given by

$$\pi_{Di} = \left(p_{Di} - \frac{c_i}{\varphi}\right) y_{Di} - w_i(f_D + \mathbb{1}_{Mi} f_{DM}).$$

Profit maximization implies the standard CES markup pricing rule $p_{Di}(\varphi) = \frac{c_i}{\rho\varphi}$, so that prices are a constant markup over marginal cost. Substituting this pricing rule into profits yields

$$\pi_{Di}(\varphi) = B_i s_i w_i^{1-\sigma} \varphi^{\sigma-1} - w_i(f_D + \mathbb{1}_{Mi} f_{DM}),$$

where $B_i \equiv \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} P_i^{\sigma-1} Y_i$ summarizes market demand. Importing firms have lower marginal costs because imported inputs reduce unit costs through sourcing substitution and input-variety effects. However, importing also requires the additional fixed cost f_{DM}. As a result, more productive firms are more likely to import.

Foreign Market Among firms serving the domestic market, some also serve the foreign market and solve an analogous problem. *Additional* profits from exporting are given by

$$\pi_{Xi} = \left(p_{Xi} - \frac{\tau_X c_i}{\varphi}\right) y_{Xi} - w_i(f_X + \mathbb{1}_{Mi} f_{XM}),$$

where exporting involves iceberg trade costs τ_X. Profit maximization implies $p_{Xi}(\varphi) = \frac{\tau_X c_i}{\rho\varphi}$, so that

$$\pi_{Xi}(\varphi) = B_j s_i (\tau_X w_i)^{1-\sigma} \varphi^{\sigma-1} - w_i(f_X + \mathbb{1}_{Mi} f_{XM}).$$

As in the domestic market, importing lowers marginal costs but requires additional fixed costs. Exporting firms therefore jointly decide whether to source imported inputs for export production. More productive firms are more likely to engage in both exporting and importing, because they are better able to cover the associated fixed costs and benefit more from lower marginal costs and access to larger markets.

Selection The profit functions in each market can be illustrated in (φ, π) space. Importing firms have steeper profit schedules but lower intercepts, reflecting that importing lowers unit costs but requires additional fixed costs. Similarly, exporting expands market access but requires firms to incur iceberg and fixed export costs. These trade-offs jointly induce selection into importing and exporting.

2.3 Equilibrium Conditions

Productivity Cutoffs Firms' sourcing and market participation decisions must reflect the trade-off between marginal cost reductions and fixed costs, so only sufficiently productive firms choose to import. In the domestic market, let φ_{Di} and φ_{DMi} denote the productivity cutoffs for operating without and with importing, respectively, so that firms with productivity above φ_{Di} serve that market, while only the more productive firms above φ_{DMi} also import intermediate inputs. In the foreign market, φ_{Xi} and φ_{XMi} denote the corresponding export cutoffs. These cutoffs are defined by zero-cutoff-profit (ZCP) conditions:

$$\begin{aligned} \varphi_{Di} &= \left(\frac{f_D}{B_i} w_i^{\sigma}\right)^{\frac{1}{\sigma-1}}, & \varphi_{DMi} &= \left(\frac{f_{DM}}{B_i}\, w_i(\tau_M w_j)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}, \\ \varphi_{Xi} &= \left(\frac{f_X}{B_j}\, w_i(\tau_X w_i)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}, & \varphi_{XMi} &= \left(\frac{f_{XM}}{B_j}\, w_i(\tau_X \tau_M w_j)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}. \end{aligned} \tag{1}$$

In what follows, we assume strict selection into both importing and exporting, so that the productivity cutoffs satisfy $\varphi_{Di} < \min\{\varphi_{DMi}, \varphi_{Xi}\} < \max\{\varphi_{DMi}, \varphi_{Xi}\} < \varphi_{XMi}$. These conditions imply a hierarchy of firm choices: less productive firms exit, intermediate firms serve only the domestic market, more productive firms engage in one additional activity (either importing or exporting), and the most productive firms both export and import.[5] In equilibrium, changes in market size shift these cutoffs through changes in wages, thereby inducing firms to reallocate across exporting and importing modes.

Free Entry The free-entry (FE) condition requires that the expected profits of potential entrants equal the entry cost, implying zero expected profits in equilibrium. Expected profits consist of contributions from both domestic and foreign market activities. Following Melitz (2003), define $J(a) \equiv \int_a^{\infty} \left[\left(\frac{\varphi}{a}\right)^{\sigma-1} - 1\right] dG(\varphi)$. As shown in Appendix A.1.1, aggregating across all operating modes for firms in country i, we can express expected profits as $w_i\left(f_D J(\varphi_{Di}) + f_{DM} J(\varphi_{DMi}) + f_X J(\varphi_{Xi}) + f_{XM} J(\varphi_{XMi})\right)$, where the first two terms correspond to domestic market activities and the latter two to foreign market activities. Equating these expected profits to the entry cost $w_i f_E$, the FE condition in country i can be written as

$$f_D J(\varphi_{Di}) + f_{DM} J(\varphi_{DMi}) + f_X J(\varphi_{Xi}) + f_{XM} J(\varphi_{XMi}) = f_E. \tag{2}$$

Together with the ZCP conditions in (1), the FE condition in (2) determines the demand shifter B_i that is consistent with zero expected profits.

Labor Market Clearing The labor-market-clearing (LMC) condition equates total labor supply to total labor demand in each country. Labor is used for (i) final-good production, (ii) input production, and (iii) firm entry in the downstream sector. However, since entry costs are exactly offset by expected profits under the FE condition, labor used for entry does not appear separately in the LMC condition. As shown in Appendix A.1.2, the LMC condition in country i can be written as

$$L_i = \underbrace{\frac{R_{Di} + R_{Xi} - E_{Di} - E_{Mi}}{w_i}}_{\text{Labor used for final-good production}} + \underbrace{\frac{E_{Di} + E_{Mj}}{w_i}}_{\text{Labor used for input production}}.$$

[5] The ordering of φ_{DMi} and φ_{Xi} is not essential, as importing and exporting are separate margins determined by distinct cost structures. The results rely only on selection into each activity and on the fact that the most productive firms engage in both.

Here, R_{Di} and R_{Xi} denote aggregate revenues from domestic and exported final goods, while E_{Di} and E_{Mi} denote expenditures on domestic and imported inputs, respectively (see Appendix A.1.2 for their expressions). In the downstream stage, labor demand equals value added, given by revenues net of input expenditures, $(R_i - E_i)/w_i$, where $R_i \equiv R_{Di} + R_{Xi}$ denotes aggregate revenue from final goods and $E_i \equiv E_{Di} + E_{Mi}$ denotes total expenditure on inputs in country i. Under CES demand and constant-markup pricing, input expenditures account for a constant share of revenues, so that $E_i/R_i = \rho$. Dividing by the wage converts these values into units of labor, since labor is the only factor of production.

By contrast, in the upstream stage, intermediate inputs are produced using labor one-for-one technology, so input expenditures translate directly into labor demand. This includes inputs supplied both domestically and to foreign firms, capturing cross-country production linkages.

Trade Balance The trade-balance (TB) condition equates net exports of final goods and intermediate inputs across countries. In country i, it can be written as

$$R_{Xi} - R_{Xj} = E_{Mi} - E_{Mj}.$$

This condition implies that a country that is a net exporter of final goods must be a net importer of inputs, reflecting balanced trade in a vertically structured world where countries exchange final goods for intermediate inputs. Thus, if country i runs a trade surplus in final-good trade, it also runs a trade deficit in input trade. We focus on cases with $R_{Xi} > 0$ and $E_{Mi} > 0$, so that no country fully specializes in either production stage. As shown later, this occurs when market sizes are not too different across countries.

Using the TB condition, the LMC condition can be simplified as $L_i = (R_{Di} + R_{Xj})/w_i = Y_i/w_i$. Here, aggregate expenditure in country i is given by $Y_i = R_{Di} + R_{Xj}$, reflecting spending on domestic and imported final goods, which is different from total revenue of firms located in country i, $R_i = R_{Di} + R_{Xi}$. Consequently, aggregate expenditure equals aggregate labor income, $Y_i = w_i L_i$.

2.4 Equilibrium Characterizations

Define the domestic-to-export revenue ratio and the domestic-to-import input expenditure ratio in country i as

$$\alpha_{Xi} \equiv \frac{R_{Di}}{R_{Xi}}, \qquad \alpha_{Mi} \equiv \frac{E_{Di}}{E_{Mi}},$$

which summarize the allocation of sales across domestic and foreign markets and the sourcing of inputs across domestic and foreign suppliers, respectively. Using these ratios to express revenues and expenditures in terms of aggregate revenues, the LMC condition can be written as

$$w_i L_i = \left(\frac{\alpha_{Xi}}{1+\alpha_{Xi}}\right) R_i + \left(\frac{1}{1+\alpha_{Xj}}\right) R_j. \tag{3}$$

Similarly, using $R_{Xi} - E_{Mi} = R_{Xj} - E_{Mj}$ together with $E_i = \rho R_i$, the TB condition becomes

$$\left(\frac{1}{1+\alpha_{Xi}} - \frac{\rho}{1+\alpha_{Mi}}\right) R_i = \left(\frac{1}{1+\alpha_{Xj}} - \frac{\rho}{1+\alpha_{Mj}}\right) R_j. \tag{4}$$

Since α_{Xi} and α_{Mi} depend on the productivity cutoffs in (1), the LMC and TB conditions in (3) and (4) can also be expressed in terms of these cutoffs, thereby jointly determining aggregate revenues R_i and wages w_i.

The equilibrium is characterized by 13 unknowns: $\varphi_{Di}, \varphi_{DMi}, \varphi_{Xi}, \varphi_{XMi}, B_i, R_i$, and w_i for $i = 1, 2$, where we choose country 2's wage as the numeraire ($w_2 = 1$). These variables are jointly determined by 13 conditions: 8 ZCP conditions in (1), 2 FE conditions in (2), 2 LMC conditions in (3), and 1 TB condition in (4).

Once these variables are determined, other objects can be expressed as functions of them. In particular, solving the domestic ZCP condition in (1), welfare per worker—equivalently, the real wage w_i/P_i—is given by

$$W_i = \left(\frac{L_i}{\sigma f_D}\right)^{\frac{1}{\sigma-1}} \rho\, \varphi_{Di}. \tag{5}$$

Hence, welfare increases with market size L_i and the domestic cutoff φ_{Di}, reflecting scale and selection effects.

3 Market Size and Vertical Specialization

3.1 Special Cases

To build intuition for the general-equilibrium mechanisms, we begin by considering two polar cases in which only one type of trade is feasible. These cases isolate the distinct channels through which market size generates the HME, whose overall implications depend on their relative importance. Throughout the analysis, we study how relative market size $L \equiv L_1/L_2$ affects key equilibrium outcomes, including the relative wage $\omega \equiv w_1/w_2$, using local comparative statics. Accordingly, we focus on proportional changes and denote $\hat{x} \equiv dx/x$.

Final-Good Trade Only Suppose first that input trade costs are prohibitively high ($\tau_M = \infty$). In this case, the import cutoffs satisfy $\varphi_{DMi} = \varphi_{XMi} = \infty$ from (1), implying that importing is infeasible ($\alpha_{Mi} = \infty$) and only final goods are tradable in equilibrium. Solving the equilibrium conditions in (1)–(4) under this restriction yields the following expressions for the domestic cutoffs $\varphi_{D1}, \varphi_{D2}$ and relative wage ω (see Appendix A.2.1):

$$\begin{aligned} \hat{\varphi}_{D1} &= -\left(\frac{\rho(1+\alpha_{X2})}{\Xi_X}\right)\hat{L}, \\ \hat{\varphi}_{D2} &= \left(\frac{\rho(1+\alpha_{X1})}{\Xi_X}\right)\hat{L}, \\ \hat{\omega} &= \left(\frac{\rho^2(\alpha_{X1}\alpha_{X2}-1)}{\Xi_X}\right)\hat{L}, \end{aligned} \tag{6}$$

where $\alpha_{X1}\alpha_{X2} - 1 > 0$ and $\Xi_X > 0$. Together with the FE condition in (2), these expressions imply

$$\hat{\varphi}_{D1} < 0 < \hat{\varphi}_{X1}, \qquad \hat{\varphi}_{X2} < 0 < \hat{\varphi}_{D2}.$$

Equation (6) reflects the standard demand-expansion mechanism underlying the HME. An increase in L raises demand for final goods produced in country 1, increasing labor demand and raising the equilibrium wage, $\hat{\omega} > 0$. The resulting increase in production costs reduces firms' competitiveness in foreign markets, $\hat{\varphi}_{X1} > 0$, shifting relatively more sales toward the domestic market. At the same time, stronger domestic demand raises revenues and allows less productive firms to survive domestically, implying $\hat{\varphi}_{D1} < 0$. Country 2 experiences the opposite adjustments. These effects of market size operate primarily through demand expansion, thereby generating the conventional HME.[6]

[6] Existing studies show that firm heterogeneity can modify the HME through export selection and agglomeration (Okubo 2009; Erhardt 2017). Related work also shows that larger markets may exhibit weaker domestic selection (Felbermayr and Jung 2018).

Input Trade Only Next, suppose that final-good trade costs are prohibitively high ($\tau_X = \infty$). In this case, the export cutoffs satisfy $\varphi_{Xi} = \varphi_{XMi} = \infty$ from (1), so that exporting is infeasible ($\alpha_{Xi} = \infty$), and only intermediate inputs are tradable in equilibrium. As before, we consider the effects of an increase in market size in country 1. Solving (1)–(4) under this restriction yields the following expressions (see Appendix A.2.2):

$$\begin{aligned}\hat{\varphi}_{D1} &= -\left(\frac{1+\alpha_{M2}}{\Xi_M}\right)\hat{L},\\ \hat{\varphi}_{D2} &= \left(\frac{1+\alpha_{M1}}{\Xi_M}\right)\hat{L},\\ \hat{\omega} &= -\left(\frac{(1+\alpha_{M1})(1+\alpha_{M2})}{\Xi_M}\right)\hat{L},\end{aligned} \tag{7}$$

where $\Xi_M > 0$. Together with the FE condition in (2), these expressions imply

$$\hat{\varphi}_{D1} < 0 < \hat{\varphi}_{DM1}, \qquad \hat{\varphi}_{DM2} < 0 < \hat{\varphi}_{D2}.$$

Equation (7) shows that the key adjustment now operates through firms' sourcing decisions. An increase in L raises demand for production inputs as output expands in country 1. Because firms rely in part on imported inputs, a portion of this additional demand is directed to country 2, so labor demand rises in both countries but more in country 2, lowering country 1's relative wage, $\hat{\omega} < 0$. As in (6), larger market size relaxes domestic selection, so that $\hat{\varphi}_{D1} < 0$. At the same time, the decline in $\omega \equiv w_1/w_2$ makes imported inputs relatively more expensive for firms in country 1, leading them to substitute toward domestic inputs and reducing the gains from importing. As a result, fewer firms can cover the fixed cost of importing, as reflected in $\hat{\varphi}_{DM1} > 0$. Country 2 experiences the opposite adjustments. These mechanisms operate mainly through sourcing linkages.[7]

Takeaway These two polar cases highlight a key tension. Market size consistently weakens domestic selection through demand expansion, but its effect on wages—and hence on the allocation of production across countries—depends on which type of trade dominates. When final-good trade dominates, market size increases domestic labor demand and raises wages. When input trade dominates, by contrast, it shifts production toward foreign suppliers, reducing domestic labor demand and lowering wages. This contrast between demand expansion and sourcing reallocation is central to the general case analyzed in the following subsections.

3.2 General Mechanism

We now turn to the general case in which both final goods and intermediate inputs are traded across borders, so that all productivity cutoffs are finite and firms jointly choose whether to export and import. We examine how an increase in market size affects equilibrium outcomes, with particular emphasis on trade patterns under vertical specialization—namely, whether a larger country becomes a net exporter of final goods as determined by firms' decisions. As the two polar cases make clear, market size operates through the two opposing forces: a demand expansion effect that raises domestic labor demand through final-good production, and a sourcing reallocation effect that shifts production toward foreign suppliers through input trade. We demonstrate that, because of these opposing forces, market size alone can generate systematic differences in vertical specialization, even in the absence of technological differences.

[7] Related work shows that input-output linkages can modify the HME, although the weak domestic selection is typically absent; see Venables (1996) and Amiti (1998) in homogeneous-firm models, and Okubo (2009) in a heterogeneous-firm model.

In general equilibrium, the relative strength of these forces is governed by the composition of trade between final goods and inputs. A key object in the model is the ratio of final-good trade to input trade, R_{Xi}/E_{Mi}. When this ratio exceeds one, final-good trade dominates input trade, so the demand expansion effect prevails and market size raises wages, as in standard trade models of HME; the limiting case corresponds to the equilibrium in (6) with $E_{Mi} = 0$. When the ratio is below one, however, input trade dominates, so the sourcing reallocation effect prevails and market size instead lowers wages; the limiting case corresponds to (7) with $R_{Xi} = 0$. When both types of trade are present, this ratio arises naturally from the equilibrium conditions and determines the sign of the wage response to market size—and thus the organization of production stages across countries under vertical specialization.

Equilibrium Structure As noted in Section 2.4, characterizing the equilibrium formally involves a system of 13 equations that jointly determine productivity cutoffs, market demand, and wages, but a full solution is not required for our purposes. Instead, we exploit the recursive structure of heterogeneous-firm models, in which productivity cutoffs are determined first, conditional on wages and expenditure, and sectoral aggregates are then recovered from these cutoffs and market demand (Melitz and Redding 2014). In our model, this allows us to use the ZCP and FE conditions in (1) and (2) to determine the productivity cutoffs $\varphi_{Di}, \varphi_{DMi}, \varphi_{Xi}, \varphi_{XMi}$ and the demand shifter B_i, taking R_i and w_i as given.

Exploiting this recursive structure, we can characterize how changes in market size affect the productivity cutoffs. More specifically, an increase in L induces a change in ω, and for a given change in ω, we can solve for the implied changes in φ_{D1} and φ_{D2} (see Appendix A.2.3):

$$\begin{aligned} \hat{\varphi}_{D1} &= -\left(\frac{\left(\frac{\theta_1}{\rho}-1\right)(1+\alpha_{M2}-\theta_2)+\left(\frac{\theta_2}{\rho}-1\right)\theta_1}{\Xi}\right)\hat{\omega}, \\ \hat{\varphi}_{D2} &= \left(\frac{\left(\frac{\theta_2}{\rho}-1\right)(1+\alpha_{M1}-\theta_1)+\left(\frac{\theta_1}{\rho}-1\right)\theta_2}{\Xi}\right)\hat{\omega}, \end{aligned} \tag{8}$$

where $\Xi > 0$ and

$$\theta_i \equiv \frac{1+\alpha_{Mi}}{1+\alpha_{Xi}} > 0.$$

Because $1 + \alpha_{Mi} - \theta_i = \alpha_{Xi}\theta_i > 0$, the coefficients in (8) depend only on the sign of $\theta_i/\rho - 1$. Further, using the definitions of α_{Xi} and α_{Mi} in Section 2.4 and $R_i/E_i = 1/\rho$ in Section 2.3, this ratio can be written as

$$\frac{\theta_i}{\rho} = \frac{R_{Xi}}{E_{Mi}}. \tag{9}$$

Equations (8) and (9) show that whether final-good trade R_{Xi} exceeds input trade E_{Mi} determines the direction of equilibrium adjustments. In particular, consistency with the limiting cases (6) and (7) implies that when $R_{Xi} > E_{Mi}$ the demand-expansion effect dominates and $\hat{\omega} > 0$, whereas when $R_{Xi} < E_{Mi}$ the sourcing-reallocation effect dominates and $\hat{\omega} < 0$.

Given the sign of $\hat{\omega}$, equation (8) determines the direction of changes in $\hat{\varphi}_{D1}$ and $\hat{\varphi}_{D2}$. When $R_{Xi} > E_{Mi}$, we have $\hat{\omega} > 0$, so higher wages raise production costs in country 1, while increased market size strengthens domestic demand. As discussed in Section 3.1, this implies $\hat{\varphi}_{D1} < 0 < \hat{\varphi}_{D2}$. When $R_{Xi} < E_{Mi}$, we have $\hat{\omega} < 0$, shifting production and reversing the wage effect. Nevertheless, larger market size relaxes domestic selection, so that again $\hat{\varphi}_{D1} < 0 < \hat{\varphi}_{D2}$.

Because R_{Xi} and E_{Mi} are determined in equilibrium, whether $R_{Xi} > E_{Mi}$ depends on the model's primitives. In particular, iceberg trade costs τ_X and τ_M play a critical role in governing the relative importance of final-good trade and input trade. We first focus on the benchmark case in which final-good trade dominates input trade.

Assumption 1. *In the initial equilibrium, final-good trade is strictly larger than input trade in the sense that $R_{Xi}/E_{Mi} > 1$, or equivalently $\theta_i/\rho > 1$.*

It is known that a large share of world trade consists of intermediate inputs (Johnson and Noguera 2012). Although this assumption need not be satisfied empirically, we adopt it to isolate the baseline mechanism. The assumption is likely to hold when input trade costs τ_M are relatively high, so that production fragmentation remains limited. We return below to examine how the results change when the assumption is violated.

Firm Selection and Trade Patterns The expressions above—in particular, equation (8)—hold for arbitrary market sizes L_1 and L_2, and therefore for any relative market size $L \equiv L_1/L_2$. To obtain sharp comparative-statics predictions, we evaluate the equilibrium at a symmetric configuration, $L_1 = L_2$, and examine how a marginal increase in L affects equilibrium outcomes.

Under Assumption 1, differentiating the ZCP conditions in (1) and using (8) yields the following ordering of productivity cutoffs in country 1:

$$\hat{\varphi}_{DM1} < \hat{\varphi}_{D1} < 0 < \hat{\varphi}_{XM1} < \hat{\varphi}_{X1}. \tag{10}$$

The inequality $\hat{\varphi}_{D1} < 0$ reflects weaker domestic selection, as market expansion increases demand and allows lower-productivity firms to operate profitably. The stronger response $\hat{\varphi}_{DM1} < \hat{\varphi}_{D1}$ should be distinguished from the level ordering $\varphi_{Di} < \varphi_{DMi}$ implied by selection into importing. It reflects a comparative-static effect: market expansion relaxes the importing cutoff more strongly because importing firms benefit from access to cheaper foreign inputs, which lowers their marginal costs relative to purely domestic firms and amplifies the relaxation of selection for importers.

In contrast, export selection becomes more stringent ($\hat{\varphi}_{X1} > 0$), as higher domestic wages raise production costs and reduce firms' competitiveness in foreign markets. Importing exporters (two-way traders) are partially insulated from this effect ($0 < \hat{\varphi}_{XM1} < \hat{\varphi}_{X1}$), as access to cheaper inputs mitigates the increase in costs.

By symmetry, country 2 exhibits the opposite pattern:

$$\hat{\varphi}_{X2} < \hat{\varphi}_{XM2} < 0 < \hat{\varphi}_{D2} < \hat{\varphi}_{DM2}. \tag{11}$$

The following proposition characterizes the effect of market size on firm selection.

Proposition 1. *Suppose that Assumption 1 holds. Starting from a symmetric equilibrium, a marginal increase in market size in one country implies that the larger country exhibits weaker domestic selection, expanded importing activity, and tighter export selection, while the smaller country exhibits the opposite adjustments.*

Proposition 1 shows that an increase in market size raises domestic demand and production in country 1, weakening domestic selection. At the same time, higher relative wages raise production costs and reduce firms' competitiveness in foreign markets, tightening export selection. The increase in relative wages also encourages substitution toward foreign inputs, lowering marginal costs for importing firms and reducing the productivity threshold for importing. As a result, importing activity expands. These forces operate in the opposite direction in country 2.

To further characterize these results, we evaluate equation (8) at symmetry. Using $\alpha_{X1} = \alpha_{X2}$, $\alpha_{M1} = \alpha_{M2}$, and $\theta_1 = \theta_2$, equation (8) reduces to

$$\hat{\varphi}_{D1} = -\left(\frac{\theta_1/\rho - 1}{\theta_1(\alpha_{X1} - 1)}\right)\hat{\omega}, \qquad \hat{\varphi}_{D2} = \left(\frac{\theta_2/\rho - 1}{\theta_2(\alpha_{X2} - 1)}\right)\hat{\omega}. \tag{12}$$

Thus, changes in domestic productivity cutoffs are symmetric in magnitude across countries, $|\hat{\varphi}_{D1}| = |\hat{\varphi}_{D2}|$. Despite this symmetry, equilibrium responses are asymmetric: a marginal increase in market size in country 1 induces opposite changes in firm selection across countries. Since the two countries are otherwise identical, the equilibrium exhibits a form of symmetry breaking in the spirit of Matsuyama (2004), whereby small differences in market size translate into systematic differences in equilibrium outcomes.

These selection patterns translate into changes in trade flows under vertical specialization. The expansion of domestic production in country 1, together with the decline in the importing cutoff, increases the demand for intermediate inputs and expands input imports E_{M1}. At the same time, tighter export selection reduces the set of firms serving foreign markets and tends to lower final-good exports R_{X1}. These forces tend to reduce the ratio R_{X1}/E_{M1}, although under Assumption 1 it remains above one. Conversely, the opposite adjustments in country 2 tend to raise R_{X2}/E_{M2}. As we show below, under Pareto productivity, the relative importance of final-good exports and input imports—captured by the ratio R_{Xi}/E_{Mi}—can be characterized explicitly and governs whether the larger country becomes a net exporter of final goods.

3.3 Parameterization of Technology

All results derived so far hold for a general productivity distribution $G(\varphi)$. To obtain sharper analytical results, we now impose additional structure on the distribution. Specifically, we assume that productivity follows a Pareto distribution with shape parameter k, with the scale parameter normalized to one:

$$G(\varphi) = 1 - \varphi^{-k}, \qquad \varphi \geq 1.$$

This parameterization yields closed-form expressions for trade flows and allows us to characterize patterns of trade and specialization across production stages.

Market-Size Effects on Trade Patterns Consider first the ratio of final-good exports to input imports in equation (9), which captures relative specialization across production stages. Under the Pareto productivity distribution, this ratio can be written as a function of productivity cutoffs:

$$\frac{R_{Xi}}{E_{Mi}} = \frac{1}{\rho}\left(\frac{1 + \frac{f_X}{f_{XM}}\left(\frac{\varphi_{XMi}}{\varphi_{Xi}}\right)^k}{1 + \frac{f_{DM}}{f_{XM}}\left(\frac{\varphi_{XMi}}{\varphi_{DMi}}\right)^k}\right). \tag{13}$$

We are interested in how market size affects this ratio through firms' joint exporting and importing decisions, reflecting productivity sourcing into these international activities. Applying the cutoff changes in (10) and (11) for (13), the value in brackets decreases in country 1 and increases in country 2. Accordingly, following an increase in relative market size $L \equiv L_1/L_2$, the ratio declines in country 1 and rises in country 2:

$$\hat{R}_{X1} - \hat{E}_{M1} < 0, \qquad \hat{R}_{X2} - \hat{E}_{M2} > 0.$$

Thus, relative specialization shifts toward greater reliance on imported intermediate inputs in the larger country and toward final-good production in the smaller country, whereby imports of intermediate inputs grow faster than final-good exports in the larger country, with the opposite pattern in the smaller country. Intuitively, market expansion raises domestic final-good production and the derived demand for intermediate inputs; because a substantial share of these inputs is sourced from abroad, input imports increase faster than final-good exports, increasing the larger country's reliance on imported inputs to support its expanded downstream production.

However, this result does not mean that the smaller country exports more final goods than the larger country. To compare exports across countries, recall that the TB condition can be expressed as $R_{X1}-E_{M1} = R_{X2}-E_{M2}$, which allows relative exports of final goods to be written as

$$\frac{R_{X1}}{R_{X2}} = \frac{1-\frac{E_{M2}}{R_{X2}}}{1-\frac{E_{M1}}{R_{X1}}}.$$

From the changes in R_{Xi}/E_{Mi}, we have that E_{M1}/R_{X1} rises while E_{M2}/R_{X2} falls following an increase in L. Substituting these changes into the expression above yields

$$\hat{R}_{X1} - \hat{R}_{X2} > 0.$$

Thus, country 1 exports more final goods and becomes relatively more specialized in downstream production, even in the absence of cross-country technological differences.

At the same time, intermediate input imports grow faster in country 1 than in country 2. In particular, combining $\hat{R}_{X1} - \hat{E}_{M1} < 0$, $\hat{R}_{X2} - \hat{E}_{M2} > 0$, and $\hat{R}_{X1} - \hat{R}_{X2} > 0$ yields

$$\hat{E}_{M1} - \hat{E}_{M2} > 0,$$

so that E_{M1}/E_{M2} increases following an increase in L. Thus, country 1 imports more intermediate inputs than country 2, reflecting the expansion of downstream production and the associated increase in input demand. This implies that country 2 becomes relatively more specialized in upstream production.

The following proposition formalizes how market size shapes trade patterns in vertical specialization.

Proposition 2. *Suppose that Assumption 1 holds and productivity follows a Pareto distribution. Starting from a symmetric equilibrium, the larger country becomes relatively more specialized in the downstream stage, while the smaller country becomes relatively more specialized in upstream production. These patterns arise even in the absence of cross-country technological differences.*

The intuition for Proposition 2 is as follows. Market expansion increases domestic production and, in turn, the derived demand for intermediate inputs, which are partly sourced from abroad. As noted in Section 2.1, this expansion influences wages through two opposing forces: the HME raises w_1 by increasing demand for domestic labor in country 1, while greater reliance on imported inputs shifts labor demand abroad. When final-good trade is greater than input trade under Assumption 1, the resulting increase in w_1 induces firms in country 1 to substitute toward imported inputs, as these become relatively cheaper than domestic inputs. In addition, the higher wage strengthens the cost advantage of imported inputs relative to domestic inputs and lowers the productivity threshold for importing, as reflected in (10), thereby expanding the set of firms that source inputs from abroad. Together, these intensive- and extensive-margin responses reinforce the rise in input imports and tilt production toward downstream activities.

The mechanism described above depends on Assumption 1, which ensures that the demand-expansion effect dominates the sourcing-reallocation effect explained in Section 3.1. Absent this condition, the latter effect may offset or dominate the former, so that the larger country's wage need not increase. In this case, the intensive- and extensive-margin responses associated with importing are weakened, reducing the tendency for a larger country to specialize more strongly in downstream activities. Nevertheless, market size still raises both final-good exports and input imports in the larger country relative to the smaller country, so that the larger country remains relatively more specialized in downstream production (see Section 3.5).

In the symmetric benchmark with $L_1 = L_2$, exports of final goods are equalized across countries, so that $R_{X1} = R_{X2}$. A marginal increase in market size therefore endogenously generates an asymmetric equilibrium in which countries specialize in downstream and upstream stages. This prediction is qualitatively consistent with the relationship between centrality and downstreamness emphasized by Antràs and de Gortari (2020), although their analysis is developed in a competitive input–output framework. In contrast, our model highlights how market size shapes this relationship through endogenous firm selection and input-sourcing decisions.

Effects of Market Size on Sales and Sourcing Patterns Next, consider the effects on the domestic revenue ratio α_{Xi} and the domestic expenditure share α_{Mi}, which summarize firms' allocation of sales across destinations and sourcing of inputs across suppliers. These objects play a central role in general equilibrium because they enter both the LMC condition in (3) and the TB condition in (4).

Consider first the domestic revenue ratio $\alpha_{Xi} \equiv R_{Di}/R_{Xi}$. Under the Pareto distribution, this is given by

$$\frac{R_{Di}}{R_{Xi}} = \frac{f_{DM}}{f_{XM}} \left(\frac{\varphi_{XMi}}{\varphi_{DMi}}\right)^k \left(\frac{1 + \frac{f_D}{f_{DM}}\left(\frac{\varphi_{DMi}}{\varphi_{Di}}\right)^k}{1 + \frac{f_X}{f_{XM}}\left(\frac{\varphi_{XMi}}{\varphi_{Xi}}\right)^k}\right). \tag{14}$$

From the ZCP condition in (1), both ratios $\varphi_{DMi}/\varphi_{Di}$ and $\varphi_{XMi}/\varphi_{Xi}$ depend on the same relative wage term (w_j/w_i) and therefore co-move in response to changes in L_1.[8] The cutoff changes in (10) and (11) determine the direction of this co-movement. As a result, in equation (14), the relative productivity cutoffs appearing in both the numerator and the denominator of the bracketed term move together, so that the effect of market size on α_{Xi} is generally ambiguous.

Equation (14) directly shows that this ambiguity reflects fixed-cost asymmetries across production modes. In particular, given the cutoff changes in (10) and (11), the two cutoff ratios entering the bracketed term in (14) move in opposite directions as market size changes. The overall effect therefore depends on the relative fixed costs across production modes and becomes monotone in L if and only if the following condition holds:

$$\frac{f_{XM}}{f_X} < \frac{f_{DM}}{f_D}. \tag{15}$$

This condition captures fixed-cost complementarity between exporting and importing: firms that already import face a lower incremental fixed cost of exporting than non-importers. In other words, engaging in both activities involves economies of scope in fixed costs. This interpretation is consistent with the market-specific bilateral economies of scope documented by Li et al. (2024).

[8]Specifically, equation (1) implies that the relative productivity cutoffs satisfy

$$\frac{\varphi_{DMi}}{\varphi_{Di}} = \frac{w_j}{w_i}\left(\frac{\tau_M^{\sigma-1} f_{DM}}{f_D}\right)^{1/(\sigma-1)}, \qquad \frac{\varphi_{XMi}}{\varphi_{Xi}} = \frac{w_j}{w_i}\left(\frac{\tau_M^{\sigma-1} f_{XM}}{f_X}\right)^{1/(\sigma-1)}.$$

If condition (15) does not hold, the relative productivity cutoffs entering equation (14) continue to co-move, but the effect of market size is no longer monotone. In this case, market expansion need not increase a country's relative exports of final goods, because importing and exporting decisions are no longer complementary. In the knife-edge case in which condition (15) holds with equality, the bracketed term in (14) becomes locally invariant to changes in market size. This corresponds to separable fixed costs across importing and exporting decisions, so that the two activities are neither complementary nor substitutable.

Taken together, an increase in L leads to the following changes in domestic and export sales:

$$\hat{R}_{D1} - \hat{R}_{X1} > 0, \qquad \hat{R}_{D2} - \hat{R}_{X2} < 0.$$

Thus, in the larger country (country 1), domestic sales grow faster than export sales, making firms relatively more oriented toward the domestic market. In contrast, in the smaller country (country 2), export sales grow faster than domestic sales, making firms relatively more export-oriented.

Next, consider the domestic expenditure share $\alpha_{Mi} \equiv E_{Di}/E_{Mi}$. Under the Pareto distribution, we obtain

$$\frac{E_{Di}}{E_{Mi}} = \frac{f_X}{f_{XM}} \left(\frac{\varphi_{XMi}}{\varphi_{Xi}}\right)^k \left(\frac{1 + \frac{f_D}{f_X}\left(\frac{\varphi_{Xi}}{\varphi_{Di}}\right)^k}{1 + \frac{f_{DM}}{f_{XM}}\left(\frac{\varphi_{XMi}}{\varphi_{DMi}}\right)^k} \right). \tag{16}$$

Using the cutoff changes in (10) and (11), together with condition (15), the expression in (16) yields a monotonic response of the domestic expenditure share to changes in market size. In particular, an increase in L generates

$$\hat{E}_{D1} - \hat{E}_{M1} < 0, \qquad \hat{E}_{D2} - \hat{E}_{M2} > 0.$$

Thus, in the larger country (country 1), spending on imported inputs grows faster than spending on domestic inputs, so firms rely more on foreign sourcing. In contrast, in the smaller country (country 2), spending on domestic inputs grows faster than spending on imported inputs, so firms rely more on domestic sourcing under condition (15).[9]

Together with the corresponding result for the domestic revenue ratio in (14), this implies that the HME holds for final-good sales but is reversed for input sourcing under vertical specialization. Larger markets become more domestically oriented in their sales of final goods, yet more dependent on imported inputs to sustain production, whereas smaller markets exhibit the opposite pattern. These sales and sourcing patterns concerns the allocation of final-good sales across destinations, and do not contradict the emergence of downstream specialization stated in Proposition 2. This asymmetry reflects fixed-cost complementarity between importing and exporting, which reinforces firms' incentives to engage in both activities and amplifies the reallocation toward foreign sourcing in larger markets.

These results are summarized in the following proposition.

Proposition 3. *Suppose that Assumption 1 holds. Starting from a symmetric equilibrium, (i) the larger country becomes more domestically oriented in its sales of final goods, while the smaller country becomes more export-oriented; and (ii) the larger country relies more heavily on imported inputs, while the smaller country relies more on domestic inputs. Consequently, the HME holds for final-good sales but is reversed for input sourcing under vertical specialization.*

[9] When there is no fixed-cost complementarity in (15), the ratio in (16) does not always satisfy the expenditure changes.

3.4 General Equilibrium

Equilibrium Relative Wage So far, we have characterized equilibrium outcomes using the ZCP and FE conditions in (1) and (2), taking the relative wage as given. We now incorporate the LMC and TB conditions in (3) and (4) to close the model. The objective is to determine the equilibrium relative wage and characterize its relationship with specialization and trade patterns in general equilibrium.

Using $\alpha_{Xi} \equiv R_{Di}/R_{Xi}$ and $R_i = R_{Di} + R_{Xi}$, aggregate revenue can be written as $R_i = (1 + \alpha_{Xi})R_{Xi}$. Substituting this into (3) for $i = 1, 2$, the LMC condition in each country can be expressed as

$$\begin{aligned} w_1 L_1 &= \alpha_{X1} R_{X1} + R_{X2}, \\ w_2 L_2 &= \alpha_{X2} R_{X2} + R_{X1}. \end{aligned} \tag{17}$$

Similarly, using $\theta_i \equiv (1 + \alpha_{Mi})/(1 + \alpha_{Xi})$ in (4), the TB condition can be rewritten as

$$\left(1 - \frac{\rho}{\theta_1}\right) R_{X1} = \left(1 - \frac{\rho}{\theta_2}\right) R_{X2}, \tag{18}$$

where the terms in parentheses are positive if and only if Assumption 1 holds. When normalizing $w_2 = 1$ as the numeraire, the equilibrium system given by (17) and (18) jointly determines the three unknowns (R_{X1}, R_{X2}, w_1). Here, using α_{Xi}, the system is expressed in terms of R_{Xi}, rather than R_i noted in Section 2.4.

Since both final goods and inputs are tradable, one country may fully specialize in the downstream stage while the other may specialize in the upstream stage. Solving (17) for R_{X1} and R_{X2} yields

$$\begin{aligned} R_{X1} &= \frac{\alpha_{X2} w_1 L_1 - w_2 L_2}{\alpha_{X1}\alpha_{X2} - 1}, \\ R_{X2} &= \frac{\alpha_{X1} w_2 L_2 - w_1 L_1}{\alpha_{X1}\alpha_{X2} - 1}. \end{aligned}$$

This shows that when market sizes are too different, two-way trade in final goods may not arise in equilibrium. However, our main interest lies in the mechanism through which market size generates trade patterns under vertical specialization. We therefore focus on interior equilibria in which neither country fully specializes in the downstream stage, $R_{Xi} > 0$ for $i = 1, 2$, which requires similar market sizes.[10]

Now we derive the general-equilibrium system that pins down relative wages. Combining the LMC conditions in (17) yields the first relationship between relative exports $R_X \equiv R_{X1}/R_{X2}$ and the relative wage $\omega \equiv w_1/w_2$, given relative market size $L \equiv L_1/L_2$:

$$R_X = \frac{\alpha_{X2}\omega L - 1}{\alpha_{X1} - \omega L} \equiv \Phi(\omega, L). \tag{19}$$

This expression shows how relative exports adjust to the relative wage in order to satisfy labor-market clearing. Because (19) is derived from the LMC condition, it reflects how wages adjust to ensure full employment in each country. For a given relative wage, a larger L increases R_X, because the larger labor force must be supported by greater production revenue, including exports. However, a higher relative wage ω raises production costs and weakens export competitiveness, partially offsetting this effect by reducing the profitability of serving foreign markets relative to domestic markets. Labor-market clearing therefore implies a negative relationship between R_X and ω.

[10] Specifically, $\frac{1}{\alpha_{X2}} < \omega L < \alpha_{X1}$. Under this restriction, $\alpha_{X1}\alpha_{X2} > 1$ follows immediately, so the denominator is also positive.

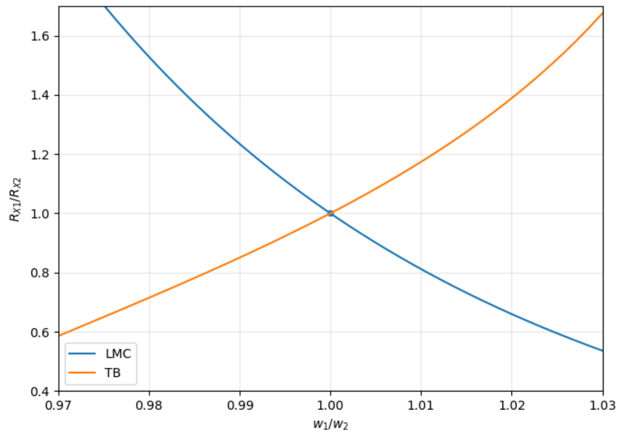


Figure 1: General-equilibrium determination in the benchmark case

The TB condition in (18) provides the second relationship between R_X and ω:

$$R_X = \frac{1 - \frac{\rho}{\theta_2}}{1 - \frac{\rho}{\theta_1}} \equiv \Psi(\omega). \tag{20}$$

This expression shows how relative exports adjust to the composition of trade across production stages in order to maintain trade balance across countries. Since $\theta_i \equiv (1+\alpha_{Mi})/(1+\alpha_{Xi})$, it captures the importance of input imports relative to final-good exports. Although (20) does not involve relative wages ω explicitly, it depends on ω indirectly through θ_i, which reflects firms' sales and sourcing decisions. A higher relative wage ω increases expenditure on imported inputs in country 1, so maintaining trade balance requires higher final-good exports, implying a positive relationship between R_X and ω.[11]

Equations (19) and (20) jointly determine the relative export ratio R_X and the relative wage ω. In principle, one could differentiate (19) and (20) to characterize the existence and uniqueness of equilibrium through the slopes of the two conditions. However, because the objects α_{X1}, α_{X2} and θ_1, θ_2 are endogenous functions of equilibrium relative wages, these equations define implicit relationships whose global properties cannot generally be characterized in closed form, even under a Pareto distribution. We therefore focus on the local properties of the two conditions around the symmetric equilibrium. Differentiating (19) and (20) and evaluating at $\omega = L = 1$ yields the following local slopes under Assumption 1 and the local regularity conditions (see Appendix A.2.4):

$$\Phi_\omega < 0, \qquad \Psi_\omega > 0.$$

Therefore, the LMC condition is downward sloping, whereas the TB condition is upward sloping, guaranteeing local existence and uniqueness of the symmetric equilibrium, although the global properties of equilibrium are difficult to characterize in this model.

Figure 1 illustrates the LMC and TB curves obtained by numerically solving equations (19) and (20) in the symmetric benchmark with $L_1 = L_2$. The figure plots the two curves under the following parameter values: $\sigma = 4$, $k = 5$, $\tau_X = 1.3$, $\tau_M = 1.5$, $f_D = 1$, $f_X = 1.4$, $f_{DM} = 1.4$, $f_{XM} = 1.2$, and $f_E = 2.5$. They are chosen

[11] A higher relative wage ω also makes foreign inputs relatively cheaper for country 1 firms and strengthens substitution toward imported inputs, which tends to make the TB curve downward sloping. Under Assumption 1, however, the resulting increase in import demand is sufficiently weak that the trade-balance adjustment through higher final-good exports dominates, implying an upward-sloping TB condition (see Appendix A.2.4).

so that Assumption 1 is satisfied ($R_{Xi}/E_{Mi} = 2.4$), yielding a configuration in the analytical characterization described above. The intersection of the two curves determines the general-equilibrium values of R_X and ω, which coincide at $(1, 1)$ under country symmetry.

Market Size in General Equilibrium Having established a unique symmetric equilibrium, we now study how market size affects relative wages, vertical specialization, and trade patterns. To formalize the general-equilibrium effects of market size, consider how an increase in L affects the equilibrium. Equation (19) shows that the LMC condition depends on L through its effect on relative labor supply. By contrast, equation (20) does not depend directly on L, so the TB condition remains unchanged for a given relative wage ω. In particular, so long as two-way trade in final goods occurs, the equilibrium has the following properties:[12]

$$\Phi_L > 0, \qquad \Psi_L = 0.$$

Thus, the equilibrium adjusts along the TB condition. In terms of Figure 1, a higher L shifts the LMC curve upward under Assumption 1, while leaving the TB curve unchanged. The intersection of the two curves therefore moves to a new equilibrium with both a higher relative export ratio R_X and a higher relative wage ω.

This general-equilibrium comparative static accords with the partial-equilibrium mechanisms established in Sections 3.2–3.3. Proposition 1 showed that, taking relative wages as given, changes in market size alter productivity cutoffs and thereby affect firms' sales and sourcing decisions. These cutoff adjustments determine the composition of trade through the domestic revenue ratio, the domestic expenditure share, and the allocation of sales across domestic and export markets. For a given relative wage, they also imply that a larger country becomes a net exporter of final goods, while simultaneously generating the sales and sourcing reallocations, as described in Propositions 2–3.

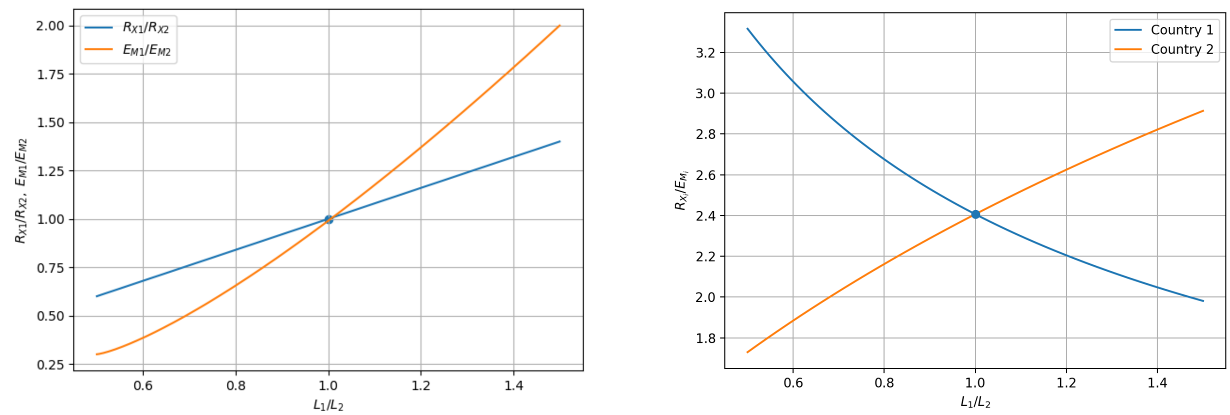
The present analysis closes the model by endogenizing the relative wage. The objects α_{X1}, α_{X2} and θ_1, θ_2 enter (19) and (20) and summarize how cutoff adjustments affect aggregate trade flows and expenditure patterns. These conditions jointly determine the equilibrium relative wage and export ratio, selecting outcomes consistent with the firm-level reallocations characterized in Propositions 1–3. General equilibrium therefore embeds the earlier partial-equilibrium mechanisms within the model's aggregate constraints: even after relative wages adjust to satisfy labor-market clearing and trade balance, the patterns of specialization and sourcing identified earlier continue to govern the direction of trade and production.

Figure 2 illustrates these general-equilibrium effects using the numerical solution above. Panel (a) plots the relationship between L and $R_X \equiv R_{X1}/R_{X2}$, $E_M \equiv E_{M1}/E_{M2}$. Consistent with Proposition 2, an increase in L raises both ratios, implying that the larger country exports more final goods while relying more heavily on imported inputs. Panel (b) shows the corresponding changes in specialization patterns by plotting the ratios R_{Xi}/E_{Mi} for $i = 1, 2$. The ratio declines for country 1 and rises for country 2, indicating that the larger country becomes relatively more downstream-oriented, whereas the smaller country becomes relatively more upstream-oriented. These patterns are consistent with Proposition 3: the HME holds for final-good trade but is reversed for input trade. Taken together, the two panels highlight how market size generates asymmetric specialization across production stages under vertical specialization.

[12]This is not merely local at symmetry, but holds globally for a given ω. Differentiating (19) with respect to L yields

$$\Phi_L = \frac{\omega(\alpha_{X1}\alpha_{X2} - 1)}{(\alpha_{X1} - \omega L)^2} > 0,$$

where the inequality follows from the interiority condition $\alpha_{X1}\alpha_{X2} > 1$ implied by two-way trade in final goods, $R_{Xi} > 0$.



(a) Sales and expenditure

(b) Sales-expenditure ratio

Figure 2: General-equilibrium effects of market size

These results highlight that an increase in relative market size raises both the relative export ratio and the relative wage of the larger country. This outcome reflects the dominance of final-good trade over input trade implied by Assumption 1. More generally, however, a positive relationship between relative market size and relative wages need not arise in general equilibrium. When input trade costs are sufficiently low, the increase in the relative wage may be attenuated—or even reversed—causing Assumption 1 to fail.

3.5 Reversal of Relative-Wage Responses

As shown in Section 3.1, market size affects relative wages through two opposing effects: a demand-expansion effect and a sourcing-reallocation effect. Assumption 1 focuses on the benchmark case, in which the demand-expansion effect dominates, so that the standard HME holds. We now consider the opposite case, in which the sourcing-reallocation effect dominates. In this equilibrium, a larger country tends to have lower relative wages, although it continues to export relatively more final goods.

Failure of HME on Relative Wages When input trade costs are sufficiently low, firms substitute strongly toward foreign inputs in response to higher wages. This strengthens the sourcing-reallocation effect, causing relative wages to decline with market size. Differentiating (19) and (20) and evaluating at $\omega = L = 1$ yields

$$\Phi_\omega < 0, \qquad \Psi_\omega < 0.$$

Therefore, while the LMC condition remains downward sloping, the TB condition becomes downward sloping. The intuition is that a higher relative wage induces firms to substitute toward cheaper foreign inputs, increasing import demand and worsening the trade balance. Restoring equilibrium requires a reduction in relative exports, implying a negative relationship between R_X and ω. Figure 3 illustrates this case under low input trade costs ($\tau_M = 1.1$), keeping all other parameter values fixed as in Figure 1. Under this parameterization, Assumption 1 no longer holds ($R_{Xi}/E_{Mi} = 0.75$), generating a strong sourcing-reallocation effect.

Let us examine how L affects the equilibrium. As in the benchmark case, this shifts the LMC curve through its effect on relative labor supply, while the TB condition remains unchanged for a given relative wage. However, since both curves are now downward sloping, the intersection moves along the TB curve in a different direction.

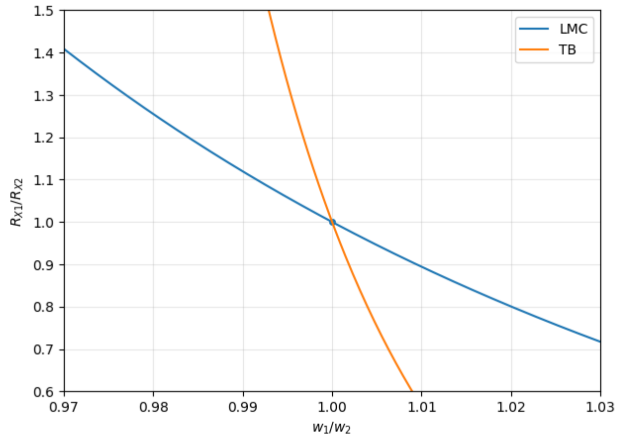


Figure 3: General-equilibrium determination under strong sourcing responses

In particular, a higher relative export ratio must be accompanied by a lower relative wage. The upward shift of the LMC curve therefore leads to a new equilibrium with a lower relative wage. As a result, an increase in L reduces ω, so that the HME on relative wages no longer holds, even though R_X rises so that the larger country continues to export relatively more final goods.

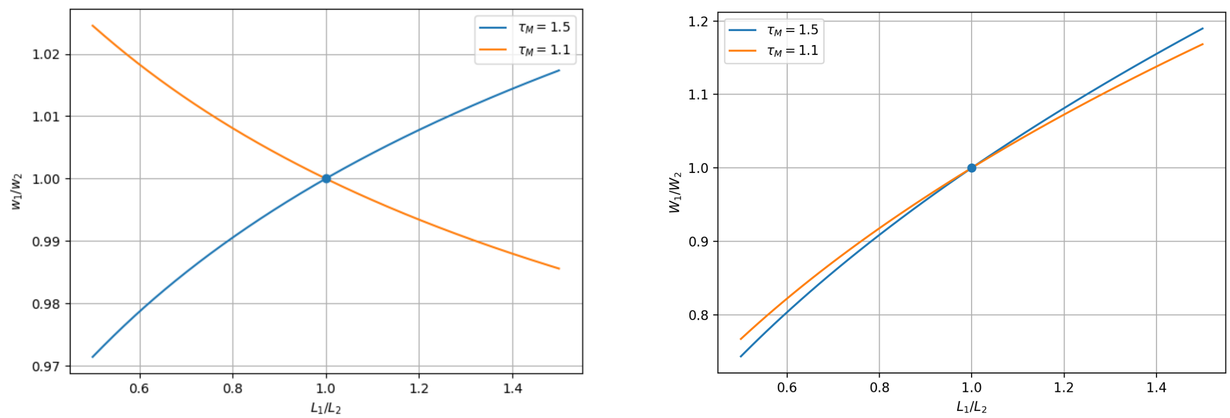
Specialization and Trade Patterns Next, we examine how the failure of the HME on relative wages is reflected in trade, sales, and sourcing patterns. Figure A.1 in Appendix A.3 presents the analogue of Figure 2 under low input trade costs, where Assumption 1 no longer holds.

Panel (a) plots the relationship in Figure 2(a) for $\tau_M = 1.1$. Even when the HME fails for relative wages, both the relative export ratio R_X and the relative import ratio E_M remain increasing in L, so that the larger country continues to export more final goods and import more intermediate inputs. This reflects the role of market size in shaping trade patterns through demand and entry, as shown in Proposition 2. Thus, the failure of the HME on wages does not overturn the direction of trade.

However, both R_X and E_M become more responsive to L than in the benchmark case with $\tau_M = 1.5$. Lower input trade costs allow firms to rely more heavily on imported inputs, thereby reducing production costs and expanding export activity in the larger country following an increase in L. At the same time, stronger reliance on foreign intermediate inputs raises the larger country's imports of intermediates relative to the smaller country. Thus, market size has a stronger effect on trade patterns when sourcing substitution becomes sufficiently strong under low input trade costs.

Panel (b) shows the relationship in Figure 2(b) for $\tau_M = 1.1$. In this case, R_{X_1}/E_{M_1} increases with L, while R_{X_2}/E_{M_2} decreases, which are opposite to the benchmark case. The intuition is that, when input trade costs are sufficiently low, firms already rely heavily on imported inputs near the symmetric equilibrium. Moreover, because the HME on wages weakens or reverses, the larger country no longer faces substantially higher wages relative to the smaller country as market size increases. As a result, firms in the larger country cannot exploit foreign wage differences through imported inputs as strongly as in the benchmark case. Consequently, market size increases final-good exports proportionally more than input imports in the larger country.

In sum, when sourcing substitution is sufficiently strong, market size continues to increase final-good exports and input imports but reverses the sales and sourcing patterns observed in the benchmark equilibrium through relative wage adjustment.



(a) Wages (b) Welfare

Figure 4: Effects of market size on wages and welfare

3.6 Market Size, Wages, and Welfare

Wages Finally, consider the welfare implications of the model. We begin by examining how market size affects relative wages. Equalizing (19) and (20) and totally differentiating around the symmetric equilibrium gives

$$\hat{\omega} = \left(\frac{\Phi_L}{\Psi_\omega - \Phi_\omega} \right) \hat{L}. \tag{21}$$

We have established that $\Phi_\omega < 0$ and $\Phi_L > 0$, while the sign of Ψ_ω depends on input trade costs. In particular, $\Psi_\omega > 0$ under Assumption 1, so that the relative wage ω is increasing in L. When Assumption 1 is violated, however, $\Psi_\omega < 0$, so that ω is decreasing in L.

Panel (a) of Figure 4 plots the relationship between relative market size and relative wages in equation (21). We use the same parameter values as in Figures 1 and 3, corresponding to two different levels of input trade costs ($\tau_M = 1.5$ or $\tau_M = 1.1$). These results are closely related to the two polar cases analyzed in Section 3.1, particularly the relative-wage responses given by the last equations in (6) and (7). When input trade costs are $\tau_M = 1.5$, the curve in Figure 4(a) is upward sloping. Thus, an increase in L raises ω, reflecting the dominance of the demand-expansion effect as in equation (6). By contrast, when input trade costs are $\tau_M = 1.1$, the curve becomes downward sloping. In this case, an increase in L lowers ω, reflecting the dominance of the sourcing-reallocation effect as in equation (7). This suggests that lower input trade costs weaken—may even reverse—the effect of market size on relative wages through sourcing linkages.

The figure highlights a key departure from the existing HME literature. When input trade costs are high, our framework generates the standard HME: larger markets export disproportionately more final goods and sustain higher relative wages (Krugman 1980). When input trade costs become sufficiently low, however, larger markets no longer sustain higher wages because firms substitute more intensively toward foreign inputs under vertical specialization. The wage response to market size therefore becomes ambiguous, resembling the endogenous wage adjustments in economic geography models (Krugman 1991). Unlike those models, however, the mechanism here arises from sourcing substitution *across* countries rather than agglomeration forces *within* countries. Since labor is immobile internationally, the model does not give rise to convergence or divergence through labor migration. Nevertheless, imported inputs allow firms to substitute indirectly toward foreign labor, thereby transmitting labor demand internationally and attenuating cross-country wage differences.

Welfare Building on this result, we turn to the welfare response to market size. From the welfare expression in (5), totally differentiating the welfare ratio $W \equiv W_1/W_2$ with respect to $L \equiv L_1/L_2$ yields

$$\hat{W} = \frac{\hat{L}}{\sigma - 1} + \hat{\varphi}_{D1} - \hat{\varphi}_{D2}.$$

The first term captures the standard scale effect of relative market size, which directly raises relative welfare. In contrast, the remaining terms capture the relative domestic selection effects that exists in firm heterogeneity: larger markets accommodate a larger mass of relatively unproductive firms, which indirectly lowers relative welfare by weakening domestic selection.

Now we evaluate welfare changes at symmetry. Equation (12) yields $\hat{\varphi}_{D1}$ and $\hat{\varphi}_{D2}$ have the same coefficient $A \equiv \frac{\theta_i/\rho - 1}{\theta_i(\alpha\chi_i - 1)}$, which is positive when Assumption 1 holds, but negative otherwise. Equation (21) further shows that $\hat{\omega}$ is positive when Assumption 1 holds, but negative otherwise. Taken together, we obtain

$$\hat{W} = \left[\frac{1}{\sigma - 1} - 2A\left(\frac{\Phi_L}{\Psi_\omega - \Phi_\omega}\right)\right]\hat{L}, \tag{22}$$

In general, we cannot analytically establish that the direct scale effect dominates the indirect selection effect. We can establish, however, that when the HME fails, the denominator $\Psi_\omega - \Phi_\omega$ becomes smaller in absolute value because $\Psi_\omega < 0$ rather than $\Psi_\omega > 0$, while $\Phi_\omega < 0$ regardless of Assumption 1. Intuitively, stronger sourcing reallocation amplifies the adverse effect of market expansion on domestic firm selection. As a result, the welfare gains from larger markets become attenuated when input trade costs are sufficiently low.

Panel (b) of Figure 4 plots the relationship between relative market size and relative welfare implied by (22) under the same parameterizations as in Figures 1 and 3. For both $\tau_M = 1.5$ and $\tau_M = 1.1$, relative welfare is increasing in relative market size, indicating that the larger country enjoys higher welfare. The slope of this relationship, however, depends on input trade costs. When input trade costs are high ($\tau_M = 1.5$), the welfare gain from market size is larger, as production is relatively localized and increases in market size translate more directly into domestic production, entry, and variety expansion. By contrast, when input trade costs are low ($\tau_M = 1.1$), part of the expansion in production is shifted abroad through input trade, attenuating local gains. Thus, while larger markets enjoy higher welfare gains, the magnitude of these gains depends on the extent of vertical fragmentation: deeper integration in input markets dampens the welfare advantage of market size by reallocating production across countries.

These results have important implications for the organization of production under globalization. A reduction in input trade costs—commonly associated with the fragmentation of production—amplifies the response of trade flows to market size while altering the distribution of income. In particular, although larger countries export more final goods, their relative wages may decline as they rely more on foreign inputs. As a result, production and income become partially decoupled: a country can expand its production and exports while experiencing a relative decline in wages. More broadly, globalization changes the organization of production under vertical specialization. Rather than reversing trade patterns, lower input trade costs shift production toward downstream activities in larger countries and upstream production in smaller countries. This vertical reorganization is accompanied by a redistribution of income across countries participating in supply chains, which lies outside standard HME logic.[13]

[13] Recent empirical work suggests that international trade may contribute to cross-country convergence in wages and income (Zhou and Bloch 2019; Damásio et al. 2024). Our model suggests that a mechanism behind this relationship may operate through lower input trade costs and vertical specialization.

4 Discussions

This section clarifies what is genuinely new relative to the existing literature based on competitive models, and shows that the theory generates empirically meaningful predictions.

4.1 Key Differences from Competitive Models

We have examined the role of market size in fragmenting production stages across countries in an environment with monopolistic competition and firm heterogeneity. We discuss the new insights generated by our framework relative to competitive models in the literature and why this distinction is important.

Monopolistic Competition In perfectly competitive models, market size does not generate the HME, and the tension between demand expansion and sourcing reallocation is absent or substantially weaker. By contrast, in our monopolistic competition model, increasing returns generate the standard HME, and market size induces endogenous wage adjustment that creates the above tension. Even within monopolistic competition frameworks, our mechanism differs from agglomeration-based explanations, such as Amiti (2005), where market size affects industrial location through vertical linkages and endogenous factor-price changes. Here, market size instead shapes firms' exporting and importing decisions, altering the composition of firms across production stages and thereby giving rise to patterns of vertical specialization.

This yields important welfare implications. In perfectly competitive models of vertical specialization, welfare effects are primarily governed by comparative advantage and production efficiency. In our framework, market size has an independent effect on welfare through demand linkages and firms' sourcing decisions. In particular, larger markets benefit from stronger demand linkages under monopolistic competition, while lower input trade costs attenuate these gains by inducing firms to substitute toward foreign inputs.

Firm Heterogeneity In homogeneous-firm models, market size affects specialization primarily through aggregate production and sourcing decisions, without altering the composition of firms across production modes (Venables 1996; Amiti 2005). By contrast, in our heterogeneous-firm model, market size additionally alters the composition of firms participating in export and import markets. More productive firms are more likely to incur the fixed costs associated with exporting and importing, so market size affects specialization not only through intensive-margin adjustments but also through extensive-margin selection across production modes. The model also generates complementarity between exporting and importing decisions documented by the literature: firms with larger export revenues have stronger incentives to source foreign inputs, while importing lowers marginal costs and further facilitates exporting. As a result, vertical specialization patterns emerge through endogenous firm-level reallocation across exporting and importing activities.

While some qualitative specialization patterns may still arise without firm heterogeneity, incorporating such heterogeneity better matches the observed complementarity between exporting and importing at the firm level and generates richer empirical implications for how market size shapes firms' global production decisions.

4.2 Empirical Implications

The discussion above suggests that the mechanisms in this paper translate into testable implications regarding the relationship between market size, firms' exporting and importing decisions, and vertical specialization across production stages. Below, we classify these implications into firm-level and aggregate-level predictions.

Firm-Level Predictions The model predicts that larger markets exhibit greater joint participation of firms in export and import activities. As market size increases, firms expand exporting activity and have stronger incentives to source foreign inputs because imported inputs reduce marginal costs and improve competitiveness in foreign markets. Consequently, market size strengthens the complementarity between exporting and importing decisions at the firm level, generating selection into global production networks.

Unlike existing firm-level studies emphasizing innovation or productivity upgrading in response to larger export markets (Lileeva and Trefler 2010; Aghion et al. 2024), our mechanism operates through firms' sourcing decisions and the complementarity between exporting and importing activities. Because the model abstracts from technological differences across regions, these predictions can be empirically examined using variation in market size across regions or provinces within a country.

Aggregate-Level Predictions The model predicts that larger markets export relatively more final goods and become relatively more specialized in downstream production stages, irrespective of the level of input trade costs. As market size increases, firms expand exporting activity while relying more intensively on imported inputs. Consequently, larger countries export relatively more final goods and import relatively more intermediate inputs. While this prediction resembles existing work (Antràs and de Gortari 2020), the underlying mechanism operates through market-size-driven firm selection and sourcing decisions rather than comparative advantage or production-network geography.

Further, lower input trade costs weaken the relationship between market size and relative wages. When input trade costs are high, larger markets sustain higher wages through the standard HME mechanism. However, lower input trade costs strengthen firms' incentives to substitute toward foreign inputs, attenuating or even reversing the positive relationship between market size and wages. The model therefore suggests a channel through which globalization may contribute to cross-country convergence in wages and income.

5 Conclusion

This paper develops a model of vertical specialization in which market size shapes firms' exporting and importing decisions under monopolistic competition and firm heterogeneity. Larger markets expand exporting activity while strengthening firms' incentives to source foreign intermediate inputs. Through this interaction, market size influences countries' specialization across production stages and participation in global production networks.

The analysis generates several results. When input trade costs are sufficiently high, the model reproduces the standard home-market effect, whereby larger countries export disproportionately more final goods and sustain higher relative wages. As input trade costs fall, however, firms substitute more intensively toward foreign inputs, weakening or even reversing the positive relationship between market size and wages. Globalization therefore changes not only the volume of trade but also the organization of production. Larger countries tend to specialize in downstream activities, whereas smaller countries become relatively more specialized in upstream production. More broadly, the model suggests that trade in intermediate inputs may attenuate cross-country wage and income differences by transmitting production and labor demand internationally.

Several extensions remain for future research. While the present framework abstracts from technological differences across countries in order to isolate the role of market size, much of the existing literature emphasizes comparative advantage and technology as determinants of vertical specialization. Combining these forces with market-size effects may provide further insights into the interaction between firm behavior, global production networks, and the international organization of production.

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A Appendix

A.1 Proofs for Section 2

A.1.1 Proof of Equation (2)

The expected operating profits in country i are given by

$$\int_{\varphi_{Di}}^{\infty} \pi_{Di}\, dG(\varphi) = \int_{\varphi_{Di}}^{\varphi_{DMi}} \left(B_i w_i^{1-\sigma}\varphi^{\sigma-1} - w_i f_D\right) dG(\varphi) + \int_{\varphi_{DMi}}^{\infty} \left(\frac{B_i}{s_i} w_i^{1-\sigma}\varphi^{\sigma-1} - w_i(f_D + f_{DM})\right) dG(\varphi),$$
$$\int_{\varphi_{Xi}}^{\infty} \pi_{Xi}\, dG(\varphi) = \int_{\varphi_{Xi}}^{\varphi_{XMi}} \left(B_j (\tau_X w_i)^{1-\sigma}\varphi^{\sigma-1} - w_i f_X\right) dG(\varphi) + \int_{\varphi_{XMi}}^{\infty} \left(\frac{B_j}{s_i} (\tau_X w_i)^{1-\sigma}\varphi^{\sigma-1} - w_i(f_X + f_{XM})\right) dG(\varphi).$$

Noting the definition of s_i, these expressions can be rewritten as

$$\int_{\varphi_{Di}}^{\infty} \pi_{Di}\, dG(\varphi) = \int_{\varphi_{Di}}^{\infty} \left(B_i w_i^{1-\sigma}\varphi^{\sigma-1} - w_i f_D\right) dG(\varphi) + \int_{\varphi_{DMi}}^{\infty} \left(B_i (\tau_M w_j)^{1-\sigma}\varphi^{\sigma-1} - w_i f_{DM}\right) dG(\varphi),$$
$$\int_{\varphi_{Xi}}^{\infty} \pi_{Xi}\, dG(\varphi) = \int_{\varphi_{Xi}}^{\infty} \left(B_j (\tau_X w_i)^{1-\sigma}\varphi^{\sigma-1} - w_i f_X\right) dG(\varphi) + \int_{\varphi_{XMi}}^{\infty} \left(B_j (\tau_X \tau_M w_j)^{1-\sigma}\varphi^{\sigma-1} - w_i f_{XM}\right) dG(\varphi).$$

Using the productivity cutoffs in (1), these expected profits can be expressed as

$$\int_{\varphi_{Di}}^{\infty} \pi_{Di}\, dG(\varphi) = w_i f_D \int_{\varphi_{Di}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{Di}}\right)^{\sigma-1} - 1\right] dG(\varphi) + w_i f_{DM} \int_{\varphi_{DMi}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{DMi}}\right)^{\sigma-1} - 1\right] dG(\varphi),$$
$$\int_{\varphi_{Xi}}^{\infty} \pi_{Xi}\, dG(\varphi) = w_i f_X \int_{\varphi_{Xi}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{Xi}}\right)^{\sigma-1} - 1\right] dG(\varphi) + w_i f_{XM} \int_{\varphi_{XMi}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{XMi}}\right)^{\sigma-1} - 1\right] dG(\varphi).$$

Defining $J(\varphi_c) \equiv \int_{\varphi_c}^{\infty} \left[\left(\frac{\varphi}{\varphi_c}\right)^{\sigma-1} - 1\right] dG(\varphi)$, the FE condition in (2) follows immediately.

A.1.2 Proof of Equations (3) and (4)

Aggregate Revenues and Expenditures We first derive the aggregate revenues. From firm-level revenues r_{Di} and r_{Xi}, the aggregate revenues of domestic and exported goods in country i are given by

$$R_{Di} = M_{Ei} \int_{\varphi_{Di}}^{\varphi_{DMi}} \left(\sigma B_i w_i^{1-\sigma}\varphi^{\sigma-1}\right) dG(\varphi) + M_{Ei} \int_{\varphi_{DMi}}^{\infty} \left(\frac{\sigma B_i}{s_i} w_i^{1-\sigma}\varphi^{\sigma-1}\right) dG(\varphi),$$
$$R_{Xi} = M_{Ei} \int_{\varphi_{Xi}}^{\varphi_{XMi}} \left(\sigma B_j (\tau_X w_i)^{1-\sigma}\varphi^{\sigma-1}\right) dG(\varphi) + M_{Ei} \int_{\varphi_{XMi}}^{\infty} \left(\frac{\sigma B_j}{s_i} (\tau_X w_i)^{1-\sigma}\varphi^{\sigma-1}\right) dG(\varphi).$$

Using the definition of s_i and defining $V(\varphi_c) \equiv \int_{\varphi_c}^{\infty} \varphi^{\sigma-1} dG(\varphi)$, these expressions can be rewritten as

$$R_{Di} = M_{Ei}\sigma\Big[B_i w_i^{1-\sigma} V(\varphi_{Di}) + B_i(\tau_M w_j)^{1-\sigma} V(\varphi_{DMi})\Big],$$
$$R_{Xi} = M_{Ei}\sigma\Big[B_j (\tau_X w_i)^{1-\sigma} V(\varphi_{Xi}) + B_j(\tau_X \tau_M w_j)^{1-\sigma} V(\varphi_{XMi})\Big].$$

Using the productivity cutoffs in (1), these can be further expressed as

$$R_{Di} = M_{Ei}\sigma w_i\Big[f_D \varphi_{Di}^{1-\sigma} V(\varphi_{Di}) + f_{DM} \varphi_{DMi}^{1-\sigma} V(\varphi_{DMi})\Big],$$
$$R_{Xi} = M_{Ei}\sigma w_i\Big[f_X \varphi_{Xi}^{1-\sigma} V(\varphi_{Xi}) + f_{XM} \varphi_{XMi}^{1-\sigma} V(\varphi_{XMi})\Big].$$

Finally, defining $F(\varphi_c) \equiv f_c \varphi_c^{1-\sigma} V(\varphi_c)$, aggregate revenues can be written compactly as

$$\begin{aligned} R_{Di} &= M_{Ei} \sigma w_i \big[F(\varphi_{Di}) + F(\varphi_{DMi})\big], \\ R_{Xi} &= M_{Ei} \sigma w_i \big[F(\varphi_{Xi}) + F(\varphi_{XMi})\big]. \end{aligned} \tag{A.1}$$

Similarly, using firm expenditures e_{Di} and e_{Mi}, with total expenditure $e_i = e_{Di} + \mathbb{1}_{Mi} e_{Mi}$ and $\tau_X e_i$ in the domestic and foreign markets, aggregate expenditures on domestic and imported inputs in country i are given by

$$\begin{aligned} E_{Di} &= M_{Ei} \int_{\varphi_{Di}}^{\varphi_{Xi}} \left(\frac{w_i}{\varphi} y_{Di} \right) dG(\varphi) + M_{Ei} \int_{\varphi_{Xi}}^{\infty} \left(\frac{\tau_X w_i}{\varphi} (y_{Di} + y_{Xi}) \right) dG(\varphi), \\ E_{Mi} &= M_{Ei} \int_{\varphi_{DMi}}^{\varphi_{XMi}} \left(\frac{w_i s_i^{1/(\sigma-1)}}{\varphi} y_{Di} \right) dG(\varphi) + M_{Ei} \int_{\varphi_{XMi}}^{\infty} \left(\frac{\tau_X w_i s_i^{1/(\sigma-1)}}{\varphi} (y_{Di} + y_{Xi}) \right) dG(\varphi). \end{aligned}$$

Using the demand functions y_{Di} and y_{Xi} and rearranging, these expressions can be written as

$$\begin{aligned} E_{Di} &= M_{Ei}(\sigma - 1) \Big[B_i w_i^{1-\sigma} V(\varphi_{Di}) + B_j (\tau_X w_i)^{1-\sigma} V(\varphi_{Xi}) \Big], \\ E_{Mi} &= M_{Ei}(\sigma - 1) \Big[B_i (\tau_M w_j)^{1-\sigma} V(\varphi_{DMi}) + B_j (\tau_X \tau_M w_j)^{1-\sigma} V(\varphi_{XMi}) \Big]. \end{aligned}$$

Using the productivity cutoffs in (1), aggregate expenditures can be expressed as

$$\begin{aligned} E_{Di} &= M_{Ei}(\sigma - 1) w_i \big[F(\varphi_{Di}) + F(\varphi_{Xi})\big], \\ E_{Mi} &= M_{Ei}(\sigma - 1) w_i \big[F(\varphi_{DMi}) + F(\varphi_{XMi})\big], \end{aligned} \tag{A.2}$$

where $F(\varphi_c)$ is defined above. Combining (A.1) and (A.2), the ratio of aggregate revenues $R_i = R_{Di} + R_{Xi}$ to aggregate expenditures $E_i = E_{Di} + E_{Mi}$ satisfy

$$\frac{R_i}{E_i} = \frac{1}{\rho}. \tag{A.3}$$

TB condition Defining $\alpha_{Xi} \equiv R_{Di}/R_{Xi}$ and $\alpha_{Mi} \equiv E_{Di}/E_{Mi}$ yields

$$\begin{aligned} \alpha_{Xi} &= \frac{F(\varphi_{Di}) + F(\varphi_{DMi})}{F(\varphi_{Xi}) + F(\varphi_{XMi})}, \\ \alpha_{Mi} &= \frac{F(\varphi_{Di}) + F(\varphi_{Xi})}{F(\varphi_{DMi}) + F(\varphi_{XMi})}. \end{aligned} \tag{A.4}$$

Since $1 + \alpha_{Xi} = R_i/R_{Xi}$ and $1 + \alpha_{Mi} = E_i/E_{Mi}$, we can write

$$R_{Xi} = \frac{R_i}{1 + \alpha_{Xi}}, \qquad E_{Mi} = \frac{E_i}{1 + \alpha_{Mi}}.$$

Substituting into the difference $R_{Xi} - E_{Mi}$ gives

$$R_{Xi} - E_{Mi} = \left(\frac{1}{1 + \alpha_{Xi}} - \frac{E_i/R_i}{1 + \alpha_{Mi}} \right) R_i.$$

Using $E_i/R_i = \rho$ from (A.3), and noting that the same relationships hold in country j, we obtain the TB condition in (4).

LMC Condition Next, we derive the LMC condition in (3). In particular, we show that

$$L_i = \frac{Y_i}{w_i} = \frac{R_{Di} + R_{Xj}}{w_i}.$$

From firm-level profits, domestic profits are given by

$$\pi_{Di} = r_{Di} - \left(e_{Di} + \mathbb{1}_{Mi} e_{Mi}\right) - w_i(f_D + \mathbb{1}_{Mi} f_{DM}),$$

and export profits by

$$\pi_{Xi} = r_{Xi} - \tau_X \left(e_{Di} + \mathbb{1}_{Mi} e_{Mi}\right) - w_i(f_X + \mathbb{1}_{Mi} f_{XM}).$$

Aggregating across firms, total profits in country i are

$$\begin{aligned} \mathit{\Pi}_i &\equiv M_{Ei} \left(\int_{\varphi_{Di}}^{\infty} \pi_{Di}(\varphi) dG(\varphi) + \int_{\varphi_{Xi}}^{\infty} \pi_{Xi}(\varphi) dG(\varphi) \right) \\ &= R_{Di} + R_{Xi} - E_{Di} - E_{Mi} - w_i F_{Pi}, \end{aligned}$$

where

$$F_{Pi} \equiv M_{Ei} \left(\int_{\varphi_{Di}}^{\infty} f_D dG(\varphi) + \int_{\varphi_{DMi}}^{\infty} f_{DM} dG(\varphi) + \int_{\varphi_{Xi}}^{\infty} f_X dG(\varphi) + \int_{\varphi_{XMi}}^{\infty} f_{XM} dG(\varphi) \right)$$

denotes total fixed production costs. The FE condition implies

$$\mathit{\Pi}_i = w_i F_{Ei},$$

where $F_{Ei} \equiv M_{Ei} f_E$ denotes total entry costs, pinning down the mass of entrants. Combining these expressions,

$$w_i(F_{Pi} + F_{Ei}) = R_{Di} + R_{Xi} - E_{Di} - E_{Mi}.$$

Labor used in final-good production equals total fixed costs, $L_i^{\text{final}} = F_{Pi} + F_{Ei}$, so that

$$L_i^{\text{final}} = \frac{R_{Di} + R_{Xi}}{w_i} - \frac{E_{Di} + E_{Mi}}{w_i}.$$

Labor is also used in input production. Input producers in country i supply both domestic inputs used by firms in country i and exported inputs used by firms in country j. Since production is linear in labor, the labor requirement equals the value of inputs produced divided by the wage. Domestic input demand is given by E_{Di}, while exports of inputs to country j are E_{Mj}. Hence, total labor used in input production in country i is

$$L_i^{\text{input}} = \frac{E_{Di} + E_{Mj}}{w_i}.$$

Total labor supply satisfies $L_i = L_i^{\text{final}} + L_i^{\text{input}}$. Substituting the expressions above gives

$$L_i = \frac{R_{Di} + R_{Xi} - E_{Mi} + E_{Mj}}{w_i}.$$

Using the TB condition, $R_{Xi} - R_{Xj} = E_{Mi} - E_{Mj}$, and rearranging, we obtain $w_i L_i = R_{Di} + R_{Xj}$. Finally, using $R_{Di} = \alpha_{Xi} R_i / (1 + \alpha_{Xi})$ and $R_{Xj} = R_j / (1 + \alpha_{Xj})$, we obtain the LMC condition in (3).

A.2 Proofs for Section 3

A.2.1 Proof of Equation (6)

When $\tau_M = \infty$, importing is infeasible, so that $\varphi_{DMi} = \varphi_{XMi} = \infty$. Then, the ZCP conditions in (1) reduce to

$$B_i w_i^{1-\sigma} \varphi_{Di}^{\sigma-1} = w_i f_D,$$
$$B_j (\tau_X w_i)^{1-\sigma} \varphi_{Xi}^{\sigma-1} = w_i f_X,$$

Similarly, noting that $J(.)$ is a strictly decreasing function, the FE condition in (2) reduces to

$$f_D J(\varphi_{Di}) + f_X J(\varphi_{Xi}) = f_E.$$

Since $E_{Mi} = 0$, the domestic-to-import input expenditure ratio defined in Section 2.4 is $\alpha_{Mi} = E_{Di}/E_{Mi} = \infty$. The LMC condition in (3) becomes $w_i L_i = R_i$, while the TB condition in (4) reduces to

$$\frac{R_i}{1+\alpha_{Xi}} = \frac{R_j}{1+\alpha_{Xj}}.$$

Combining the two conditions yields

$$\frac{w_i L_i}{1+\alpha_{Xi}} = \frac{w_j L_j}{1+\alpha_{Xj}}.$$

We now consider the effect of an increase in L_1, holding L_2 fixed. Since $L \equiv L_1/L_2$, the resulting comparative statics correspond to changes in relative market size in the main text. Differentiating the ZCP conditions,

$$\begin{aligned} \hat{B}_1 + (\sigma-1)\hat{\varphi}_{D1} &= \sigma\hat{\omega}, \qquad & \hat{B}_2 + (\sigma-1)\hat{\varphi}_{D2} &= 0, \\ \hat{B}_2 + (\sigma-1)\hat{\varphi}_{X1} &= \sigma\hat{\omega}, \qquad & \hat{B}_1 + (\sigma-1)\hat{\varphi}_{X2} &= 0, \end{aligned} \tag{A.5}$$

In contrast, the FE condition does not directly involve L. Differentiating this condition and using $J'(a)a = -(\sigma-1)a^{1-\sigma}V(a)$, from the definitions of $J(.)$ and $V(.)$ in Appendix A.1.2, we obtain

$$F(\varphi_{Di})\hat{\varphi}_{Di} + F(\varphi_{Xi})\hat{\varphi}_{Xi} = 0,$$

where $F(.)$ is defined in Appendix A.1.2, which is a strictly decreasing function. Since $\varphi_{DMi} = \varphi_{XMi} = \infty$ imply $F(\varphi_{DMi}) = F(\varphi_{XMi}) = 0$ in (A.1), we obtain

$$\frac{F(\varphi_{Di})}{F(\varphi_{Xi})} = \frac{R_{Di}}{R_{Xi}} = \alpha_{Xi},$$

where α_{Xi} is the domestic-to-export revenue ratio defined in Section 2.4. Thus, we obtain

$$\begin{aligned} \hat{\varphi}_{X1} &= -\alpha_{X1}\hat{\varphi}_{D1}, \\ \hat{\varphi}_{X2} &= -\alpha_{X2}\hat{\varphi}_{D2}. \end{aligned} \tag{A.6}$$

Finally, rewriting the TB condition as $\omega L = \frac{1+\alpha_{Xi}}{1+\alpha_{Xj}}$ and differentiating it with respect to L yields

$$\hat{\omega} = -\beta_{X1}\hat{\varphi}_{D1} + \beta_{X2}\hat{\varphi}_{D2} - \hat{L}, \tag{A.7}$$

where

$$\beta_{Xi} \equiv \frac{\alpha_{Xi}}{1+\alpha_{Xi}} \left[\sigma - 1 + \gamma_{Di} + (\sigma - 1 + \gamma_{Xi})\alpha_{Xi}\right],$$
$$\gamma_{Di} \equiv -\frac{d\ln V(\varphi_{Di})}{d\ln\varphi_{Di}}; \qquad \gamma_{Xi} \equiv -\frac{d\ln V(\varphi_{Xi})}{d\ln\varphi_{Xi}}.$$

Substituting (A.6) and (A.7) into (A.5) yields

$$(\beta_{X1}+\rho)\hat{\varphi}_{D1} - (\beta_{X2} - \rho\alpha_{X2})\hat{\varphi}_{D2} = -\hat{L},$$
$$-(\beta_{X1}-\rho\alpha_{X1})\hat{\varphi}_{D1} + (\beta_{X2}+\rho)\hat{\varphi}_{D2} = \hat{L}.$$

Solving this system and substituting back into (A.7) yields (6), where

$$\Xi_X \equiv (\beta_{X1}+\rho)(\beta_{X2}+\rho) - (\beta_{X1}-\rho\alpha_{X1})(\beta_{X2}-\rho\alpha_{X2}) > 0.$$

Here we need to verify that $\alpha_{X1}\alpha_{X2} > 1$ that appears in the numerator of (6). Using R_{Di} and R_{Xi} in (A.1),

$$\alpha_{Xi} = \frac{R_{Di}}{R_{Xi}} = \frac{f_D\varphi_{Di}^{1-\sigma}V(\varphi_{Di})}{f_X\varphi_{Xi}^{1-\sigma}V(\varphi_{Xi})}.$$

From the ZCP conditions, the productivity cutoff ratio is given by

$$\left(\frac{\varphi_{Xi}}{\varphi_{Di}}\right)^{\sigma-1} = \frac{B_i}{B_j}\tau_X^{\sigma-1}\frac{f_X}{f_D}.$$

Substituting this into α_{Xi} derived above yields

$$\alpha_{Xi} = \tau_X^{\sigma-1}\frac{B_i}{B_j}\frac{V(\varphi_{Di})}{V(\varphi_{Xi})}.$$

Hence,

$$\alpha_{X1}\alpha_{X2} = \tau_X^{2(\sigma-1)}\frac{V(\varphi_{D1})V(\varphi_{D2})}{V(\varphi_{X1})V(\varphi_{X2})}.$$

Since $\varphi_{Xi} > \varphi_{Di}$ and $V(\varphi)$ is strictly decreasing, we have $V(\varphi_{Di})/V(\varphi_{Xi}) > 1$, implying the desired result.

A.2.2 Proof of Equation (7)

When $\tau_X = \infty$, exporting is infeasible, so that $\varphi_{Xi} = \varphi_{XMi} = \alpha_{Xi} = \infty$. In this case, the ZCP conditions in (1) and the FE condition in (2) reduce to

$$B_i w_i^{1-\sigma}\varphi_{Di}^{\sigma-1} = w_i f_D,$$
$$B_i(\tau_M w_j)^{1-\sigma}\varphi_{DMi}^{\sigma-1} = w_i f_{DM},$$
$$f_D J(\varphi_{Di}) + f_{DM}J(\varphi_{DMi}) = f_E.$$

Further, as in Appendix A.2.1, combining the LMC condition in (3), the TB condition in (4) can be written as

$$\frac{w_i L_i}{1+\alpha_{Mi}} = \frac{w_j L_j}{1+\alpha_{Mj}}.$$

Differentiating the ZCP conditions with respect to L yields the system of equations similar to (A.5):

$$\begin{aligned} \hat{B}_1 + (\sigma-1)\hat{\varphi}_{D1} &= \sigma\hat{\omega}, \qquad & \hat{B}_2 + (\sigma-1)\hat{\varphi}_{D2} &= 0, \\ \hat{B}_1 + (\sigma-1)\hat{\varphi}_{DM1} &= \hat{\omega}, \qquad & \hat{B}_2 + (\sigma-1)\hat{\varphi}_{DM2} &= (\sigma-1)\hat{\omega}. \end{aligned} \tag{A.8}$$

Analogously to (A.6), differentiating the FE condition and rearranging, we obtain

$$\begin{aligned} \hat{\varphi}_{DM1} &= -\alpha_{M1}\hat{\varphi}_{D1}, \\ \hat{\varphi}_{DM2} &= -\alpha_{M2}\hat{\varphi}_{D2}, \end{aligned} \tag{A.9}$$

where

$$\alpha_{Mi} = \frac{F(\varphi_{Di})}{F(\varphi_{DMi})} = \frac{E_{Di}}{E_{Mi}}.$$

Finally, analogously to (A.7) differentiating the TB condition with respect to L yields

$$\hat{\omega} = -\beta_{M1}\hat{\varphi}_{D1} + \beta_{M2}\hat{\varphi}_{D2} - \hat{L}, \tag{A.10}$$

where

$$\begin{aligned} \beta_{Mi} &\equiv \frac{\alpha_{Mi}}{1+\alpha_{Mi}}\left[\sigma - 1 + \gamma_{Di} + (\sigma - 1 + \gamma_{Mi})\alpha_{Mi}\right], \\ \gamma_{Mi} &\equiv -\frac{d\ln V(\varphi_{DMi})}{d\ln \varphi_{DMi}}. \end{aligned}$$

Substituting (A.9) and (A.10) into (A.8) yields

$$\begin{aligned} (\alpha_{M1} + \beta_{M1} + 1)\hat{\varphi}_{D1} - \beta_{M2}\hat{\varphi}_{D2} &= -\hat{L}, \\ -\beta_{M1}\hat{\varphi}_{D1} + (\alpha_{M2} + \beta_{M2} + 1)\hat{\varphi}_{D2} &= \hat{L}. \end{aligned}$$

Solving this system and substituting back into (A.8) yields (7), where

$$\Xi_M \equiv (\alpha_{M1} + \beta_{M1} + 1)(\alpha_{M2} + \beta_{M2} + 1) - \beta_{M1}\beta_{M2} > 0.$$

A.2.3 Proof of Equation (8)

From the ZCP conditions in (1), selection into *exporting* for a given sourcing mode requires

$$\begin{aligned} \left(\frac{\varphi_{Xi}}{\varphi_{Di}}\right)^{\sigma-1} &= \frac{B_i}{B_j}\frac{\tau_X^{\sigma-1} f_X}{f_D} > 1, \\ \left(\frac{\varphi_{XMi}}{\varphi_{DMi}}\right)^{\sigma-1} &= \frac{B_i}{B_j}\frac{\tau_X^{\sigma-1} f_{XM}}{f_{DM}} > 1. \end{aligned} \tag{A.11}$$

Similarly, selection into *importing* within a given operating market requires

$$\begin{aligned} \left(\frac{\varphi_{DMi}}{\varphi_{Di}}\right)^{\sigma-1} &= \left(\frac{w_j}{w_i}\right)^{\sigma-1} \frac{\tau_M^{\sigma-1} f_{DM}}{f_D} > 1, \\ \left(\frac{\varphi_{XMi}}{\varphi_{Xi}}\right)^{\sigma-1} &= \left(\frac{w_j}{w_i}\right)^{\sigma-1} \frac{\tau_M^{\sigma-1} f_{XM}}{f_X} > 1. \end{aligned} \tag{A.12}$$

Differentiating the domestic ZCP conditions in (1) and rearranging yields

$$\frac{\hat{B}_1 - \hat{B}_2}{\sigma - 1} = \frac{1}{\rho}\hat{\omega} - \hat{\varphi}_{D1} + \hat{\varphi}_{D2}.$$

Further, differentiating the exporting cutoff conditions in (A.11) gives

$$\hat{\varphi}_{X1} - \hat{\varphi}_{D1} = \hat{\varphi}_{XM1} - \hat{\varphi}_{DM1} = \frac{\hat{B}_1 - \hat{B}_2}{\sigma - 1},$$
$$\hat{\varphi}_{X2} - \hat{\varphi}_{D2} = \hat{\varphi}_{XM2} - \hat{\varphi}_{DM2} = -\frac{\hat{B}_1 - \hat{B}_2}{\sigma - 1}.$$

Similarly, differentiating the importing cutoff conditions in (A.12) yields

$$\hat{\varphi}_{DM1} = \hat{\varphi}_{D1} - \hat{\omega}, \qquad \hat{\varphi}_{DM2} = \hat{\varphi}_{D2} + \hat{\omega}. \tag{A.13}$$

Using the above expressions, we obtain

$$\begin{aligned} \hat{\varphi}_{X1} &= \frac{1}{\rho}\hat{\omega} + \hat{\varphi}_{D2}, \qquad \hat{\varphi}_{XM1} = \left(1 + \frac{1}{\rho}\right)\hat{\omega} + \hat{\varphi}_{D2}, \\ \hat{\varphi}_{X2} &= -\frac{1}{\rho}\hat{\omega} + \hat{\varphi}_{D1}, \qquad \hat{\varphi}_{XM2} = \left(1 - \frac{1}{\rho}\right)\hat{\omega} + \hat{\varphi}_{D1}. \end{aligned} \tag{A.14}$$

Equations (A.13) and (A.14) show that $\hat{\varphi}_{DMi}, \hat{\varphi}_{Xi}, \hat{\varphi}_{XMi}$ can be expressed in terms of $\hat{\varphi}_{Di}$ and $\hat{\omega}$.

Next, differentiating the FE condition in (2),

$$F(\varphi_{Di})\hat{\varphi}_{Di} + F(\varphi_{DMi})\hat{\varphi}_{DMi} + F(\varphi_{Xi})\hat{\varphi}_{Xi} + F(\varphi_{XMi})\hat{\varphi}_{XMi} = 0.$$

Substituting (A.13) and (A.14) into this, we get

$$(F(\varphi_{D1}) + F(\varphi_{DM1}))\hat{\varphi}_{D1} + (F(\varphi_{X1}) + F(\varphi_{XM1}))\hat{\varphi}_{D2} + \left[\frac{F(\varphi_{X1}) + F(\varphi_{XM1})}{\rho} - (F(\varphi_{DM1}) + F(\varphi_{XM1}))\right]\hat{\omega} = 0,$$
$$(F(\varphi_{D2}) + F(\varphi_{DM2}))\hat{\varphi}_{D2} + (F(\varphi_{X2}) + F(\varphi_{XM2}))\hat{\varphi}_{D1} - \left[\frac{F(\varphi_{X2}) + F(\varphi_{XM2})}{\rho} - (F(\varphi_{DM2}) + F(\varphi_{XM2}))\right]\hat{\omega} = 0.$$

Now divide by $F(\varphi_{DMi}) + F(\varphi_{XMi})$. Using α_{Xi} and α_{Mi} in (A.4) for the definition of θ_i,

$$\theta_i \equiv \frac{1 + \alpha_{Mi}}{1 + \alpha_{Xi}} = \frac{F(\varphi_{Xi}) + F(\varphi_{XMi})}{F(\varphi_{DMi}) + F(\varphi_{XMi})}, \qquad 1 + \alpha_{Mi} - \theta_i = \frac{F(\varphi_{Di}) + F(\varphi_{DMi})}{F(\varphi_{DMi}) + F(\varphi_{XMi})}.$$

The above FE condition therefore can be rewritten as

$$(1 + \alpha_{M1} - \theta_1)\hat{\varphi}_{D1} + \theta_1\hat{\varphi}_{D2} = -\left(\frac{\theta_1}{\rho} - 1\right)\hat{\omega},$$
$$\theta_2\hat{\varphi}_{D1} + (1 + \alpha_{M2} - \theta_2)\hat{\varphi}_{D2} = \left(\frac{\theta_2}{\rho} - 1\right)\hat{\omega}.$$

Solving these two equations for $\hat{\varphi}_{D1}$ and $\hat{\varphi}_{D2}$ yields (8), where

$$\Xi \equiv (1 + \alpha_{M1} - \theta_1)(1 + \alpha_{M2} - \theta_2) - \theta_1\theta_2 > 0.$$

A.2.4 Proof of Equations (19) and (20)

We characterize the slopes of equations (19) and (20) locally around the symmetric equilibrium. Because α_{Xi}, α_{Mi}, and θ_i are endogenous functions of equilibrium wages, we write them as $\alpha_{Xi}(\omega)$, $\alpha_{Mi}(\omega)$, and $\theta_i(\omega)$ below. We focus only on their local slopes at symmetry with $\omega \equiv w_1/w_2 = 1$ and $L \equiv L_1/L_2 = 1$.

Slope of the LMC Condition Differentiating equation (19) with respect to ω and evaluating at symmetry, where $\omega = L = 1$, $\alpha_{X1}(1) = \alpha_{X2}(1)$, and $\alpha'_{X2}(1) = -\alpha'_{X1}(1)$, yields

$$\Phi_\omega = \frac{\alpha_{X1}(1) + 1 - 2\alpha'_{X1}(1)}{\alpha_{X1}(1) - 1}.$$

Since the interiority condition implies $\alpha_{X1}(1) > 1$, the denominator is positive. Therefore,

$$\Phi_\omega < 0 \quad \Longleftrightarrow \quad 2\alpha'_{X1}(1) > \alpha_{X1}(1) + 1,$$

where $\alpha'_{X1}(1) > 0$. The sign reflects the response of sales decisions to the relative wage. As ω approaches zero, country 1 becomes extremely competitive relative to country 2. In this case, maintaining positive final-good trade requires country 2 firms to become increasingly domestically oriented. Indeed, for the numerator of (19), $\alpha_{X2}(\omega)\omega L - 1$, remaining positive, this implies that $\alpha_{X2}(\omega) \to \infty$ and $\alpha_{X1}(\omega) \to 0$ (by symmetry) as $\omega \to 0$. Accordingly, an increase in ω reduces country 2's domestic orientation while increasing country 1's domestic orientation locally around the symmetric equilibrium.

The LMC condition is therefore downward sloping if this endogenous reallocation of sales toward domestic markets is sufficiently strong. Intuitively, a higher relative wage ω raises export cutoffs and shifts firms toward domestic sales in country 1. When this reallocation is strong enough, the relative export ratio falls with ω, so

$$\Phi_\omega < 0.$$

Under the parameterizations of Figures 1 and 3 in Sections 3.4 and 3.5, the condition for $\Phi_\omega < 0$ is satisfied, ensuring that the LMC condition is downward sloping locally around the symmetric equilibrium.

Slope of the TB Condition Differentiating equation (20) with respect to ω and evaluating at symmetry, where $\omega = L = 1$, $\theta_1(1) = \theta_2(1)$, and $\theta'_2(1) = -\theta'_1(1)$, yields

$$\Psi_\omega = -\frac{2\rho}{\theta_1(1)^2} \frac{\theta'_1(1)}{1 - \rho/\theta_1(1)}.$$

Further, differentiating $\theta_1(\omega)$ and evaluating at symmetry, where $\alpha_{X1}(1) = \alpha_{X2}(1)$ and $\alpha_{M1}(1) = \alpha_{M2}(1)$, gives

$$\theta'_1(1) = \frac{\alpha'_{M1}(1) - \theta_1(1)\alpha'_{X1}(1)}{1 + \alpha_{X1}(1)}.$$

Hence, the sign of $\theta'_1(1)$ (and consequently the sign of Ψ_ω) depends not only on the response of sales decisions to the relative wage, $\alpha'_{X1}(1) > 0$, but also on that of sourcing decisions, $\alpha'_{M1}(1) < 0$. The intuition for the sign for $\alpha'_{Mi}(1)$ is parallel to that for $\alpha'_{Xi}(1)$. As ω approaches zero, country 1 becomes extremely competitive relative to country 2. Thus, firms in country 2 increasingly substitute toward imported inputs from country 1. Maintaining positive input trade therefore requires country 2 firms to become increasingly dependent on foreign sourcing, implying $\alpha_{M2}(\omega) \to 0$ and $\alpha_{M1}(\omega) \to \infty$ (by symmetry) as $\omega \to 0$. This implies that an increase in ω reduces foreign sourcing in country 2 while increasing foreign sourcing in country 1. Taken together, we obtain $\theta_1'(1) < 0$ around the symmetric equilibrium.

From the expression for Ψ_ω, the sign of the TB slope is therefore determined by the sign of $1 - \rho/\theta_1(1)$. When input trade costs τ_M are sufficiently high, Assumption 1 holds. In this case, $1 - \rho/\theta_1(1) > 0$, so that

$$\Psi_\omega > 0.$$

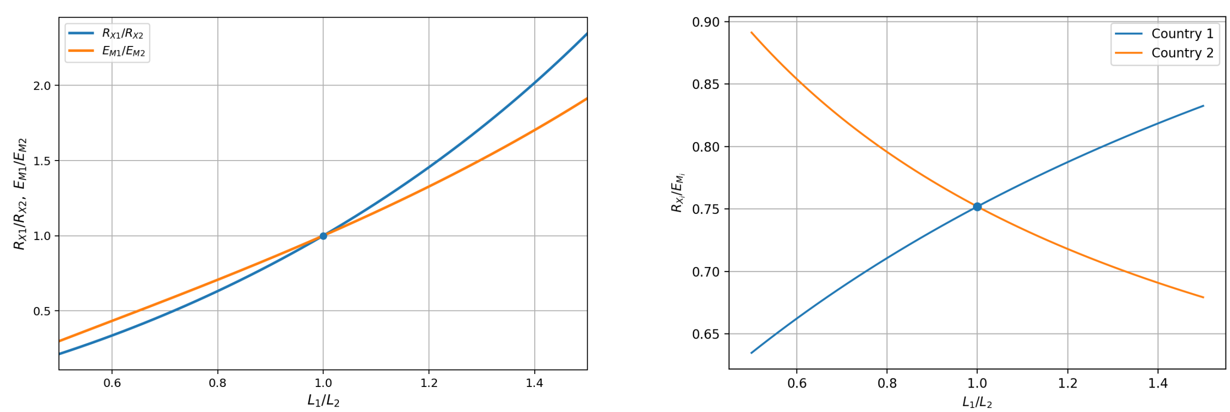
When input trade costs are sufficiently low, however, Assumption 1 is violated. Then, $1 - \rho/\theta_1(1) < 0$, so that

$$\Psi_\omega < 0.$$

This establishes that the sign of Ψ_ω depends on the level of input trade costs around the symmetric equilibrium, as claimed in Sections 3.4 and 3.5.

A.3 Additional Figures for Section 3.5

Figure A.1 reports additional numerical illustrations corresponding to the comparative statics discussed in Section 3.5. The figure is the counterpart of Figure 2 generated under the same parameterization, with the key difference being whether input trade costs are sufficiently high ($\tau_M = 1.5$) or sufficiently low ($\tau_M = 1.1$).



(a) Sales and expenditure

(b) Sales-expenditure ratio

Figure A.1: General-equilibrium effects of market size: failure of the home-market effect