Supplementary Note

This Note shows that our findings in section 3 are robust to a variety of alternative specifications of the model.

A Presence of Foreign Consumers

For a sharper characterization, we have assumed in the main analysis that all consumers are in Home. This assumption is however not crucial to our result. Suppose that a fraction $\mu \in [0, 1)$ of the consumers reside in Home while the remaining fraction, $(1 - \mu)$, reside in Foreign. In addition, all consumers have an identical demand for final goods, $Q = Q(P)$, and Home and Foreign markets are segmented. Following the literature, we can then show that if $\hat{P}(t)$ is the equilibrium price in Home, it is also the equilibrium price in Foreign.

Under this environment, the optimal tariff, $t = t(\beta)$, implicitly solves the following:

$$t = -\frac{P'(\hat{Q}(t))\hat{Q}(t)(2 + \hat{\epsilon})}{s} \left(\frac{s(1 - \mu) + 1 + \hat{\epsilon}}{2 + \hat{\epsilon}} - \beta \right).$$

where $\hat{Q}(t)$ and $P(\hat{Q}(t))$ respectively denote the aggregate world output and price in a Cournot equilibrium for a given t. Observe that, as in Proposition 3.1, (i) $t(\beta) > 0$ if β is less than a threshold value, and (ii) $t'(\beta) < 0$. The only difference lies in the possibility of $t(\beta) > 0$: The optimal tariff is more likely to be positive under $\mu < 1$ since the negative effect of tariff on consumer surplus receives less weight.

B Strategic Interactions between Governments

We have assumed in the main analysis that only the Home government sets a tariff rate and the Foreign government does not engage actively in trade policy. Here we show that the negative relationship between bargaining power and tariffs is robust under a non-cooperative tariff setting by the two governments.

Consider an environment in which the governments of both countries, Home and Foreign, choose tariff rates non-cooperatively to maximize the welfare of their respective countries. To illustrate this strategic interaction in a simple fashion, assume that $\mu = 0$, i.e., all consumers are in the Foreign country. Let t_F denote the Foreign tariff rate on imports of final goods and let t_H denote the Home tariff rate on imports of intermediate inputs. For a given pair of tariffs (t_H, t_F), let $\hat{Q}(t_H, t_F)$ and $\hat{P}(t_H, t_F) \equiv P(\hat{Q}(t_H, t_F))$ denote the equilibrium aggregate output and price. Then, the welfare of Home and Foreign, denoted by W_H and W_F respectively, are given by:

$$\begin{aligned}
W_H &= \beta(\hat{P}(t_H, t_F) - c - t_H - t_F)\hat{Q}(t_H, t_F) + t_H\hat{Q}(t_H, t_F), \\
W_F &= \int_0^{\hat{Q}(t_H, t_F)} P(y)\mathrm{d}y - \hat{P}(t_H, t_F)\hat{Q}(t_H, t_F) \\
&\quad + (1 - \beta)(\hat{P}(t_H, t_F) - c - t_H - t_F)\hat{Q}(t_H, t_F) + t_F\hat{Q}(t_H, t_F).
\end{aligned}$$

Let us now turn to Nash equilibrium tariffs in this two-country setting (see the Appendix to this Note for detailed derivations).

Proposition B.1

(i) *The country with lower bargaining power sets a higher tariff. More formally,*

$$t_H(\beta) \gtreqless t_F(\beta) \iff \beta \lesseqgtr \frac{1}{2}.$$

where $t_H(\beta)$ and $t_F(\beta)$ respectively denote the optimal tariffs for Home and Foreign.

(ii) *Suppose that $\frac{\partial^2 W_i}{\partial t_i^2} + \frac{\partial^2 W_j}{\partial t_j \partial t_i} < 0$ holds for $i \neq j \in \{H, F\}$.*[1] *Then, the optimal tariff in Home (Foreign) is monotonically decreasing (increasing) in the bargaining power of Home firms, i.e.,*

$$\frac{\mathrm{d}t_H}{\mathrm{d}\beta} < 0, \quad \frac{\mathrm{d}t_F}{\mathrm{d}\beta} > 0.$$

There are two caveats to this extension. First, though it is assumed that all consumers reside in Foreign, the fraction of consumers in Home, μ, does not qualitatively affect the main result. Second, we have assumed away an export tax – not banned by WTO but illegal by U.S. law – on intermediate inputs imposed by the Foreign government. If we allow for this possibility, the Foreign government would face a trade-off between the terms-of-trade improvement and a reduction in Foreign firms' profits in determining the optimal export tax. This trade-off is similar to the one faced by the Home government in determining optimal import tariff. Hence, we expect that when Foreign firms' bargaining power is high (low), the optimal export tax is negative (positive), and it is monotonically decreasing with Foreign firms' bargaining power. Results similar to the ones reported in Proposition B.1 would continue to hold.

Example: Consider the class of inverse demand functions: $P(Q) = a - Q^d$, where $d > 0$. The optimal tariffs are:

$$t_H(\beta) = \frac{(a-c)d(d+1)}{s+d^2}\left(\frac{d}{d+1} - \beta\right), \quad t_F(\beta) = \frac{(a-c)d(d+1)}{s+d^2}\left(\frac{d}{d+1} - (1-\beta)\right).$$

First, consider the linear demand function for which $\epsilon = d - 1 = 0$. The optimal tariffs for Home and Foreign are respectively shown as bold lines HH and FF in Figure S.1. As expected from Proposition B.1,

(i) HH lies above (below) FF when Home firms have less (more) bargaining power;

[1] Note that, while $\frac{\partial^2 W_i}{\partial t_i^2} < 0$ from the second-order condition, whether $\frac{\partial^2 W_j}{\partial t_j \partial t_i}$ is positive or negative depends on the strategic relationship between Home and Foreign tariffs. It can be immediately seen that this condition necessarily holds if Home and Foreign tariffs are *strategic substitutes*, i.e., $\frac{\partial^2 W_j}{\partial t_j \partial t_i} < 0$.

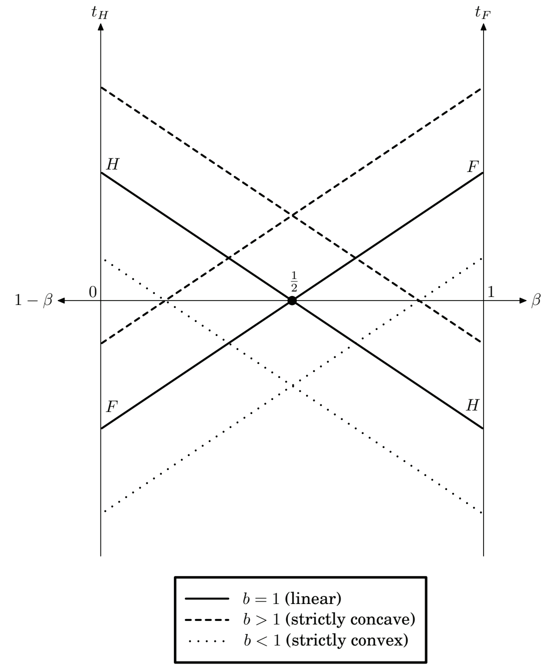

FIGURE S.1 – t_H, t_F and β

(ii) HH and FF are downward sloping in the bargaining power of Home (β) and Foreign ($1-\beta$) respectively.

Both (i) and (ii) are preserved when $\epsilon > 0$ (see the dashed line) as well as when $\epsilon < 0$ (see the dotted line). The only qualitative difference between these cases is in the likelihood of import tariffs and import subsidies. Under linear demand (i.e., $\epsilon = 0$), free trade is optimal if both countries have equal bargaining power. Otherwise, if bargaining power is unequal, the country with less bargaining power sets an import tariff while the other country offers an import subsidy. As both HH and FF shift up for $\epsilon > 0$, compared to linear demand, the optimal tariff is more likely to be positive for strictly concave demand functions ($d > 1$). The opposite is true for strictly convex demand functions ($d < 1$).

C Ad Valorem Tariff

In the main text we assumed that Foreign firms face a specific tariff on intermediate input. Here we show that the negative relationship between β and optimal tariff goes through when Foreign firm faces an ad valorem tariff on intermediate input.

Everything is same as in the model described in section 3 except that, instead of specific tariff of t per unit, each Foreign firm pays ad valorem tariff $\tau > 0$. Consider a pair $i(= 1, 2, ..., s)$ where a Home firm H_i pays r_i for one unit of intermediate input while a Foreign firm F_i receives R_i. The two prices, r_i and R_i, are connected by ad valorem tariff as follows:

$$r_i = (1+\tau)R_i.$$

The relevant utility functions for the analysis of the bargaining are H_i's profit, $\pi_{H_i} \equiv \left[P\left(q_i + \sum_{j\neq i}^{s} q_j\right) - r_i\right] q_i$ and, F_i's profit, $\pi_{F_i} \equiv (\frac{r_i}{1+\tau} - c)q_i$. As in section 3.1 we can show that for all $s > 1$ the unique bargaining outcome is given by $\hat{r}_1 = ... = \hat{r}_s \equiv \hat{r}$ and $\hat{q}_1 = ... = \hat{q}_s \equiv \hat{q}$, where $\hat{r}$ (> 0) and $\hat{q}$ (> 0) are determined by (C.1) and (C.2) below:

$$\hat{q} = -\frac{P(\hat{Q}) - C}{P'(\hat{Q})}, \tag{C.1}$$

$$\hat{r} = (1-\beta)P(\hat{Q}) + \beta C, \tag{C.2}$$

where $C = (1+\tau)c$ and $Q = \hat{Q}$ uniquely solves the following:

$$sP(Q) + P'(Q)Q = sC. \tag{C.3}$$

The lemma stated below is effectively the ad valorem tariff analog of Lemma 3.1. Only part (iii) is weaker than its counterpart in Lemma 3.1.

Lemma C.1

(i) *For a given ad valorem tariff rate τ, the aggregate output $\hat{Q}$ and the final-good price $\hat{P} \equiv P(\hat{Q})$ are independent of β; i.e.,* $\mathrm{d}\hat{Q}/\mathrm{d}\beta = \mathrm{d}\hat{P}/\mathrm{d}\beta = 0$.

(ii) *For a given bargaining power β, an increase in the ad valorem tariff rate lowers output and raises prices; i.e.,* $\mathrm{d}\hat{Q}/\mathrm{d}\tau < 0$, $\mathrm{d}\hat{P}/\mathrm{d}\tau > 0$ *and* $\mathrm{d}\hat{r}/\mathrm{d}\tau > 0$.

(iii) *Let $\hat{R} \equiv \frac{\hat{r}}{1+\tau}$ denote the price received by a Foreign firm in equilibrium (for each unit of the intermediate input). Then,* $\mathrm{d}\hat{R}/\mathrm{d}\tau < 0$ *for all logconcave demand functions*.

As in Lemma 3.1(iii), an ad valorem tariff improves the input terms of trade for all logconcave demand functions. Let $\hat{Q}(\tau,\beta)$, $\hat{P}(\tau,\beta)$ and $\hat{r}(\tau,\beta)$ respectively denote the equilibrium output, the price of final goods and the price of intermediate inputs for a given τ and β; similarly, let $\hat{R}(\tau,\beta) = \frac{\hat{r}(\tau,\beta)}{T}$ for a given τ and β. Note also that, as in Lemma 3.1(i), equilibrium output does not depend on β in the short run, and thus we use $\hat{Q}(\tau)$ and $\hat{P}(\tau)$ to denote the equilibrium output and price respectively.

In the first stage, the Home government chooses a tariff rate τ to maximize Home welfare (W_H):

$$W_H \equiv \underbrace{\int_0^{\hat{Q}(\tau)} P(y)\mathrm{d}y - \hat{P}(\tau)\hat{Q}(\tau)}_{\text{Consumer surplus}} + \underbrace{\left(\hat{P}(\tau) - \hat{r}(\tau,\beta)\right)\hat{Q}(\tau)}_{\text{Home profits}} + \underbrace{(\hat{r}(\tau,\beta) - \hat{R}(\tau,\beta))\hat{Q}(\tau)}_{\text{Tariff revenues}}.$$

Simplifying the above expression further gives

$$W_H \equiv \int_0^{\hat{Q}(\tau)} P(y)\mathrm{d}y - \hat{R}(\tau,\beta)\hat{Q}(\tau).$$

Differentiating W_H with respect to τ and rearranging, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}\tau} = (\hat{P}(\tau) - \hat{R}(\tau,\beta))\frac{\mathrm{d}\hat{Q}(\tau)}{\mathrm{d}\tau} - \frac{\mathrm{d}\hat{R}(\tau,\beta)}{\mathrm{d}\tau}\hat{Q}(\tau). \tag{C.4}$$

The first term captures the welfare loss due to the tariff-induced output reduction. The price-cost margin, $\hat{P}(\tau) - \hat{R}(\tau,\beta)$, multiplied by the amount of output lost, $\frac{\mathrm{d}\hat{Q}(\tau)}{\mathrm{d}\tau}$, is the magnitude of welfare loss. The second term, $-\frac{\mathrm{d}\hat{R}(\tau,\beta)}{\mathrm{d}\tau}\hat{Q}(\tau)$, captures the welfare gains arising from the terms-of-trade improvement ($\frac{\mathrm{d}\hat{R}(\tau,\beta)}{\mathrm{d}\tau} < 0$). As in section 3.2, the optimal tariff rate strikes a balance between the two.

Let $\tau(\beta)$ denote the optimal ad valorem tariff. Substituting the expressions for $\frac{\mathrm{d}\hat{Q}(\tau)}{\mathrm{d}\tau}$ and $\frac{\mathrm{d}\hat{R}(\tau,\beta)}{\mathrm{d}\tau}$ from Lemma C.1 into (C.4) and rearranging we get:

$$\frac{\mathrm{d}W_H}{\mathrm{d}\tau} = \left(\frac{\tau\hat{P}}{1+\tau} + \frac{\beta(\hat{P} - c(1+\tau))}{1+\tau}\right)\frac{sc}{P'(\hat{Q})(s+1+\hat{\epsilon})} + \frac{(1-\beta)\hat{P}\hat{Q}}{(1+\tau)^2}\left(1 - \frac{sc(1+\tau)}{(s+1+\hat{\epsilon})\hat{P}}\right). \tag{C.5}$$

When $\beta = 1$, the second term in (C.5) is zero, i.e., the terms-of-trade motive vanishes. Only the harmful effect of the tariff – output reduction – remains. An import subsidy raises Home welfare by increasing output. Indeed we find that $\frac{\mathrm{d}W_H}{\mathrm{d}\tau}\big|_{\beta=1} = \frac{sc(\hat{P}-c)}{P'(\hat{Q})(s+1+\hat{\epsilon})} < 0$ for all $\tau \geq 0$, which explains why an import subsidy is optimal in this case ($\tau(1) < 0$). When $\beta = 0$, optimal tariff is strictly positive ($\tau(0) > 0$) since $\frac{\mathrm{d}W_H}{\mathrm{d}\tau}\big|_{\beta=0} = \frac{sc\tau\hat{P}}{P'(\hat{Q})(s+1+\hat{\epsilon})} + \frac{\hat{P}\hat{Q}}{(1+\tau)^2}\left(1 - \frac{sc(1+\tau)}{(s+1+\hat{\epsilon})\hat{P}}\right) > 0$ for all $\tau \leq 0$.

The above discussion suggests that Home's optimal tariff is positive when Home firms' bargaining power is low, and is negative when their bargaining power is high. Moreover, there is a range of values for β such that the optimal tariff is strictly decreasing in β. Formally, as in section 3.4, we get:

$$\mathrm{sgn}\frac{\mathrm{d}\tau(\beta)}{\mathrm{d}\beta} = \mathrm{sgn}\frac{\partial^2 W_H}{\partial\beta\partial\tau}.$$

From (C.5), we have that $\frac{\partial^2 W_H}{\partial\beta\partial\tau} = -\frac{\hat{P}\hat{Q}}{(1+\tau)^2} < 0$, and $\tau(\beta)$ is monotonically decreasing in β. Since $\tau(0) > 0$, $\tau(1) < 0$ and $\frac{\mathrm{d}\tau(\beta)}{\mathrm{d}\beta} < 0$, we have the following result:

Proposition C.1 *Let $\tau(\beta)$ denote the optimal ad valorem tariff. Then, the following holds:*

(i) There exists $\hat{\beta} \in (0,1)$ such that

$$\tau(\beta) \gtreqless 0 \quad \Leftrightarrow \quad \beta \lesseqgtr \hat{\beta}.$$

(ii) $\tau(\beta)$ is monotonically decreasing in β.

D Tariffs on Final Good and Intermediate Input

In section 3.4 we mentioned that an increase in bargaining power of Home firms decreases tariff on intermediate input but increases tariff on final good. Below we demonstrate this claim formally.

Consider an environment where Home specializes in final goods but Foreign produces and exports both final goods and intermediate inputs to Home. Suppose Home and Foreign respectively have m_1 and m_2 final-good producers who have identical production technologies. Foreign has n potential intermediate-input producers. Assume that $n > m(\equiv m_1+m_2)$ and the matching function is $s(m,n) = \min\{m,n\}$. This implies $s(m,n) = m$ and all final-good producers in Home and Foreign find upstream partners. As before Home firms' bargaining power is β vis-a-vis a Foreign firm. Here we also assume that all firms in the same country have equal bargaining power. Thus, when a Foreign final-good producer is matched with a Foreign intermediate-input producer, the surplus is equally split among them as they have the same bargaining power. A welfare maximizing Home government imposes a specific tariff of t^I and t^F on intermediate input and final good respectively.

Let $\hat{q}_1$ and $\hat{q}_2$ respectively denote the equilibrium output produced by a Home-Foreign pair and a Foreign-Foreign pair, and let $\hat{r}_1$ and $\hat{r}_2$ respectively denote the equilibrium unit price of intermediate input by each pair. Proceeding as in section 3.1 we can show that

$$\hat{q}_1 = -\frac{P(\hat{Q}) - c - t^I}{P'(\hat{Q})}, \quad \hat{r}_1 = (1-\beta)P(\hat{Q}) + \beta(c + t^I), \tag{D.1}$$

$$\hat{q}_2 = -\frac{P(\hat{Q}) - c - t^F}{P'(\hat{Q})}, \quad \hat{r}_2 = 0.5P(\hat{Q}) + 0.5(c + t^F), \tag{D.2}$$

where $Q = \hat{Q}$ uniquely solves the following:

$$(m_1 + m_2)P(Q) + P'(Q)Q = (m_1 + m_2)c + m_1 t^I + m_2 t^F. \tag{D.3}$$

Let $\pi_1 \equiv (\hat{P} - c - t^F)\hat{q}_1$ denote the equilibrium profit of a Home-Foreign pair where $\hat{P} = P(\hat{Q})$. As equilibrium output of a pair is decreasing in own cost, $\hat{q}_1$ is decreasing in t^I. Furthermore, $\hat{P} - t^I$ is decreasing in t^I for all logconcave demand functions considered in the paper. These imply that $\pi_1 \equiv (\hat{P} - c - t^I)\hat{q}_1$ is decreasing in t^I. An increase in t^F reduces $\hat{q}_2$ and raises $\hat{q}_1$ (since outputs are strategic substitutes under Assumption 1'). Furthermore, $\hat{P}$ is increasing in t^F. These imply that $\pi_1 \equiv (\hat{P} - c - t^I)\hat{q}_1$ is increasing in t^F. Thus,

$$\frac{\partial \pi_1}{\partial t^I} < 0, \quad \frac{\partial \pi_1}{\partial t^F} > 0. \tag{D.4}$$

Now let us turn to welfare. Home welfare, denoted by W_H is given by:

$$W_H = \underbrace{\int_0^{\hat{Q}(t^I,t^F)} P(y)\mathrm{d}y - \hat{P}(t^I,t^F)\hat{Q}(t^I,t^F)}_{\text{Consumer surplus}} + \underbrace{\beta m_1 \pi_1(t^I,t^F)}_{\text{Home profits}} + \underbrace{t^I m_1 \hat{q}_1(t^I,t^F) + t^F m_2 \hat{q}_2(t^I,t^F)}_{\text{Tariff revenues}}$$

First assume that t^F is exogenously given and thus the Home government sets only t^I to maximize welfare. Differentiating W_H with respect to t^I we get

$$\frac{\partial W_H}{\partial t^I} = -P'(\hat{Q})\hat{Q}\frac{\partial \hat{Q}}{\partial t^I} + \beta m_1 \frac{\partial \pi_1}{\partial t^I} + m_1 \hat{q}_1 + t^I m_1 \frac{\partial \hat{q}_1}{\partial t^I} + t^F m_2 \frac{\partial \hat{q}_2}{\partial t^I}. \tag{D.5}$$

Solving $\frac{\partial W_H}{\partial t^I} = 0$ yields the optimal tariff on intermediate input which we denote as $t^I(t^F, \beta)$. As in section 3.4, we find that

$$\mathrm{sgn}\frac{\mathrm{d}t^I(t^F,\beta)}{\mathrm{d}\beta} = \mathrm{sgn}\frac{\partial^2 W_H}{\partial \beta \partial t^I} = m_1 \mathrm{sgn}\frac{\partial \pi_1}{\partial t^I},$$

where the last equality comes from (D.5). Since $\frac{\partial \pi_1}{\partial t^I} < 0$ from (D.4), it follows that

$$\frac{\mathrm{d}t^I(t^F,\beta)}{\mathrm{d}\beta} < 0. \tag{D.6}$$

Now assume that t^I is exogenously given and thus the Home government sets only t^F to maximize welfare. Differentiating W_H with respect to t^F we get

$$\frac{\partial W_H}{\partial t^F} = -P'(\hat{Q})\hat{Q}\frac{\partial \hat{Q}}{\partial t^F} + \beta m_1 \frac{\partial \pi_1}{\partial t^F} + m_2 \hat{q}_2 + t^F m_2 \frac{\partial \hat{q}_2}{\partial t^F} + t^I m_1 \frac{\partial \hat{q}_1}{\partial t^F}. \tag{D.7}$$

Let $t^F(t^I, \beta)$ denote the optimal tariff on final good, i.e., the value of t^F that solves $\frac{\partial W_H}{\partial t^F} = 0$. Using analogous reasoning as before, we find that

$$\mathrm{sgn}\frac{\mathrm{d}t^F(t^I,\beta)}{\mathrm{d}\beta} = \mathrm{sgn}\frac{\partial^2 W_H}{\partial \beta \partial t^F} = m_1 \mathrm{sgn}\frac{\partial \pi_1}{\partial t^F}.$$

Since $\frac{\partial \pi_1}{\partial t^F} > 0$ from (D.4), it follows that

$$\frac{\mathrm{d}t^F(t^I,\beta)}{\mathrm{d}\beta} > 0. \tag{D.8}$$

Proposition D.1 below restates (D.6) and (D.8) in words.

Proposition D.1

(i) *Optimal Home tariff on intermediate inputs is strictly decreasing in β when Home tariff on final goods is exogenously given.*

(ii) *Optimal Home tariff on final goods is strictly increasing in β when Home tariff on intermediate inputs is exogenously given.*

How does an exogenous reduction in t^F affect optimal t^I? Or, how does an exogenous increase in t^I affect optimal t^F? To answer these questions we need to know whether t^I and t^F are complements ($\frac{\partial^2 W_H}{\partial t^I \partial t^F} > 0$) or substitutes ($\frac{\partial^2 W_H}{\partial t^I \partial t^F} < 0$). Assume that the inverse demand function is linear, i.e., $P = a - Q$. It is straightforward to show that Home welfare is given by

$$W_H = \frac{(m_1\hat{q}_1 + m_2\hat{q}_2)^2}{2} + \beta m_1 \hat{q}_1^2 + t^I m_1 \hat{q}_1 + t^F m_2 \hat{q}_2,$$

where

$$\begin{aligned}\hat{q}_1 &= \frac{a - c - (m_1+1)t^I + m_2 t^F}{m_1 + m_2 + 1},\\ \hat{q}_2 &= \frac{a - c - (m_1+1)t^F + m_1 t^I}{m_1 + m_2 + 1}.\end{aligned}$$

Using these expressions we find that

$$\frac{\partial^2 W_H}{\partial t^I \partial t^F} = \frac{3 + 2m_1 + 2m_2(1-\beta) - 2\beta}{(m_1 + m_2 + 1)^2} > 0.$$

Thus t^I and t^F are strategic complements; an exogenous reduction in t^F lowers t^I and vice versa. From a policy point of view, this finding suggests that successful negotiation on tariff reductions in one sector can prompt unilateral tariff reductions in the vertically linked sector.

E Alternative Bargaining

The main analysis in section 3 has assumed that each Home-Foreign pair i bargains simultaneously over the input price r_i and the level of output q_i. We can consider a variant of our model where each pair $i\,(= 1, 2, ..., s)$ first bargains over the input price r_i alone, and subsequently, each H_i chooses q_i taking $\boldsymbol{r} \equiv (r_1, r_2, ..., r_s)$ as given. This sequence is in the spirit of the right-to-manage model in the labor union literature, where firms and union first bargain over the wage, and subsequently, each firm chooses the employment level.

We consider a three-stage game where the sequence of events is as follows: (i) In stage 1, the Home government sets a tariff rate t per unit of imported input; (ii) Stage 2 involves bargaining between H_i and F_i in which each pair i bargains over the input price r_i, taking other pairs' input prices $\boldsymbol{r} = (r_1, r_2, ..., r_s)$ as given; (iii) Stage 3 involves Cournot competition in the final-good sector in which each H_i chooses its own output, q_i, taking the bargained prices of the intermediate input and other Home firms' output as given.

E.1 Bargaining

Consider first the third-stage Cournot competition. Each Home firm H_i chooses the quantity of output q_i to maximize $\left[P\left(q_i + \sum_{j\neq i}^{s} \hat{q}_j\right) - r_i\right] q_i$, where r_i is given from the second stage. Under Assumption 1 or 1', there exists a unique equilibrium $\hat{\boldsymbol{q}} \equiv (\hat{q}_1, \hat{q}_2, ..., \hat{q}_s)$ where

$$\hat{q}_i = -\frac{P(\hat{Q}) - r_i}{P'(\hat{Q})}, \quad i = 1, 2, ..., s \tag{E.1}$$

and $Q = \hat{Q}$ solves the following:

$$sP(\hat{Q}) + P'(\hat{Q})\hat{Q} = \sum_{j=1}^{s} r_j. \tag{E.2}$$

In the second stage, Nash bargaining between H_i and F_i determines the input price so that $\hat{r}_i = \arg\max_{r_i}[(P(\hat{Q}) - r_i)]^{\beta}[(r_i - c - t)]^{1-\beta}\hat{q}_i$. We can show that there exists a symmetric equilibrium $\hat{r}_1 = ... = \hat{r}_s \equiv \hat{r}$ where

$$\hat{r} = \frac{P(\hat{Q}) + B(c+t)}{1+B}, \tag{E.3}$$

and

$$B \equiv \frac{\beta + 1 - \frac{s(1+\beta)+\epsilon}{s(s+1+\epsilon)}}{1-\beta} > 0.$$

Note that (E.1) and (E.3) are respectively corresponding to (3.1) and (3.2) in the original model. Rearranging (E.3) yields

$$\frac{P(\hat{Q}) - \hat{r}}{\hat{r} - c - t} = B. \tag{E.3'}$$

Recall from (3.2') that the right-hand side of (E.3') equals $\frac{\beta}{1-\beta}$ (i.e., the ratio of bargaining power between Home and Foreign firms) in the original model. Loosely speaking, thus, B can be viewed as the modified ratio of bargaining power between Home and Foreign firms. The following lemma records some comparative statics results (with respect to bargaining power and tariff rates).

Lemma E.1

(i) *For a given tariff rate t, the aggregate output $\hat{Q}$ is increasing in β and the final-good price $\hat{P} \equiv P(\hat{Q})$ is decreasing in it; i.e., $\partial\hat{Q}/\partial\beta > 0$ and $\partial\hat{P}/\partial\beta < 0$.*

(ii) *For a given bargaining power β, an increase in the tariff rate lowers output and raises prices; i.e., $\partial\hat{Q}/\partial t < 0$, $\partial\hat{P}/\partial t > 0$ and $\partial\hat{r}/\partial t > 0$.*

While $\hat{Q}$ is independent of β in the original model, it is increasing in β in the current setup. In contrast, the signs of $\frac{\partial\hat{Q}}{\partial t}$ and $\frac{\partial\hat{r}}{\partial t}$ are the same as in the original model.

E.2 Tariffs

In the first stage, the Home government chooses a tariff rate t to maximize Home welfare, which is exactly the same as the original model. Hence tariff gives rise to two opposing effects (a tariff-induced output reduction effect and a terms-of-trade improvement effect) on Home welfare in the current model. However, since the nature of alternative bargaining has a different impact on the equilibrium variables through B that measures the relative bargaining strengths of Home and Foreign firms, the optimal tariff is also affected by B.

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ and simplifying, we have the following proposition.

Proposition E.1

Let $t(\beta)$ denote the optimal tariff. At $t = t(\beta)$ the following holds:

$$t = -\frac{P'(\hat{Q})\hat{Q}}{sB}(-B + 1 + \epsilon). \tag{E.4}$$

$\hat{Q}$ and $\hat{\epsilon}$ respectively are the aggregate output and elasticity of slope evaluated at $t = t(\beta)$. Furthermore,

(i) *there exists $\hat{\beta}^{AB}$ such that*

$$t(\beta) \gtreqless 0 \quad \Leftrightarrow \quad \beta \lesseqgtr \hat{\beta}^{AB} \equiv \frac{1 + s\epsilon + \epsilon + \epsilon^2}{2s + 1 + s\epsilon + 3\epsilon + \epsilon^2},$$

(ii) *$t(\beta)$ is monotonically decreasing in β.*

Note that $\hat{\beta}^{AB}$ and $\hat{\beta}$ are usually different. For instance, consider linear demand ($\epsilon = 0$). Substituting $\epsilon = 0$ gives $\hat{\beta}^{AB} = \frac{1}{2s+1}$, whereas the corresponding cutoff is $\hat{\beta} = \frac{1}{2}$ in the original model. Consequently, for all $s \geq 1$, a tariff is less likely to improve welfare in the alternative bargaining setup.

Appendix to the Supplementary Note

Proofs for Section B

B.1 Proof of Proposition B.1

Consider (i) first. Simplifying the first-order conditions, $\frac{\partial W_H}{\partial t_H} = 0$ and $\frac{\partial W_F}{\partial t_F} = 0$, gives:

$$t_H = -\frac{P'(\hat{Q}(t_H, t_F))\hat{Q}(t_H, t_F)(2+\hat{\epsilon})}{s}\left(\frac{1+\hat{\epsilon}}{2+\hat{\epsilon}} - \beta\right), \tag{B.1}$$

$$t_F = -\frac{P'(\hat{Q}(t_H, t_F))\hat{Q}(t_H, t_F)(2+\hat{\epsilon})}{s}\left(\frac{1+\hat{\epsilon}}{2+\hat{\epsilon}} - (1-\beta)\right), \tag{B.2}$$

where $\hat{\epsilon} \equiv \epsilon(\hat{Q}(t_H, t_F))$. Clearly $t_H = t_H(\beta)$ and $t_F = t_F(\beta)$ satisfy (B.1) and (B.2). Subtracting (B.2) from (B.1) gives

$$t_H - t_F = -\frac{P'(\hat{Q}(t_H, t_F))\hat{Q}(t_H, t_F)(2+\hat{\epsilon})}{s}(1-2\beta).$$

Since $-P'(t_H, t_F)\hat{Q}(t_H, t_F)(2+\hat{\epsilon}) > 0$, the above equation implies (i).

For the proof of (ii), totally differentiating $\partial W_H/\partial t_H = 0$ and $\partial W_F/\partial t_F = 0$ and rewriting them in a matrix form gives

$$\begin{bmatrix} \frac{\partial^2 W_H}{\partial t_H^2} & \frac{\partial^2 W_H}{\partial t_H \partial t_F} \\ \frac{\partial^2 W_F}{\partial t_F \partial t_H} & \frac{\partial^2 W_F}{\partial t_F^2} \end{bmatrix} \begin{bmatrix} \frac{\mathrm{d}t_H}{\mathrm{d}\beta} \\ \frac{\mathrm{d}t_F}{\mathrm{d}\beta} \end{bmatrix} = \begin{bmatrix} -\frac{\partial^2 W_H}{\partial t_H \partial \beta} \\ -\frac{\partial^2 W_F}{\partial t_F \partial \beta} \end{bmatrix}.$$

Applying Cramer's rule, we get

$$\frac{\mathrm{d}t_H}{\mathrm{d}\beta} = \frac{\left(-\frac{\partial^2 W_H}{\partial t_H \partial \beta}\right)\left(\frac{\partial^2 W_F}{\partial t_F^2}\right) - \left(\frac{\partial^2 W_H}{\partial t_H \partial t_F}\right)\left(-\frac{\partial^2 W_F}{\partial t_F \partial \beta}\right)}{\Delta}, \quad \frac{\mathrm{d}t_F}{\mathrm{d}\beta} = \frac{\left(\frac{\partial^2 W_H}{\partial t_H^2}\right)\left(-\frac{\partial^2 W_F}{\partial t_F \partial \beta}\right) - \left(-\frac{\partial^2 W_H}{\partial t_H \partial \beta}\right)\left(\frac{\partial^2 W_F}{\partial t_F t_H}\right)}{\Delta},$$

where $\Delta \equiv \left(\frac{\partial^2 W_H}{\partial t_H^2}\right)\left(\frac{\partial^2 W_F}{\partial t_F^2}\right) - \left(\frac{\partial^2 W_H}{\partial t_H \partial t_F}\right)\left(\frac{\partial^2 W_F}{\partial t_F \partial t_H}\right) > 0$ follows from the stability condition. It is easy to check that

$$\frac{\partial^2 W_H}{\partial t_H \partial \beta} = -\frac{(2+\hat{\epsilon})\hat{Q}(t_H, t_F)}{s+1+\hat{\epsilon}} < 0, \quad \frac{\partial^2 W_F}{\partial t_F \partial \beta} = \frac{(2+\hat{\epsilon})\hat{Q}(t_H, t_F)}{s+1+\hat{\epsilon}} > 0.$$

Substituting these expressions in $\frac{\mathrm{d}t_H}{\mathrm{d}\beta}$ and $\frac{\mathrm{d}t_F}{\mathrm{d}\beta}$ above, we find that

$$\frac{\mathrm{d}t_H}{\mathrm{d}\beta} = \frac{\left(\frac{(2+\hat{\epsilon})\hat{Q}(t_H, t_F)}{s+1+\hat{\epsilon}}\right)\left(\frac{\partial^2 W_F}{\partial t_F^2} + \frac{\partial^2 W_H}{\partial t_H \partial t_F}\right)}{\Delta}, \quad \frac{\mathrm{d}t_F}{\mathrm{d}\beta} = \frac{\left(-\frac{(2+\hat{\epsilon})\hat{Q}(t_H, t_F)}{s+1+\hat{\epsilon}}\right)\left(\frac{\partial^2 W_H}{\partial t_H^2} + \frac{\partial^2 W_F}{\partial t_F \partial t_H}\right)}{\Delta}.$$

It follows that $\frac{\mathrm{d}t_H}{\mathrm{d}\beta} < 0$ and $\frac{\mathrm{d}t_F}{\mathrm{d}\beta} > 0$ as long as $\frac{\partial^2 W_i}{\partial t_i^2} + \frac{\partial^2 W_j}{\partial t_j \partial t_i} < 0$ for $i \neq j \in \{H, F\}$. □

Proofs for Section C

C.1 Proof of Lemma C.1

Proof: (i) It immediately follows from noting that $\hat{Q}$, i.e. the value of Q that solves (C.3) does not depend on β.

(ii) Totally differentiating (C.2) and (C.3) yields

$$\begin{aligned}
\frac{\mathrm{d}\hat{Q}}{\mathrm{d}\tau} &= \frac{sc}{P'(\hat{Q})(s+1+\hat{\epsilon})}, \\
\frac{\mathrm{d}\hat{P}}{\mathrm{d}\tau} &= P'(\hat{Q})\frac{\mathrm{d}\hat{Q}}{\mathrm{d}\tau} = \frac{sc}{s+1+\hat{\epsilon}}, \\
\frac{\mathrm{d}\hat{r}}{\mathrm{d}\tau} &= (1-\beta)\frac{\mathrm{d}\hat{P}}{\mathrm{d}\tau} + \beta c = \frac{c(s+\beta(1+\hat{\epsilon}))}{s+1+\hat{\epsilon}},
\end{aligned}$$

where $\hat{\epsilon}$ is

$$\epsilon(Q) \equiv \frac{P''(Q)Q}{P'(Q)}$$

evaluated at $Q=\hat{Q}$. From Assumption 1' it follows that $\epsilon \geq -1$, which in turn implies that $s+1+\hat{\epsilon}>0$. The claim directly follows from noting that $P'(\hat{Q})<0$ and $s+\beta(1+\hat{\epsilon})>0$.

(iii) We have that $\hat{R} = \frac{\hat{r}}{1+\tau} = (1-\beta)\frac{\hat{P}}{1+\tau} + \beta c$ Differentiating the above expression with respect to τ and rearranging we get

$$\frac{\mathrm{d}\hat{R}}{\mathrm{d}\tau} = (1-\beta)\frac{\mathrm{d}\frac{\hat{P}}{1+\tau}}{\mathrm{d}\tau} = \frac{(1-\beta)\hat{P}}{(1+\tau)^2}\left(\frac{sc(1+\tau)}{(s+1+\hat{\epsilon})\hat{P}} - 1\right).$$

In equilibrium $c(1+\tau)<\hat{P}$ must hold. Furthermore, if $1+\hat{\epsilon}>0$, we have that $\frac{s}{s+1+\hat{\epsilon}}<1$ and thus $\frac{sc(1+\tau)}{(s+1+\hat{\epsilon})\hat{P}}<1$, which implies the result. □

Proofs for Section E

E.1 Proof of Lemma E.1

In this proof, we first show that the modified ratio of bargaining power between Home and Foreign firms, $B = \frac{\beta+1-\frac{s(1+\beta)+\epsilon}{s(s+1+\epsilon)}}{1-\beta}$, is strictly increasing in β. Using this result, we next detail proofs of Lemma E.1.

Rewrite B as

$$B = \frac{(1+\beta)s(s+\epsilon)-\epsilon}{(1-\beta)s(s+1+\epsilon)}.$$

Since $s(s+1+\epsilon)(>0)$ is independent of β, $\operatorname{sgn}\frac{\mathrm{d}B}{\mathrm{d}\beta} = \operatorname{sgn}\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}\beta}$, where $\mathcal{B} \equiv \frac{(1+\beta)s(s+\epsilon)-\epsilon}{1-\beta}$. Differentiating $\mathcal{B}$ with respect to β gives

$$\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}\beta} = \frac{2s(s+\epsilon)-\epsilon}{(1-\beta)^2}.$$

Since the numerator in the right-hand side of the above equation is increasing in s, it suffices for the desired result to show that $\left.\frac{\mathrm{d}\mathcal{B}}{\mathrm{d}\beta}\right|_{s=1} = \frac{2+\epsilon}{(1-\beta)^2} > 0$.

Next, we turn to proofs of Lemma E.1:

(i) Since $\frac{\partial\hat{Q}}{\partial\beta} = \frac{\partial\hat{Q}}{\partial B}\frac{\mathrm{d}B}{\mathrm{d}\beta}$ and $\frac{\mathrm{d}B}{\mathrm{d}\beta}>0$, $\operatorname{sgn}\frac{\partial\hat{Q}}{\partial\beta} = \operatorname{sgn}\frac{\partial\hat{Q}}{\partial B}$. Substituting (E.3) into (E.2) and rewriting it, we get

$$sBP(\hat{Q}) + (1+B)P'(\hat{Q})\hat{Q} - sB(c+t) = 0.$$

Differentiating this equation with respect to B gives

$$\begin{aligned}
\frac{\partial\hat{Q}}{\partial B} &= \frac{\hat{Q}}{B[1+\epsilon+B(s+1+\epsilon)]}, \\
\frac{\partial\hat{P}}{\partial B} &= P'(\hat{Q})\frac{\partial\hat{Q}}{\partial B} = \frac{P'(\hat{Q})\hat{Q}}{B[1+\epsilon+B(s+1+\epsilon)]}, \\
\frac{\partial\hat{r}}{\partial B} &= \frac{P'(\hat{Q})}{B(1+B)}\left[\frac{1}{s} + \frac{1}{1+\epsilon+B(s+1+\epsilon)}\right].
\end{aligned}$$

Then, the result follows from noting that $B>0$, $P'(\hat{Q})<0$, and $1+\epsilon>0$.

(ii) Differentiating (E.2) and (E.3) with respect to t gives

$$\frac{\partial \hat{Q}}{\partial t} = \left[\frac{s}{(s+1)P'(\hat{Q}) + P''(\hat{Q})\hat{Q}}\right]\frac{\partial \hat{r}}{\partial t}, \quad \frac{\partial \hat{r}}{\partial t} = \frac{1}{1+B}\left\{P'(\hat{Q})\frac{\partial \hat{Q}}{\partial t} + B\right\}.$$

Solving for $\frac{\partial \hat{Q}}{\partial t}$ and $\frac{\partial \hat{r}}{\partial t}$ from the above expressions, we get

$$\begin{aligned}
\frac{\partial \hat{Q}}{\partial t} &= \frac{sB}{P'(\hat{Q})(1+\epsilon+B(s+1+\epsilon))}, \\
\frac{\partial \hat{P}}{\partial t} &= P'(\hat{Q})\frac{\partial \hat{Q}}{\partial t} = \frac{sB}{1+\epsilon+B(s+1+\epsilon)}, \\
\frac{\partial \hat{r}}{\partial t} &= \frac{(s+1+\epsilon)B}{1+\epsilon+B(s+1+\epsilon)}.
\end{aligned}$$

As in (i), the claim directly follows from noting that $B > 0$, $P'(\hat{Q}) < 0$, and $1+\epsilon > 0$. □

E.2 Proof of Proposition E.1

First, we derive the optimal tariff. Differentiating W_H with respect to t gives

$$\left.\frac{\mathrm{d}W_H}{\mathrm{d}t}\right|_{t=t(\beta)} = \hat{Q}\left[\frac{-B+1+\epsilon}{1+\epsilon+B(s+1+\epsilon)}\right] + t\left[\frac{sB}{P'(\hat{Q})(1+\epsilon+B(s+1+\epsilon))}\right].$$

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t}\big|_{t=t(\beta)} = 0$ and simplifying, we have (E.4).

Next, we turn to the properties of the optimal tariff:
(i) Given $P'(\hat{Q}) < 0$, it immediately follows from (E.4) that

$$t \gtreqless 0 \iff B \lesseqgtr 1+\epsilon \iff \beta \lesseqgtr \hat{\beta}^{AB},$$

where the last relationship comes from the proof of Proposition E.1.

(ii) Note $\frac{\mathrm{d}t}{\mathrm{d}\beta} = \frac{\mathrm{d}t}{\mathrm{d}B}\frac{\mathrm{d}B}{\mathrm{d}\beta}$. We have already shown that $\frac{\mathrm{d}B}{\mathrm{d}\beta} > 0$, which means that $\mathrm{sgn}\frac{\mathrm{d}t}{\mathrm{d}\beta} = \mathrm{sgn}\frac{\mathrm{d}t}{\mathrm{d}B}$. Total differentiation of (E.4) with respect to B gives

$$\frac{\mathrm{d}t}{\mathrm{d}B} = -\frac{P'(\hat{Q})\hat{Q}}{sB^2}\left[(1+\epsilon-B)\frac{\mathrm{d}\hat{Q}}{\mathrm{d}B}\frac{B}{\hat{Q}} - 1\right](1+\epsilon). \tag{E.5}$$

Moreover, from (E.1) and (E.2), we get

$$\frac{\mathrm{d}\hat{Q}}{\mathrm{d}B} = \frac{s}{P'(\hat{Q})(s+1+\epsilon)}\frac{\mathrm{d}r}{\mathrm{d}B}, \quad \frac{\mathrm{d}\hat{r}}{\mathrm{d}B} = \frac{P'(\hat{Q})}{1+B}\left(\frac{\mathrm{d}\hat{Q}}{\mathrm{d}B} + \frac{\hat{Q}}{sB} + \frac{B}{P'(\hat{Q})}\frac{\mathrm{d}t}{\mathrm{d}B}\right). \tag{E.6}$$

Substituting (E.6) into (E.5) and evaluating $t = t(\beta)$, we get:

$$\frac{\mathrm{d}t}{\mathrm{d}B} = \left\{\frac{(1+\epsilon)(s+2+\epsilon)}{sB^2[(1+\epsilon)(2+\epsilon)+sB]}\right\}P'(\hat{Q})\hat{Q}. \tag{E.7}$$

The result follows from noting that $1+\epsilon > 0$, $s+2+\epsilon > 0$, $B > 0$, and $P'(\hat{Q}) < 0$. □