Tariffs, Vertical Specialization and Oligopoly[*]

Tomohiro Ara[†]　　　　Arghya Ghosh[‡]

Fukushima University　　University of New South Wales

September 2015

Abstract

We examine optimal tariffs in an environment with vertical specialization where the Home country specializes in final goods and the Foreign country specializes in intermediate inputs. A matched Home-Foreign pair bargains simultaneously over the input price and the level of output, and competes *à la* Cournot with other matched pairs in markets. We find that the optimal Home tariff rate is strictly decreasing in the bargaining power of Home firms, and an increase in the Home firms' bargaining power might therefore raise Foreign profits. Under an endogenous market structure with entry followed by matching, the relationship between bargaining power and output is non-monotone if the demand function is strictly concave or convex. This in turn induces a non-monotone relationship between the optimal tariff and bargaining power for a class of demand functions. For linear demand, free trade is optimal irrespective of bargaining power. We show that non-monotonicity result is retained under endogenous bargaining power.

Keywords: Tariffs, Oligopoly, Bargaining Power, Outsourcing, Free Entry
JEL Classification Numbers: F12, F13

[†]Faculty of Economics and Business Administration, Fukushima University, Fukushima 960-1296, Japan. *Email address*: tomohiro.ara@gmail.com

[‡]*Corresponding author*: School of Economics, UNSW Business School, University of New South Wales, Sydney, NSW, 2052, Australia. *Email address*: a.ghosh@unsw.edu.au

1 Introduction

The fragmentation of production chains and vertical specialization have led to rapid growth in intermediate input trade in recent years (Hummels, Ishii, and Yi, 2001; Yeats, 2001; Yi, 2003), largely as a result of lower tariffs accompanied by trade liberalization across countries. This input trade growth takes place largely through foreign outsourcing rather than through foreign direct investment (FDI). Hanson, Mataloni, and Slaughter (2005), for example, find that FDI undertaken by U.S. multinationals has grown very rapidly, yet somewhat less than foreign outsourcing by U.S. firms. Documenting the enormous growth of manufacturing exports from China, Spencer (2005) shows that processing exports, which occur through international outsourcing between foreign buyers and independent Chinese subcontractors, constitute a large part of those manufacturing exports.[1]

The increasing importance of international outsourcing and FDI in input trade gives rise to new models of vertical specialization which embed organizational structures in an imperfectly competitive framework. Though these new models have matched positive features of reality quite well, barring a few papers (discussed later in the Introduction), little attention has been paid to the welfare implications and trade policy in these models with vertical relationships that fragment the production process across countries.[2] Our paper takes a step towards filling this gap by explicitly considering tariffs in this setting.

Presence of cross-border vertical relationships softens the mercantilist us-versus-them argument that underpins much of the discussion on trade policy and trade agreements. As production stages become more fragmented across the globe, countries' trade interests become more aligned. Thus, cross-border vertical relationships are expected to act towards lowering trade barriers. For example, in the presence of vertical FDI, governments of source countries have an incentive to improve market access for imported inputs from foreign affiliates of source country firms (Blanchard, 2007). Using U.S. firm-level data on foreign affiliate activity and detailed measures of U.S. trade policy, Blanchard and Matschke (forthcoming) provide evidence that is indeed the case. Our work is complementary to these papers in that we also consider trade policy in the presence of vertical linkages. However, our primary focus is on outsourcing rather than FDI. Furthermore, while the analysis of our basic framework (section 3) delivers a similar message, i.e., vertical linkages can somewhat blunt the terms-of-trade motive for tariff, the effects are more nuanced in the enriched framework (sections 4 and 5) with bargaining, matching and entry-exit considerations.

---

[1]See Jones, Kierzkowski, and Lurong (2005) and Kimura and Ando (2005) for further evidence on the shift from intra-firm trade to arm's length trade in fragmented goods. Kimura and Ando (2005, Table 8), for example, find in Japanese multinationals that the share of intra-firm transactions in total purchases in East Asia decreased from 44% to 33%, while the share of arm's length transactions increased from 52% to 65% during 1995–1998.

[2]For instance, in a survey of the recent literature, Antràs and Rossi-Hansberg (2009, p.61) note that "although the literature on organizations and trade has been largely concerned with matching positive features of reality ..... much less attention has been given to the normative and policy implications of changes in the international organization of production."

1

Like horizontal specialization (e.g., a classical Ricardian world), vertical specialization creates gains from trade. We take the existence of these gains and the pattern of specialization as given. Without loss of generality, we assume that a Foreign country specializes in intermediate-input production whereas a Home country specializes in final-good production, importing inputs from the Foreign country. To facilitate exposition, we hereafter use Foreign and Home for the Foreign country and the Home country respectively. Firms first enter incurring a fixed cost and then seek partners from other stages of production. A matched Home-Foreign pair then bargains simultaneously over the input price and the level of output, and compete with other matched pairs in a Cournot market.

While the existence of input price allows us to draw analogy to terms-of-trade in the trade literature, effectively we consider a generalized Nash bargaining framework between Home and Foreign firms. The surplus generated by each pair is split between Home and Foreign firms according to their bargaining power. Bargaining strength of Home and Foreign firms affects the distribution of gains from trade within matched Home-Foreign pairs, which in turn affects welfare of Home and its trade policy. Bargaining power of Home firms can depend on a host of factors including relative size of Home and Foreign, number of Home and Foreign firms, and share of Home in Foreign export basket among others. Trade agreements of Home and Foreign with other countries also play a role in determining the firms' outside options which in turn affect the bargaining outcomes. Possible determinants of bargaining power (including the ones described above) and empirical evidence on the links between bargaining power and trade policy have been discussed in Limão (2006) and Olarreaga and Özden (2005). We discuss the existing evidence on bargaining power and trade policy and its relationship with our findings in section 3.4.

We remain agnostic regarding the source of bargaining power for most parts of the paper, especially in sections 3 and 4. In section 3, we assume that the bargaining power is exogenously given and the same for all Home firms. Exogenous bargaining power is apt for section 3 where the number of firms is fixed. The same bargaining power for all firms is plausible as well since all downstream firms belong to the same country (Home), all upstream firms belong to the same country (Foreign), and all firms in each country have identical technologies. In section 4, we allow for free entry of Home and Foreign firms but still retain the assumption of exogenous bargaining power. While the assumption of exogenous bargaining power is not ideal in the presence of entry and exit, it allows us to separate the effects arising due to endogenous market structure from the effects arising from endogenous bargaining power. In a matching environment, the number of Home firms (buyers of intermediate input) and Foreign firms (sellers of intermediate input) seems to be plausible determinants of bargaining power. In section 5, assuming that each buyer's bargaining power varies negatively with the number of buyers and positively with the number of sellers, we show that the results from section 4 (with exogenous bargaining power) go through with suitable restrictions on the bargaining power functions.

When the market structure is exogenously given (i.e., the number of matched pairs is fixed), optimal Home tariff is strictly positive when Home firms' bargaining power is low. A robust

finding is that optimal Home tariff is strictly decreasing in the bargaining power of Home firms. An increase in the Home tariff rate leads to a less-than-proportionate increase in the price of intermediate inputs for all demand functions that are strictly logconcave. Since Home imports intermediate inputs, this works as a terms-of-trade improvement for Home. Counteracting this welfare gain from the terms-of-trade improvement is welfare loss due to tariff induced reduction in output. These two effects are present in all oligopoly models, with or without vertical relationships. The difference lies in the effect of tariffs on Home profits. In single-stage oligopoly models, tariffs on imported final good raises Home profits as Home and Foreign firms produce substitutes. However, in models with vertical relationships like ours, tariffs on intermediate input reduces Home profits since the intermediate input and the final good are complements in production. When Home firms have no bargaining power, for example, their profits are nil and only the terms-of-trade motive is present, which induces the Home government to set a positive tariff rate on imports. As the bargaining power of Home firms increases, so do Home firms' profits, but the adverse effect of a tariff on Home profits increases and hence the optimal tariff declines accordingly. In the extreme case where Home firms have full bargaining power, the optimal tariff becomes negative.

An interesting by-product of our analysis is the possibility of a positive relationship between Home bargaining power and Foreign profits. Because of the induced lower tariffs, we find that an increase in Home bargaining power not only benefits Home firms but it can also raise the profits of Foreign firms.

The story is different when the market structure is endogenously determined through free entry and random matching. Free entry implies that the expected Home profit is zero and only the terms-of-trade effect remains. However, an improvement in terms-of-trade becomes less likely, since in addition to directly increasing prices, an increase in tariffs also indirectly increases prices by reducing the number of matched pairs. Due to the latter effect, we find that terms-of-trade do not necessarily improve for all logconcave demand functions; improvement in terms-of-trade only occurs for strictly concave demand functions, implying that the optimal tariff is positive (negative) for strictly concave (convex) demand. For the special case of linear demand, the optimal tariff is zero irrespective of the bargaining power of firms. In this setup, the importance of demand curvature notwithstanding, bargaining power plays a key role because it affects the thickness of the market – the number of firms entering in each stage of production. If the bargaining power of any one side is high, fewer firms enter on the other side of the market. Thus, compared to a case in which the bargaining power of Foreign and Home firms is similar, matching is worse and output is lower if the bargaining power between Home and Foreign firms differs significantly. This non-monotone relationship between output and bargaining power in turn leads to a non-monotone relationship between the optimal tariff and bargaining power for a class of logconcave demand functions. We show that this non-monotone relationship holds regardless of whether the bargaining power of respective firms is exogenous (in section 4) or endogenous (in section 5).

As far as we are aware, the only papers that examine trade policy in the context of trade in

intermediate products and international outsourcing (i.e. offshoring) are Ornelas and Turner (2008, 2012) and Antràs and Staiger (2012a). Ornelas and Turner analyze the effect of tariffs on an incomplete-contract setting with international outsourcing where Foreign suppliers make relationship-specific investments for the needs of Home firms. Prior to investments by Foreign suppliers, Home firms decide whether to vertically integrate Foreign suppliers by incurring a fixed cost. By affecting the investment levels and organizational choices, they show that a reduction in tariff can lead to a significant increase in trade volume, well beyond that which could be explained by standard trade models.[3] Contractual incompleteness and relationship-specific investments are also at the heart of Antràs and Staiger (2012a). Taking the possibility of offshoring in intermediate inputs as given, they provide the first analysis of trade agreements in the presence of offshoring.

As is clear from the description above, offshoring is also present in the current setup. To highlight the novel interaction between bargaining power and trade policy, we abstract away from contractual incompleteness and relationship-specific investments, thereby allowing for complete contracts. This is not to say however that incomplete contracts or relationship-specific investments are not important. In fact, one can presume that relationship-specific investments or switching costs could also exist in the background of our model, which prompts us to look beyond regular markets. We discuss in the concluding section how free entry and random matching in our framework might actually play a similar role with that of relationship-specific investments in an incomplete-contract framework.

In terms of modeling, our paper is closely related to Antràs and Staiger (2012b) who also explore a matching market with complete contracts. Like us, they consider random matching between sellers and buyers, and assume that there is no possibility of rematching so that the outside option is normalized to zero. They show that unilateral trade policies and the rationale for trade agreements arising from a bargaining setup are different from those arising from a market-competition setup. Despite the similarity in the assumption of contractual completeness and random matching, there are some key differences between their work and ours. First, we consider bargaining as a mode of transaction in the intermediate-input market rather than in the final-good market. Second, the two papers address fundamentally different policy issues. Antràs and Staiger unravel a subtle difference between market competition and bargaining in terms of their impact on trade agreements. While our focus is narrower in the sense that we do not consider strategic interactions in the policy space, we illustrate how bargaining power can play an important role in affecting trade policy. In the presence of an exogenous market structure, bargaining power in itself has no effect on output; it only affects output through its impact on trade policy. With an endogenous market structure, in contrast, bargaining power affects output in a non-monotone fashion, which in turn yields a non-monotone relationship

---

[3]In contrast to Ornelas and Turner (2008, 2012), we assume away firms' incentives to undertake intra-firm trade (FDI), i.e., firms' choices between vertical FDI and international outsourcing. However, unlike Ornelas and Turner, we consider endogenous market structure (see sections 4 and 5) and analyze how import tariffs affect the firms' entry incentives into the upstream and downstream markets.

between bargaining power and tariff.

A handful of papers have considered trade policy in the context of vertical oligopolies. In an international vertical oligopoly setting, Ishikawa and Lee (1997), Ishikawa and Spencer (1999), and Chen, Ishikawa and Yu (2004) analyze the strategic interaction between Foreign firms and Home firms and examine the effect of export subsidies for an imported intermediate input on social welfare. In somewhat different contexts, Spencer and Qiu (2001) and Qiu and Spencer (2002) also develop a model of informal procurement within the vertical relationship between Japanese automakers and Japanese parts suppliers (so-called "Keiretsu") and consider the effect of market-opening trade policy (e.g., voluntary import expansion). In these papers, however, bargaining power of vertically related firms does not play a key role in the analysis. In contrast, bargaining power is critical in our model as it affects how economic rents are distributed between Home firms and Foreign firms and hence it affects trade policy. Moreover, free entry and random matching, which are at the core of our analysis with an endogenous market structure, are not considered in these works.

2 Model

Consider a setting with two countries, Home and Foreign, specializing respectively in final-good production and in intermediate-input production. Foreign has n upstream firms, $F_1, F_2, ..., F_n$. Home has m downstream firms, $H_1, H_2, ..., H_m$; each procures the intermediate input from an upstream Foreign firm to produce the final good. There are two ways to procure an intermediate input: intra-firm trade (FDI) and arm's length trade (outsourcing). In both cases, contracts are used to specify the delivery of input and Home firms bargain with Foreign firms regarding the terms of their contracts.[4] As emphasized in the Introduction, much of the increase in trade in intermediate inputs can be attributed to the increase in outsourcing, and we thus restrict our attention to outsourcing in this paper.[5]

Upon entry, each firm seeks a partner from the other stage of production. Entry cost is K_H for Home firms and K_F for Foreign firms. Matching randomly occurs between Home and Foreign firms. We assume that one-to-one matching takes place in outsourcing.[6] Let $s = s(m, n)$ denote the number of pairs that are formed in this matching process, where $s(m, n) \leq \min\{m, n\}$

---

[4]Contract types used here are equivalent to *quantity-forcing contracts*, by which bargaining firms can choose the quantity that maximizes their joint profits, splitting them according to their respective bargaining power.

[5]Focusing on one mode of procurement is in line with the finding in the horizontal FDI literature with homogenous firms, where, in equilibrium, either all foreign firms choose FDI or all of them choose exports. An exception is Cole and Davies (2011). They develop a strategic tariff setting where foreign firms differ in fixed costs and both modes of entry – exports and FDI – arise in equilibrium: firms with low fixed costs choose FDI while firms with high fixed costs choose exports.

[6]Following Grossman and Helpman (2002), we focus on one-to-one matching for tractability. Whereas the previous literature assumes relationship-specific investments that lock firms into one-to-one relationships, our model abstracts from them. It is possible, however, to assume that such investments or switching costs exist in the background of our model; we elaborate on this in later sections where matching plays a crucial role.

and $s(m, n)$ is increasing in both of its arguments. More properties of this matching function are described in section 4.1.

One unit of the final good requires one unit of the intermediate input. The unit cost of production for the intermediate input is $c(> 0)$. We assume that after procuring intermediate inputs at a negotiated price, Home firms can transform these inputs to final products without incurring additional costs. This assumption is made for analytical simplicity and has no serious implications for our main results.

There is a unit mass of identical consumers with a quasi-linear utility function, $U(Q) + y$, where y is a competitively produced numeraire good and Q is a homogeneous final good produced using intermediate inputs. Assuming income to be high enough, maximizing $U(Q) + y$ subject to the budget constraint gives demand for the homogeneous product, $Q = Q(P)$, such that (i) $Q(P) = 0$ for $P \geq \bar{P}(> c)$, (ii) $Q(P)$ is twice continuously differentiable, and (iii) $Q'(P) < 0$ for all $P \in (0, \bar{P})$. These assumptions guarantee the existence of the Cournot equilibrium. We will often work with inverse demand functions. The above assumptions regarding $Q(P)$ imply that the inverse demand function $P = P(Q)$ is twice continuously differentiable and $P'(Q) < 0$ for all $Q \geq 0$.[7] For a sharper characterization, we assume that final goods are consumed only in Home and the Foreign government does not undertake trade policy, but none of the key results relies on these assumptions.

The timing of the game is as follows. First, the Home government sets a specific tariff rate, t, to maximize Home welfare which consists of consumer surplus, aggregate Home profits and tariff revenues. Second, Home and Foreign firms enter and random matching takes place between them. In section 3, we bypass the second stage and assume that the number of matched pairs is fixed. Thus, in this section, tariffs have no effect on the market structure. In sections 4 and 5, we assume that after observing tariff rates, firms enter the markets; then they search for a partner (from another stage of production) and $s = s(m, n)$ matched pairs are formed. Thus, in these sections, the market structure is endogenously affected by tariffs. Lastly, bargaining over the input price and the level of output takes place within a pair and Cournot competition occurs across matched pairs in the Home market.

3   Exogenous Market Structure

We first consider an environment where the market structure is given. Entry costs K_H and K_F have been sunk and matching has taken place. We treat the number of matched pairs $s = s(m, n)$ as fixed and invariant to the tariff rate. Formal proofs for all propositions and lemmas are relegated to the appendix.

---

[7]An important implicit assumption in our framework with complete specialization is prohibitive cost of Home (Foreign) in producing intermediate (final) goods. Let c_i and d_i respectively denote the unit cost of intermediate input and unit cost of transformation (from intermediate input to final good) where $i = H(ome), F(oreign)$. We have implicitly assumed that $c_F = c < \bar{P} \leq c_H$ and $d_H = 0 < \bar{P} \leq d_F$ which gives rise to complete specialization in our model.

3.1 Bargaining

Consider a third-stage bargaining game. Each pair i consisting of a Home firm H_i and a Foreign firm F_i bargains simultaneously over the terms of their contract (r_i, q_i), where a Home firm H_i purchases q_i units of the intermediate input from a Foreign firm F_i at the unit price of r_i, and then produces q_i units of the final product.

We characterize the outcome of the bargaining using a generalized Nash bargaining solution in which every Home firm has the same bargaining power denoted by $\beta \in [0, 1]$. The outcome of the third-stage bargaining is s pairs of input prices and quantities $(\hat{r}_i, \hat{q}_i)$ $(i = 1, 2, ..., s)$ which satisfy the following condition: $(r_i, q_i) = (\hat{r}_i, \hat{q}_i)$ is the Nash solution to the bargaining problem between H_i and F_i, given that both expect $(\hat{r}_j, \hat{q}_j)$ $(j \neq i)$ to be agreed upon between H_j and F_j. The relevant utility functions for the analysis of the bargaining are H_i's profit, $\pi_{H_i} \equiv \left[P\left(q_i + \sum_{j \neq i}^{s} q_j \right) - r_i \right] q_i$ and, F_i's profit, $\pi_{F_i} \equiv (r_i - c - t)q_i$. For simplicity, we assume that the disagreement point is zero for both parties.

Then, $(r_i, q_i) = (\hat{r}_i, \hat{q}_i)$ uniquely solves the following maximization problem:

$$(\hat{r}_i, \hat{q}_i) = \arg \max_{r_i, q_i} \left[\underbrace{\left\{ P\left(q_i + \sum_{j \neq i}^{s} \hat{q}_j \right) - r_i \right\} q_i}_{\pi_{H_i}} \right]^{\beta} \left[\underbrace{(r_i - c - t)q_i}_{\pi_{F_i}} \right]^{1 - \beta},$$

subject to

$$\pi_{H_i} \geq 0 \quad \text{and} \quad \pi_{F_i} \geq 0.[8]$$

The assumption below ensures that the solution to the maximization problem is unique.

Assumption 1 The demand function $Q(P)$ is logconcave.

The equivalent assumption in terms of inverse demand function is:

Assumption 1' $P'(Q) + QP''(Q) \leq 0$ for all $Q \geq 0$.

Assumption 1 holds if and only if marginal revenue is steeper than demand. In the trade literature, this assumption is first introduced in Brander and Spencer (1984a, b) who show that when the Home country imports from a Foreign monopolist with constant marginal cost, a small tariff improves welfare if and only if Assumption 1' holds.[9]

---

[8]The two constraints $\pi_{H_i} \geq 0$ and $\pi_{F_i} \geq 0$ imply that the outside option of each firm – Home as well as Foreign – is zero. Qualitatively, the results will not change if the outside option is not zero but a constant. One way to interpret the zero outside option is that if the firms are not matched, they earn zero profit in the Home market.

[9]Deneckre and Kovenock (1999) and Anderson and Renault (2003) show that strict concavity of $(Q(P))^{-1}$ (which is weaker than the logconcavity of $Q(P)$) is sufficient to ensure the uniqueness of the Cournot equilibrium. Nevertheless, we use Assumption 1 to ensure non-negative tariffs for some $\beta \geq 0$. In their work on dynamic merger review, Nocke and Whinston (2010) use a slightly stronger assumption than Assumption 1': $P'(Q) + QP''(Q) < 0$ for all $Q \geq 0$.

In our framework, in addition to guaranteeing uniqueness, Assumption 1' ensures that the optimal tariff is non-negative at least for some $\beta \geq 0$. A convenient way to state Assumption 1' is in terms of elasticity of slope which is defined as $\epsilon(Q) \equiv \frac{P''(Q)Q}{P'(Q)}$. Observe that $\epsilon(Q) \geq -1 \Leftrightarrow P'(Q) + QP''(Q) \leq 0$. This condition is sufficient to prove the main results. For a sharper characterization – see especially section 4.3 – we shall occasionally focus on a class of demand functions which not only satisfy Assumption 1 but also satisfy the following:

Assumption 2 $\gamma(Q) \equiv \frac{\epsilon'(Q)Q}{\epsilon(Q)} \leq 1 \Leftrightarrow \epsilon(Q) = \frac{P''(Q)Q}{P'(Q)} \geq \alpha(Q) \equiv \frac{P'''(Q)Q}{P''(Q)}$.

Assumption 2 implies that the curvature of inverse demand is greater than the curvature of slope of inverse demand for all $Q \geq 0$.[10] Any inverse demand function with constant elasticity of slope (e.g., linear, constant elasticity, and semi-log among others) satisfies Assumption 2 since $\epsilon(Q) - \alpha(Q) = 1$ holds whenever $\epsilon(Q)$ is constant (and both $\epsilon(Q)$ and $\alpha(Q)$ are well-defined).

Now back to bargaining. For all $s \geq 1$, the unique bargaining outcome is given by $\hat{r}_1 = ... = \hat{r}_s \equiv \hat{r}$ and $\hat{q}_1 = ... = \hat{q}_s \equiv \hat{q}$, where $\hat{r}\ (> 0)$ and $\hat{q}\ (> 0)$ are determined by (3.1) and (3.2) below:

$$\hat{q} = -\frac{P(\hat{Q}) - c - t}{P'(\hat{Q})}, \tag{3.1}$$

$$\hat{r} = (1 - \beta)P(\hat{Q}) + \beta(c + t), \tag{3.2}$$

where $Q = \hat{Q}$ uniquely solves the following:

$$sP(Q) + P'(Q)Q = s(c + t). \tag{3.3}$$

Solving the bargaining problem is equivalent to solving the following sequence of decisions. First, each H_i chooses q_i to maximize the joint profit:

$$\pi_i \equiv \pi_{H_i} + \pi_{F_i} = \left[P\left(q_i + \sum_{j \neq i}^{s} \hat{q}_j \right) - c - t \right] q_i.$$

This maximization problem yields (3.1). Then, each matched pair $i(= 1, 2, ..., s)$ divides this joint profit between themselves according to bargaining power which implies

$$\left(P(\hat{Q}) - \hat{r} \right) \hat{q} = \beta(P(\hat{Q}) - c - t)\hat{q};$$
$$(\hat{r} - c - t)\hat{q} = (1 - \beta) \left(P(\hat{Q}) - c - t \right) \hat{q}.$$

Canceling $\hat{q}$ from both sides of each equation and rewriting it, we get (3.2). Note that equation (3.2) can be written as

$$\frac{P(\hat{Q}) - \hat{r}}{\hat{r} - c - t} = \frac{\beta}{1 - \beta}, \tag{3.2'}$$

---

[10]Cowan (2007) shows that the ratio of the slope curvature to demand curvature ($\alpha(Q)/\epsilon(Q)$ in our setting) plays an important role in the welfare analysis, and a critical value of this ratio is 1.

which implies that the ratio of price-cost margin between Home and Foreign firms is exactly the same as the ratio of their bargaining power. The following lemma records some important comparative statics results for future reference.

Lemma 3.1

(i) *For a given tariff rate t, the aggregate output $\hat{Q}$ and the final-good price $\hat{P} \equiv P(\hat{Q})$ are independent of β; i.e., $\mathrm{d}\hat{Q}/\mathrm{d}\beta = \mathrm{d}\hat{P}/\mathrm{d}\beta = 0$.*

(ii) *For a given bargaining power β, an increase in the tariff rate lowers output and raises prices; i.e., $\mathrm{d}\hat{Q}/\mathrm{d}t < 0$, $\mathrm{d}\hat{P}/\mathrm{d}t > 0$ and $\mathrm{d}\hat{r}/\mathrm{d}t > 0$.*

(iii) *Let $r^* \equiv \hat{r} - t$ denote the price received by a Foreign firm in equilibrium (for each unit of the intermediate input). Then,*

$$\frac{\mathrm{d}r^*}{\mathrm{d}t} \lesseqgtr 0 \quad \Leftrightarrow \quad \frac{\mathrm{d}\hat{P}}{\mathrm{d}t} \lesseqgtr 1 \quad \Leftrightarrow \quad 1 + \epsilon(\hat{Q}(t)) \gtreqless 0.$$

Not surprisingly, $\hat{r}$ increases as t increases. However, $\frac{\mathrm{d}\hat{r}}{\mathrm{d}t} - 1 \leq 0$ or equivalently $\frac{\mathrm{d}r^*}{\mathrm{d}t} \leq 0$ as long as the demand is logconcave. For all such demand functions, the pass-through of tariff to an intermediate-input price faced by Home producers is less than complete. Foreign firms absorb part of the tariff increase which acts like a terms-of-trade gain for Home. While r^* is an input price internal to the firms, a reduction in r^* hurts Foreign firms and benefits Home firms. Hence, we refer to a decrease in r^* as an improvement in terms-of-trade in the paper, though we are aware that r^* is more like firms' terms-of-trade. The terms-of-trade improvement creates a rationale for Home to set a positive tariff.

Note that we introduce the concept of an input price since it can be interpreted in a similar fashion to the terms-of-trade. Nothing is lost – except for the interpretation – if each Home-Foreign pair jointly decides on the output level and uses generalized Nash bargaining to share the surplus. Unlike the vertical oligopoly models developed by Ishikawa and Lee (1997) and Ishikawa and Spencer (1999), there is no double marginalization effect at work in our model.

3.2 Tariffs

Let $\hat{Q}(t, \beta)$, $\hat{P}(t, \beta)$ and $\hat{r}(t, \beta)$ respectively denote the equilibrium output, the price of final goods and the price of intermediate inputs for a given t and β. Since the equilibrium output does not depend on β in the short run, we use $\hat{Q}(t)$ and $\hat{P}(t)$ to denote the equilibrium output and price respectively. In the first stage, the Home government chooses a tariff rate t to maximize Home welfare (W_H):

$$W_H \equiv \underbrace{\int_0^{\hat{Q}(t)} P(y)\mathrm{d}y - \hat{P}(t)\hat{Q}(t)}_{\text{Consumer surplus } (CS)} + \underbrace{\left(\hat{P}(t) - \hat{r}(t, \beta)\right)\hat{Q}(t)}_{\text{Home profits } (\Pi_H)} + \underbrace{t\hat{Q}(t)}_{\text{Tariff revenues } (TR)}.$$

Expressing $\hat{r}(t, \beta) - t$ as $r^*(t, \beta)$ and simplifying the above expression further gives

$$W_H \equiv \int_0^{\hat{Q}(t)} P(y)\mathrm{d}y - r^*(t, \beta)\hat{Q}(t).$$

Differentiating W_H with respect to t and rearranging, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = (\hat{P}(t) - r^*(t, \beta))\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} - \frac{\mathrm{d}r^*(t, \beta)}{\mathrm{d}t}\hat{Q}(t).$$

The first term captures the welfare loss due to the tariff-induced output reduction. Home consumers value the good at $\hat{P}(t)$ while effectively it costs $r^*(t, \beta)(< \hat{P}(t))$ to produce (from Home's perspective). This price-cost margin, $\hat{P}(t) - r^*(t, \beta)$, multiplied by the amount of output lost, $\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t}$, is the magnitude of welfare loss. The second term, $-\frac{\mathrm{d}r^*(t, \beta)}{\mathrm{d}t}\hat{Q}(t)$, captures the welfare gains arising from the terms-of-trade improvement ($\frac{\mathrm{d}r^*(t, \beta)}{\mathrm{d}t} < 0$). The optimal tariff rate strikes a balance between the two – welfare gains from the terms-of-trade improvement and welfare losses from the reduction in output. As we show below, the bargaining power parameter β plays an important role in delineating the relative importance of the two effects, which in turn helps to determine the sign of the optimal tariff.

Using the expression for $\frac{\mathrm{d}r^*(t, \beta)}{\mathrm{d}t}$ and the fact that $\hat{P}(t) - r^*(t, \beta) = \beta(\hat{P}(t) - c - t) + t$, we can express $\frac{\mathrm{d}W_H}{\mathrm{d}t}$ as follows:

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = \beta(\hat{P}(t) - c - t)\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} + (1 - \beta)\left(1 - \frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t}\right)\hat{Q}(t) + t\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t}. \tag{3.4}$$

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ and solving for t gives the expression for the optimal tariff which is presented later in Proposition 3.1. Here we focus on the sign of the optimal tariff. Using (3.4) and noting that $\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} < 0$ we find that the optimal tariff is strictly positive (negative) if and only if the following holds:

$$\beta(\hat{P}(t) - c - t)\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} + (1 - \beta)\left(1 - \frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t}\right)\hat{Q}(t) > (<)0. \tag{3.5}$$

Suppose first $\beta = 1$. Then $(1 - \beta)(1 - \frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t}) = 0$, i.e., the terms-of-trade motive vanishes. Only the harmful effect of the tariff – output reduction – remains. An import subsidy raises Home welfare by increasing output and we find that, indeed, the optimal tariff is negative. More generally, when $\beta = 1$, Home captures all profits in the bargaining stage. The situation is like a domestic, single-stage, Cournot oligopoly with s firms. A positive subsidy increases welfare in an oligopoly setup by narrowing the wedge between price and marginal cost, which explains why an import subsidy is optimal in this case.

When $\beta = 0$, Home firms have no bargaining power. This is equivalent for Home to importing the final good from Foreign and its welfare is composed of consumer surplus and tariff

revenues. In such cases, the sign of the optimal tariff is determined exclusively by the terms-of-trade motive, or equivalently by the sign of $1 - \frac{d\hat{P}(t)}{dt}$. As the pass-through from tariff to domestic prices is incomplete for all logconcave demand functions, $1 - \frac{d\hat{P}(t)}{dt} > 0$ holds, which implies that the optimal tariff is strictly positive.

The above discussion suggests that optimal tariff for Home is positive when Home firms' bargaining power is low, and is negative when their bargaining power is high. Furthermore, invoking the standard continuity argument (in terms of β), it follows that there is a range of values for β such that the optimal tariff is strictly decreasing in β. Analyzing (3.4) further gives a more precise characterization.

Proposition 3.1

Let $t(\beta)$ denote the optimal tariff. At $t = t(\beta)$ the following holds:

$$t = -\frac{P'(\hat{Q}(t))\hat{Q}(t)(2 + \hat{\epsilon})}{s} \left(\frac{1 + \hat{\epsilon}}{2 + \hat{\epsilon}} - \beta \right). \tag{3.6}$$

$\hat{Q}(t)$ and $\hat{\epsilon}$ respectively are the aggregate output and elasticity of slope evaluated at $t = t(\beta)$. Furthermore,

(i) *there exists $\hat{\beta}$ such that*

$$t(\beta) \gtreqless 0 \quad \Leftrightarrow \quad \beta \lesseqgtr \hat{\beta} \equiv \frac{1 + \hat{\epsilon}}{2 + \hat{\epsilon}},$$

(ii) *$t(\beta)$ is monotonically decreasing in β.*

Home welfare W_H consists of consumer surplus (*CS*), tariff revenues (*TR*), and Home profits (Π_H) – see the expression for W_H in the beginning of this subsection. The increase in t affects all three. However, since $\hat{Q}(t)$ and $\hat{P}(t)$ are independent of β, *CS* and *TR* are also independent of β. Using (3.2), express Home profits as a share of aggregate profits: $\Pi_H = (\hat{P}(t) - \hat{r}(t, \beta))\hat{Q}(t) = \beta(\hat{P}(t) - c - t)\hat{Q}(t)$. An increase in tariff reduces aggregate profits $(\hat{P}(t) - c - t)\hat{Q}(t)$ as both $\hat{P}(t) - c - t$ and $\hat{Q}(t)$ decline with an increase in t. The higher bargaining power of Home firms, the greater the reduction in Home profits and consequently the lower the optimal tariff.[11]

As an illustrative example, consider the following class of inverse demand functions: $P(Q) = a - Q^d, d > 0$. Observe that $d = 1$ for linear demand and $d > (<)1$ for strictly concave (convex) demand. The elasticity of slope is constant and denoted by $\epsilon = d - 1$. Applying (3.6) yields

$$t = \frac{(a - c)d(d + 1)}{s + d(d + 1)(1 - \beta)} \left(\frac{d}{d + 1} - \beta \right).$$

---

[11]We find that the negative relationship between bargaining power and optimal tariff is robust to a variety of alternative specifications of the model, including (a) presence of Foreign consumers, (b) strategic interactions between governments regarding tariff rates, (c) ad valorem tariffs, (d) Home tariffs on both intermediate inputs and final goods, and (e) an alternative bargaining mechanism where bargaining within a upstream-downstream pair is only over the input price. Details are available in the Supplementary Note of this paper.

For linear demand ($\epsilon = d - 1 = 0$), the optimal tariff is positive if and only if $\beta < \frac{1}{2}$. The optimal tariff rate is more likely to be positive when demand is concave ($\epsilon > 1$).

The negative relationship between β and optimal tariff rate $t(\beta)$ bears resemblance with the relationship between international ownership and Home tariffs studied in Blanchard (2010). In an environment with horizontal FDI, the paper shows that an increase in the degree of Home ownership of Foreign firms can prompt a welfare maximizing Home government to set lower tariff (than it would have set otherwise). Loosely speaking, our short run relationship between β and $t(\beta)$ mimics Blanchard's findings if we treat our framework as one with vertically integrated Foreign firms and interpret β as the proxy for degree of Home ownership in Foreign firms.

How does our findings on intermediate-input tariffs compare with the results on final-good tariffs which have been the main focus of the existing literature on trade policy? To fix ideas, consider the pre-fragmentation world where intermediate inputs and final goods are produced in the same location and within the same firm. Assume that a Foreign monopolist produces intermediate inputs at a unit cost c, transforms them into final goods at a unit cost $d_F \in [0, \bar{P} - c)$ and exports the final good to Home. As before, the Home government sets a specific tariff t on Foreign imports. It is well known that a small tariff on integrated Foreign monopolist improves welfare if and only if the terms of trade, $P^* \equiv \hat{P} - t$, improves with tariff (see, e.g., Helpman and Krugman, 1989), i.e.,

$$\frac{\mathrm{d}P^*}{\mathrm{d}t} < 0.$$

In the post-fragmentation world, the final good is produced at Home as it can transform one unit of intermediate input costlessly into one unit of final good. Here, a small tariff on Foreign intermediate inputs improves welfare if and only if

$$\frac{\mathrm{d}P^*}{\mathrm{d}t} + \beta < 0.$$

While the magnitudes of $\frac{\mathrm{d}P^*}{\mathrm{d}t}$ in the pre- and post-fragmentation world are not necessarily the same (unless $d_F = 0$ or demand is linear), $\frac{\mathrm{d}P^*}{\mathrm{d}t}$ is negative in both cases for all logconcave demand functions. In the post-fragmentation world however there is a new effect. An increase in input tariff reduces joint profit of the monopoly Home-Foreign pair. Since a β fraction of joint profit accrues to Home, Home profit reduces with tariff. As the terms-of-trade improvement effect of tariff is somewhat blunted by profit reduction effect, tariffs on intermediate inputs are less likely to improve Home welfare. Indeed, for high values of β, imposing a tariff on intermediate input worsens welfare. In the Supplementary Note, we further investigate the interaction between final-good trade and intermediate-input trade policies.

3.3 Profits

Recall that Home and Foreign profits respectively are given by

$$\Pi_H = [P(\hat{Q}) - \hat{r}]\hat{Q} = \beta\Pi, \quad \Pi_F = (\hat{r} - c - t)\hat{Q} = (1 - \beta)\Pi,$$

where $\Pi \equiv \Pi_H + \Pi_F = [P(\hat{Q}) - c - t]\hat{Q}$ is the aggregate joint profits. Differentiating Π_H and Π_F with respect to β yields

$$\frac{\mathrm{d}\Pi_H}{\mathrm{d}\beta} = \Pi + \beta\frac{\partial\Pi}{\partial t}\cdot\frac{\mathrm{d}t}{\mathrm{d}\beta},$$

$$\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} = \underbrace{-\Pi}_{\text{Share effect}} + \underbrace{(1 - \beta)\frac{\partial\Pi}{\partial t}\cdot\frac{\mathrm{d}t}{\mathrm{d}\beta}}_{\text{Size effect}}.$$

An increase in β has two effects on Π_i ($i \in \{H, F\}$). First, by increasing the share of Home firms in the joint profits, an increase in β raises Π_H and reduces Π_F. We call this the *share effect*, which is positive for Home firms and negative for Foreign firms. Note that this effect exists even when the tariff is exogenously set. Second, an increase in β reduces $t(\beta)$ which in turn leads to higher joint profits Π. We call this indirect effect the *size effect*, which benefits both Home and Foreign firms. Since both the size effect and the share effect are positive for Home firms, Π_H increases as β increases. Surprisingly, we find that Π_F can increase with an increase in β.

Proposition 3.2

An increase in Home firms' bargaining power might lead to higher Foreign profits. For demand functions with constant elasticity of slope (i.e., $\epsilon = \frac{P''(Q)Q}{P'(Q)}$ is constant), $\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} > 0$ if

$$\beta < \max\left\{0, 1 - \frac{s}{2 + \epsilon}\right\}.$$

Proposition 3.2 suggests that an indirect increase in Foreign profits due to a lower tariff (induced by higher β) might outweigh a direct decrease in Foreign profits due to a lower share of joint profits (i.e., lower $1 - \beta$). This situation is more likely to arise when the number of matched pairs s is small or the curvature of the inverse demand ϵ is large. To see this clearly, consider $P(Q) = a - Q^d$ for which $\epsilon = d - 1$. For linear demand ($d = 1$), an increase in β leads to higher Π_F if the market structure is a bilateral monopoly (i.e., $s = 1$). As d increases, the counterintuitive possibility arises for higher values of s as well. Note that irrespective of the market structure, there always exists d high enough such that $\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} > 0$ holds.

3.4 Discussions

Third-country setup: So far, we have considered a framework where all final-good producers and consumers are located in the same country (Home). An alternative framework widely used in the oligopolistic trade literature is a third-country setup where consumers and producers are based in different countries (Brander and Spencer, 1985). In the absence of consumers, optimal trade policies of the producers' countries are dictated by profit-shifting motives. We show that the negative relationship between bargaining power and optimal tariff holds even in this setup as well.

Suppose there are two downstream firms, H_1 and H_2, located in two different countries, 1 and 2 say. All consumers reside in a different country, 3 say. As before, all intermediate-input producers are located in country F(oreign). A downstream firm H_i procures intermediate input from Foreign firm F_i, produces final good, and sell in country 3. Assume that the inverse demand function $P(Q)$ and production technologies are the same as in section 3.1. Bargaining within each pair i as well as the Cournot competition between the pair remains qualitatively the same as before. As H_1 and H_2 are from different countries, bargaining power of H_1 and H_2 vis-a-vis their Foreign upstream counterparts (denoted by β_1 and β_2 respectively) are not necessarily equal.

Let t_1 and t_2 denote the specific tariff rates on Foreign intermediate inputs imposed by the governments of countries 1 and 2 respectively. Proceeding as in section 3.1 we find that:

$$\hat{q}_i = -\frac{P(\hat{Q}) - c - t_i}{P'(\hat{Q})},$$

$$\hat{r}_i = (1 - \beta_i)P(\hat{Q}) + \beta_i(c + t_i),$$

where $Q = \hat{Q}$ uniquely solves the following:

$$2P(Q) + P'(Q)Q = 2c + t_1 + t_2.$$

For given tariff rates, joint profit of a downstream-upstream pair i is $\pi_i = (P(\hat{Q}) - c - t_i)\hat{q}_i$, while profit of a downstream firm i (within a pair i) is $\pi_{H_i} = \beta\pi_i = \beta(P(\hat{Q}) - c - t_i)\hat{q}_i$. Then

$$\frac{\partial \pi_i}{\partial t_i} = \left(\frac{\partial P(\hat{Q})}{\partial t_i} - 1\right)\hat{q}_i - P'(\hat{Q})\hat{q}_i\frac{\partial \hat{q}_i}{\partial t_i}$$

$$= \hat{q}_i\left(P'(\hat{Q})\frac{\partial \hat{q}_j}{\partial t_i} - 1\right)$$

$$= -\frac{(4 + (1 + \hat{\theta}_j)\hat{\epsilon})\hat{q}_i}{3 + \hat{\epsilon}},$$

where $\theta_j \equiv \frac{q_j}{Q}$. From Assumption 1' (i.e., $\epsilon \geq -1$), it follows that $\frac{\partial \pi_i}{\partial t_i} < 0$. We show below that this negative relationship between profit and tariff rate, which holds quite generally, underpins the negative relationship between bargaining power and optimal tariff.

For simplicity, assume that only country 1's government is active. In particular, assume that country 1 chooses a tariff rate $t_1 = t$ on Foreign intermediate input to maximize its welfare while country 2 maintains free trade (i.e., $t_2 = 0$). Country 1's welfare is given by

$$W_1 = \underbrace{\beta_1 \pi_1(t)}_{\text{Profits of downstream firm 1}} + \underbrace{t\hat{q}_1(t)}_{\text{Tariff revenues}}. \tag{3.7}$$

Setting $\frac{dW_1}{dt} = 0$ and solving for t gives the expression for the optimal tariff t^*. Differentiating $\frac{dW_1}{dt}\big|_{t=t^*} = 0$ with respect to β_1 and rearranging we get

$$\frac{dt^*}{d\beta_1} = -\frac{\frac{\partial^2 W_1}{\partial \beta_1 \partial t}}{\frac{\partial^2 W_1}{\partial t}}.$$

Assuming that the second-order condition is satisfied, i.e. $\frac{\partial^2 W_1}{\partial t^2} < 0$, it follows that

$$\operatorname{sgn}\frac{dt^*}{d\beta_1} = \operatorname{sgn}\frac{\partial^2 W_1}{\partial \beta_1 \partial t} = \operatorname{sgn}\frac{d\pi_1}{dt}. \tag{3.8}$$

Since $\frac{d\pi_1}{dt} < 0$ it follows that optimal tariff of country 1 is decreasing in the β_1. This result is similar to Proposition 3.1(ii). Like Proposition 3.1(i), we also find that optimal tariff t^* is strictly positive if and only if β_1 is less than a critical threshold $\hat{\beta}_1$ for the following reasoning. When $\beta_1 = 1$ and Foreign upstream firms have no bargaining power, our setup is equivalent to the Brander-Spencer (1985) setup with final-good producers only. The welfare expression in (3.7) reduces to $W_1 = \pi_1(t) + t\hat{q}_1(t)$ as in Brander and Spencer (1985). Given the full bargaining power of downstream firms, import subsidy offered to Foreign upstream firm F_1 acts as an export subsidy to country 1's firm H_1. Just as an export subsidy is optimal in Brander and Spencer (1985), we find that import subsidy is optimal ($t^* < 0$) in our framework. When $\beta_1 = 0$, welfare of H_1 consists only of tariff revenues. Naturally, a positive tariff $t^* > 0$ maximizes H_1's welfare. These findings at the two extremes, $t^* < 0$ for $\beta_1 = 1$ and $t^* > 0$ for $\beta_1 = 0$, together with (3.8) imply that there exists a unique cutoff $\hat{\beta}_1$ such that

$$t^* \gtreqless 0 \quad \Leftrightarrow \quad \beta_1 \lesseqgtr \hat{\beta}_1.$$

Now consider a more general setup with several downstream firms in countries 1 and 2 and consumers in three countries 1, 2, and 3. All upstream firms are in Foreign. Country 1's welfare (given by W_1) consists of consumer surplus, tariff revenues, and profits of its own downstream firms in all markets. Let Π_1 denote aggregate profits for all downstream-upstream pairs that involve downstream firms from country 1. Like (3.8), we find that

$$\operatorname{sgn}\frac{dt^*}{d\beta_1} = \operatorname{sgn}\frac{\partial^2 W_1}{\partial \beta_1 \partial t} = \operatorname{sgn}\frac{d\Pi_1}{dt}.$$

As both $\hat{Q}$ and $\hat{q}_i$ are independent of bargaining power, consumer surplus and tariff revenues are independent of β_1. Joint profit, Π_1, continues to be decreasing in t_1 which in turn implies the negative relationship between β_1 and t^*.

Related evidence on bargaining power and trade policy: There exists empirical evidence that bargaining power can impact trade policy. The importance of bargaining power in negotiating tariff reductions is known in the literature on trade agreements. Determinants of bargaining power however are not usually modelled. Limão (2006) and Olarreaga and Özden (2005) are two exceptions. Hypothesizing that the countries with larger GDP would have relatively stronger bargaining power, Limão (2006) finds that exporters from larger countries indeed tend to receive a lower tariff for the goods exported to the U.S. Similarly, hypothesizing that the sectors with a smaller number of importers would have stronger bargaining power, Olarreaga and Özden (2005) find that the U.S. government tends to set a lower tariff in sectors with higher concentration of importers. In particular, they argue that despite preferential access to the U.S., exporters from African countries often do not enjoy benefits from tariff reduction because of their low bargaining power.[12]

The findings in the aforementioned work suggest that Foreign firms with low bargaining power face high Home tariffs. This might seem at odds with our finding that bargaining power of Foreign firms and Home tariffs are positively related. The findings however do not contradict each other once we accept the fact the above papers study tariffs on final goods whereas our paper studies tariffs on intermediate inputs. To see this more clearly, let us extend the setup in section 3.1 to allow for Foreign final-good producers, so that Home imports final goods as well as intermediate inputs from Foreign. Let t^I and t^F respectively denote the Home tariff rate on intermediate inputs and final goods, both of which are imported from Foreign. In this environment, we find that an increase in the bargaining power of Home firms leads not only to lower Home tariff on intermediate inputs (as in Proposition 3.1), but also to higher Home tariff on final goods. Thus if the government is allowed to choose tariff on final goods, the theoretical finding is consistent with the empirical evidence in the above papers. In addition, we also find that the two tariff rates, t^I and t^F, are strategic complements when the demand is linear. This finding is heartening from a policy point of view as it implies that successful negotiation on tariff reduction in one sector prompts unilateral tariff reduction in the vertically linked sector. Details are presented in the Supplementary Note.

Indirect evidence of our link between bargaining power and tariff in the context of vertical relationships is partly reflected in recent work by Blanchard and Matschke (forthcoming) on vertical FDI. When a multinational firm owns export-oriented (i.e., offshoring) affiliates abroad, a source country (i.e., a multinational's home country) has an incentive to improve market access by offering lower tariffs or preferential access to a host country. Using firm-level panel

---

[12]Hasan, Mitra and Ramaswamy (2007) find evidence in the opposite direction (from tariff to bargaining power). Using industry-level data disaggregated by states, they show that trade liberalization in India in the early 1990s led to reduction of bargaining power of workers.

data on U.S. foreign affiliate activity and detailed measures of U.S. trade policy, they show that is indeed the case. Vertical FDI in their paper is akin to full bargaining power of Home firms ($\beta = 1$) in our paper: tariffs are indeed lowest for $\beta = 1$ in our framework which is in line with the empirical finding reported in Blanchard and Matschke.

4   Endogenous Market Structure

In section 3, we have assumed that the number of Home and Foreign firms is fixed. Since m and n are fixed, the number of matched pairs, $s = s(m, n)$, is fixed as well and in particular it does not vary with tariff rates. Now we consider an environment where m and n are endogenously determined and tariffs are set prior to entry decisions. Here, in addition to the the direct effect on quantities and prices, tariffs also indirectly affect quantities and prices by influencing the market structure.

In the context of single-stage oligopoly models, Horstmann and Markusen (1986), Venables (1985) and more recently Etro (2011) and Bagwell and Staiger (2012a, b) all have shown that the endogenous market structure can drastically alter the optimal trade policy obtained from the exogenous market structure. Like these preceding papers, we also find that free entry can affect the relationship between bargaining power and optimal tariff. In particular, we show that the monotone relationship between bargaining power and tariff no longer holds. The particular nature of non-monotonicity depends on the curvature of demand function.

The timing of events is as outlined in the last paragraph of section 2. First, the Home government chooses a tariff rate t, following which entry occurs. Suppose m Home firms and n Foreign firms enter their respective market. Subsequently, $s = s(m, n)$ pairs are formed through random matching. Unmatched firms immediately exit. Finally, after matching, each pair i consisting of a Home firm and a Foreign firm chooses (r_i, q_i) to solve the maximization problem stated prior to Assumption 1 in section 3.1.

Let us start with the last stage. The bargaining problem and the Cournot competition works exactly the same way as before. Accordingly, as in section 3, the unique equilibrium in this stage is characterized by $\hat{q}_1 = \hat{q}_2 = ... = \hat{q}_s \equiv \hat{q}$ and $\hat{r}_1 = \hat{r}_2 = ... = \hat{r}_s \equiv \hat{r}$, satisfying

$$\hat{q} = -\frac{P(\hat{Q}) - c - t}{P'(\hat{Q})},$$
$$\hat{r} = (1 - \beta)P(\hat{Q}) + \beta(c + t),$$

where $Q = \hat{Q}$ satisfies the following for any given s:

$$sP(Q) + P'(Q)Q = s(c + t). \tag{4.1}$$

The third-stage subgame outcome is thus identical to that in section 3. Here, since number of Home and Foreign firms are endogenous, search for partner firms becomes important. In what

follows, we analyze the effect of entry and matching on equilibrium.

4.1 Entry and Matching

In the second stage, the number of matched pairs $s = s(m, n)$ is endogenously determined as there is free entry of firms. Recall from section 2 that the entry costs of a Home firm and a Foreign firm respectively are K_H and K_F. Firms are risk-neutral and entry occurs until the post-entry profit equals K_H (K_F) for a Home (Foreign) firm.

Consider first entry by Home firms. If m Home firms enter and only $s = s(m, n)$ firms are matched, the probability of finding a Foreign partner for each Home firm is s/m. If successful, it receives $\beta(P(Q) - c - t)q$; otherwise, it receives 0. Thus the expected post-entry profit for a Home firm is $\frac{s}{m}\beta(P(Q) - c - t)q$. Following the standard practice in the oligopoly literature, we treat m as a continuous variable. This implies that in free entry equilibrium, the expected post-entry profit must equal K_H for a Home firm:

$$\frac{s}{m}\beta(P(Q) - c - t)q = K_H. \tag{4.2}$$

By analogous reasoning we can establish that the expected post-entry profit must equal K_F for a Foreign firm:

$$\frac{s}{n}(1 - \beta)(P(Q) - c - t)q = K_F. \tag{4.3}$$

An equilibrium in this model is a vector $(Q, m, n) = (\hat{Q}, \hat{m}, \hat{n})$ such that (4.1) − (4.3) holds. Rather than working with $\hat{m}$ and $\hat{n}$ we find it more convenient to work with the number of matched pairs, $s = s(m, n)$, and the ratio, $z = \frac{n}{m}$, which captures the relative thickness of Foreign firms. To proceed further, we need to specify the property of the matching function. Following the literature (e.g., Grossman and Helpman, 2002), we assume that the matching function satisfies the following properties:

$$s(\lambda m, \lambda n) = \lambda s(m, n), \tag{4.4}$$

$$\frac{\partial s(m, n)}{\partial m} > 0 \text{ and } \frac{\partial s(m, n)}{\partial n} > 0, \tag{4.5}$$

$$\frac{\partial^2 s(m, n)}{\partial m^2} < 0 \text{ and } \frac{\partial^2 s(m, n)}{\partial n^2} < 0. \tag{4.6}$$

Equation (4.4) means constant-returns-to-scale in matching. Furthermore, equations (4.4) − (4.6) imply complementarity or supermodularity in matching, i.e., $\frac{\partial^2 s(m,n)}{\partial m \partial n} > 0$. Supermodular and log-supermodular functions are routinely used in matching environments (e.g., Shimer and Smith, 2000). See Costinot (2009) for the application of log-supermodularity to trade settings.

Letting $\lambda = 1/m$ in (4.4), we have that

$$s(m, n) = m \cdot s\left(1, \frac{n}{m}\right) \equiv mS(z),$$

where $z \equiv n/m$ measures the relative number of upstream and downstream firms entering in each stage of production, that is, the relative thickness of the vertically related markets. From the definition of $S(z)$ it follows that $S(z) \leq 1$. We further assume that

$$\lim_{z \to 0} S(z) = 0. \tag{4.7}$$

Using the properties of the matching function $s(m, n)$ given in (4.4) – (4.6), this normalized matching function $S(z)$ can be shown to satisfy the following properties:

$$S'(z) > 0, \quad S''(z) < 0, \quad S(z) > zS'(z). \tag{4.8}$$

As an illustrative example, consider $s(m, n) = \frac{mn}{m+n}$. We can express $s(m, n) = mS(z)$ where $S(z) = \frac{z}{1+z}$. It is easy to check that $s(m, n)$ satisfies (4.4) – (4.6) and $S(z)$ satisfies (4.7) and (4.8).

4.2   Equilibrium and Comparative Statics

Equilibrium: An equilibrium in this framework with endogenous market structure is a vector $(Q, s, z) = (\hat{Q}, \hat{s}, \hat{z})$ which solves the following system of equations:

$$s(P(Q) - c - t) + QP'(Q) = 0, \tag{4.9}$$

$$-\frac{\beta S(z)P'(Q)Q^2}{s^2} = K_H, \tag{4.10}$$

$$z = \left(\frac{1-\beta}{\beta}\right)\left(\frac{K_H}{K_F}\right). \tag{4.11}$$

Equation (4.9) is the same as (4.1). Equation (4.10) restates the zero-profit condition for Home firms, (4.2), by using (4.1) and the following substitutions: $m = \frac{S(z)}{s}$ and $n = \frac{zS(z)}{s}$. Equation (4.11), capturing the relationship between bargaining power and relative thickness of Foreign firms, follows from (4.2) and (4.3). It implies that the proportion of Foreign firms ($z = \frac{n}{m}$) declines as the bargaining power of Home firms increases. Bargaining power thus affects the probability of finding a partner firm and the aggregate output through the relative thickness of the vertically related markets.

An advantage of working with (4.9) – (4.11) is the separability between $\hat{z}$ and $(\hat{Q}, \hat{s})$. To see this, observe that $\hat{z}$ is determined by (4.11) alone. Taking $z = \hat{z}$ as given, solving (4.9) and (4.10) then gives $\hat{Q}$ and $\hat{s}$. Figure 1 graphically represents these relationships. ZZ in Figure 1(a) depicts (4.11) which captures the negative relationship between the relative thickness of Foreign firms z and the bargaining power of Home firms β. Taking β (and consequently $\hat{z}$) as given, Figure 1(b) depicts (4.9) and (4.10) which are given by AA and BB respectively. The fact that AA is flatter than BB follows from noting that

$$\left.\frac{\mathrm{d}Q}{\mathrm{d}s}\right|_{AA} = \frac{q}{s+1+\epsilon} < \left.\frac{\mathrm{d}Q}{\mathrm{d}s}\right|_{BB} = \frac{Q}{2+\epsilon}.$$
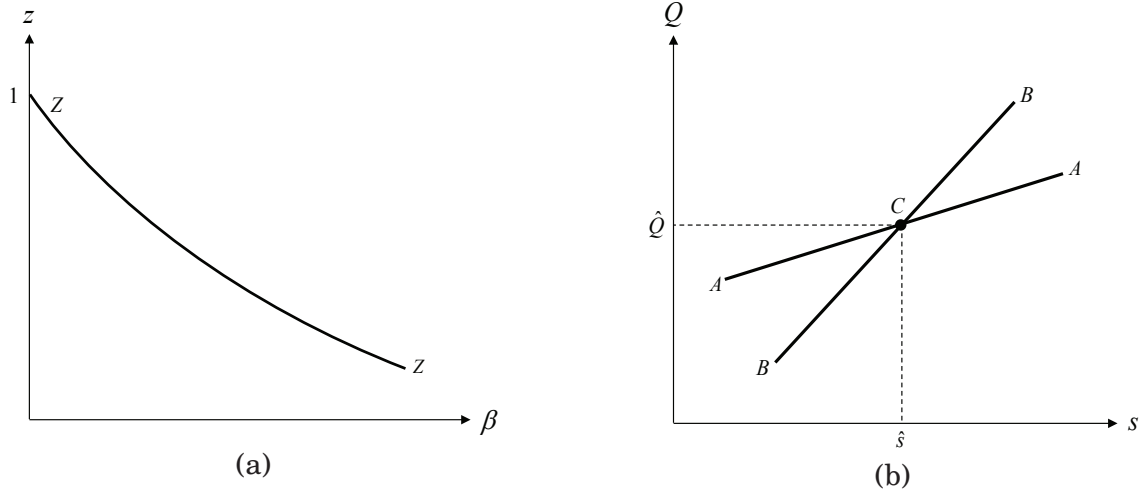
19

FIGURE 1 – Equilibrium outcomes

Point C, the intersection of AA and BB, separately determines $(\hat{s}, \hat{Q})$.

Effect of a change in tariff rate: First we consider the effect of a change in tariff rate t. Observe that t only appears in (4.9). Hence, only AA is affected by a change in t. From (4.9) it follows that as t increases, s must increase for any given Q. Consequently, as illustrated in Figure 2, AA shifts to the right and both $\hat{Q}$ and $\hat{s}$ declines. Recall from section 3, a tariff lowers equilibrium output $\hat{Q}$ and raises equilibrium price $\hat{P}$ even when the number of matched pairs is exogenously given. Here, a tariff discourages entry which in turn reduces the number of matched pairs $\hat{s}$. This effect lowers output and raises price even further. The following lemma summarizes some important comparative statics results that arise from Figure 2.

Lemma 4.1

(i) *An increase in the tariff rate lowers the number of matched pairs and output, and raises prices in equilibrium; i.e., $\frac{\partial \hat{s}}{\partial t} < 0$, $\frac{\partial \hat{Q}}{\partial t} < 0$, $\frac{\partial \hat{P}}{\partial t} > 0$ and $\frac{\partial \hat{r}}{\partial t} > 0$.*

(ii) *Let $r^* \equiv \hat{r} - t$ denote the price received by a Foreign firm in equilibrium (for each unit of the intermediate input). Then,*

$$\frac{\partial r^*}{\partial t} \lesseqqgtr 0 \quad \Leftrightarrow \quad \frac{\partial \hat{P}}{\partial t} \lesseqqgtr 1 \quad \Leftrightarrow \quad \epsilon(\hat{Q}(t)) \gtreqqless 0.$$

Lemma 4.1 (ii) says that an increase in tariff improves the terms-of-trade, i.e., lowers r^*, if and only if the demand is concave. Recall that when the market structure is exogenous, tariff reduces r^* for all logconcave demand functions ($\epsilon(\hat{Q}) \geq -1$). When the market structure is endogenous, in contrast, tariff reduces r^* only for concave demand functions ($\epsilon(\hat{Q}) \geq 0$). This suggests that terms-of-trade improvement is less likely with endogenous market structure. The
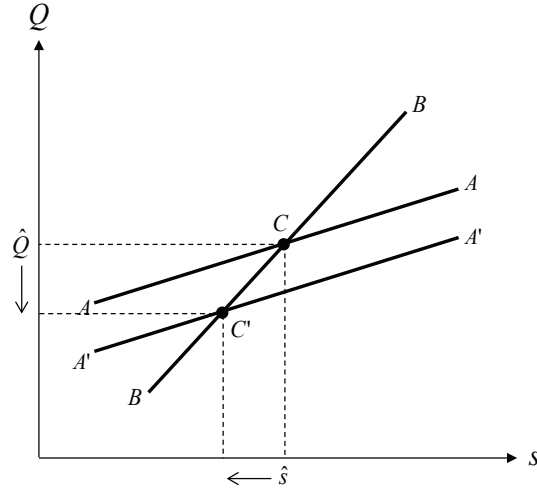
20

FIGURE 2 – Effect of an increase in t

reasoning is simple. The implicit terms-of-trade is given by

$$r^* = \hat{r} - t = (1 - \beta)(\hat{P} - t) + \beta c.$$

As t increases, $\hat{P}$ increases under both exogenous and endogenous market structures. However, in response to an increase in t, $\hat{P}$ increases more under the endogenous market structure (as explained in the paragraph preceding Lemma 4.1). Thus, a reduction in $\hat{P} - t$, and accordingly a reduction in r^* becomes less likely.

Effect of a change in bargaining power: Recall that in the exogenous market structure, a change in β has no effect on the aggregate output $\hat{Q}$ or the final-good price $\hat{P} = P(\hat{Q})$ (see Lemma 3.1(i)). Here we show that this no longer holds and the relationship between $\hat{Q}$ and β is non-monotone. First we present a technical result which underpins this non-monotonicity.

Lemma 4.2 *There exists $\tilde{\beta} \in (0, 1)$ such that*

$$\frac{\partial \beta S(\hat{z})}{\partial \beta} \gtreqless 0 \quad \Leftrightarrow \quad \beta \lesseqgtr \tilde{\beta},$$

where $\beta S(\hat{z})$ denotes the expected profit share of each Home firm.

Each Home firm's expected share of profit (within a Home-Foreign pair) is given by its bargaining power β times its probability of being matched with a Foreign partner ($\frac{\hat{s}}{\hat{m}}(\equiv S(\hat{z}))$). As β increases, the relative number of Home firms increases which in turn lowers $S(\hat{z})$, i.e., each Home firm's probability of being matched with a Foreign firm. Thus the expected profit share of each Home firm $\beta S(\hat{z})$ can increase or decrease with an increase in β. Lemma 4.2 says that the expected share increases if and only if β is less than a threshold value $\tilde{\beta}$. For example,
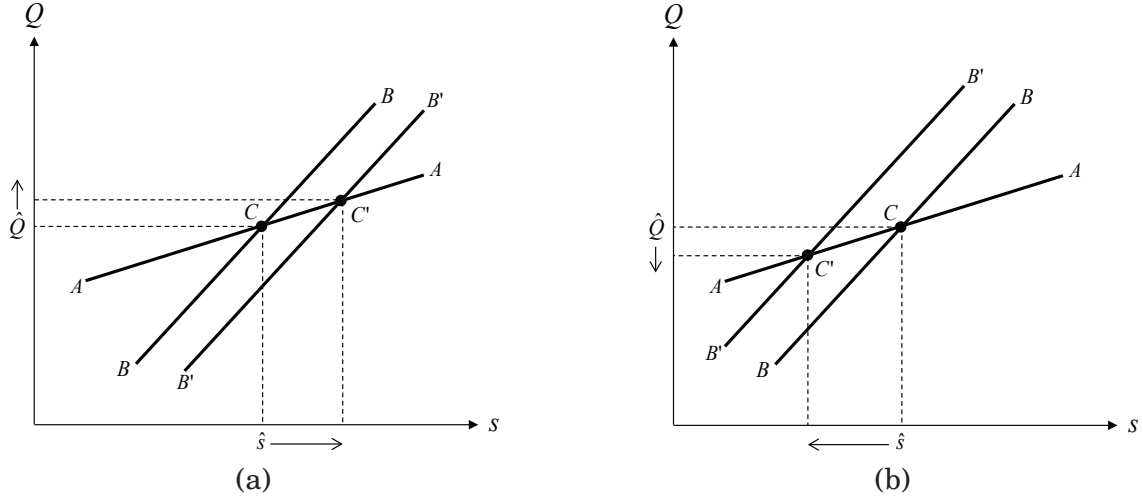
FIGURE 3 – Effect of an increase in β

consider $\beta < \tilde{\beta}$. From Lemma 4.2 we know that $\beta S(\hat{z})$ increases as β increases in this range. From (4.10) it follows that, for a given Q, s must increase. Thus BB shifts to the right and consequently both $\hat{Q}$ and $\hat{s}$ increase. Exactly the opposite holds when $\beta > \tilde{\beta}$. Figures 3(a) and 3(b) respectively illustrate the effect of an increase in β when $\beta < \tilde{\beta}$ and $\beta > \tilde{\beta}$.

The discussion above implies the following non-monotonicity result.

Lemma 4.3 *The relationship between aggregate output $\hat{Q}$ and bargaining power β is non-monotone. In particular, there exists $\tilde{\beta}$ such that*

$$\frac{\partial \hat{Q}}{\partial \beta} \gtreqqless 0 \quad \Leftrightarrow \quad \beta \lesseqqgtr \tilde{\beta}.$$

A similar non-monotone relationship holds between the number of matched pairs $\hat{s}$ and β.

To better appreciate the non-monotone relationship between $\hat{Q}$ and β, suppose $K_H = K_F$ and β is low. It follows from (4.11) that $\hat{m} < \hat{n}$, i.e., there are fewer Home firms. An increase in β encourages entry by Home firms and prompts exit by Foreign firms. This improves matching (because there were fewer Home firms to start with) and leads to an increase in the aggregate output. On the other hand, if β is high to start with, $\hat{m} > \hat{n}$. By inducing the entry of Home firms and the exit of Foreign firms, a further increase in β worsens the mismatch which in turn leads to lower aggregate output. As we show in the next subsection, the non-monotone relationship between $\hat{Q}$ and β carries over to the non-monotone relationship between optimal tariff t and β. The particular nature of non-monotonicity, however, depends on the curvature of the demand function.

4.3 Tariffs

Let $\hat{Q}(t,\beta)$ and $\hat{P}(t,\beta)$ respectively denote the equilibrium output and the price of final goods for a given t and β. Note from Lemma 4.3 that the equilibrium output (and hence the price) does depend on β in the long run. In the first stage, the Home government chooses a tariff rate t to maximize Home welfare. As profits are zero under free entry, Home welfare (W_H) effectively consists of consumer surplus and tariff revenues only. Then Home welfare is given by

$$W_H = \underbrace{\left[\int_0^{\hat{Q}(t,\beta)} P(y)\mathrm{d}y - P(\hat{Q}(t,\beta))\hat{Q}(t,\beta)\right]}_{\text{Consumer surplus}} + \underbrace{t\hat{Q}(t,\beta).}_{\text{Tariff revenues}}$$

Differentiating W_H with respect to t and rearranging, we get

$$\frac{\mathrm{d}W_H}{\mathrm{d}t} = \left(1 - \frac{\mathrm{d}\hat{P}(t,\beta)}{\mathrm{d}t}\right)\hat{Q}(t,\beta) + t\frac{\mathrm{d}\hat{Q}(t,\beta)}{\mathrm{d}t} = \frac{\mathrm{d}r^*(t,\beta)}{\mathrm{d}t}\frac{\hat{Q}(t,\beta)}{1-\beta} + t\frac{\mathrm{d}\hat{Q}(t,\beta)}{\mathrm{d}t}. \tag{4.12}$$

Setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ and solving for t gives the expression for the optimal tariff which is presented later in Proposition 4.1. Since $\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} < 0$, it follows from (4.12) that the optimal tariff is strictly positive (negative) if and only if the terms-of-trade improve for Home, i.e. r^* declines. From Lemma 4.1(ii) we already know that this happens only if inverse demand is concave. Proposition 4.1 presents these findings and provides a sharper characterization.

Proposition 4.1

Let $t(\beta)$ denote the optimal tariff. At $t = t(\beta)$ the following holds:

$$t(\beta) = \left(\frac{-P'(\hat{Q}(t(\beta),\beta))\hat{q}(t(\beta),\beta)}{2}\right) \cdot \hat{\epsilon}. \tag{4.13}$$

$\hat{Q}(t,\beta)$ and $\hat{\epsilon}$ respectively are the aggregate output and elasticity of slope evaluated at $t = t(\beta)$. Furthermore,

(i) *$t(\beta)$ is strictly positive (negative) if the inverse demand is strictly concave (convex). Note that $t(\beta) = 0$ for linear demand.*

(ii) *Suppose Assumptions 1 and 2 hold. Unless the inverse demand function is linear ($\epsilon = 0$), $t(\beta)$ changes non-monotonically with β. More specifically, the following holds:*

$$\mathrm{sgn}\,\frac{\mathrm{d}t}{\mathrm{d}\beta} = \mathrm{sgn}\,P''(\hat{Q}(t(\beta),\beta))\frac{\partial\hat{Q}}{\partial\beta}.$$

As equilibrium expected profits are zero, the terms-of-trade motive is the only thing to consider in determining the sign of the optimal tariff. An increase in tariff improves the terms-
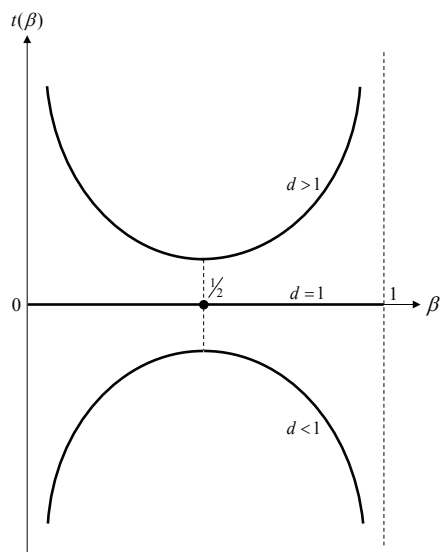
FIGURE 4 – Non-monotonicity of $t(\beta)$

of-trade if and only if the demand is concave, which explains Proposition 4.1(i): the optimal policy is tariff (subsidy) with strictly concave (convex) demand functions.

What should we make of the fact that (a) demand curvature matters for the sign of the optimal tariff and (b) the relationship between bargaining power and tariff differs between the two cases – endogenous and exogenous market structures? Our reading of the literature suggests that, in terms of the dependence of optimal policy on demand curvature, our results have a similar flavor to some of the existing results in the trade literature. For example, the classic result that the sign of the optimal tariff in the presence of a foreign monopoly depends on whether there is incomplete pass-through, which in turn depends on whether the demand curve is flatter than the marginal revenue curve (Brander and Spencer, 1984a,b; Helpman and Krugman, 1989, Chapter 4). Concerning the difference in results between endogenous and exogenous market structures, our finding is in the line with Horstmann and Markusen (1986) and Venables (1985), who have shown that in the single-stage oligopoly models, entry can alter optimal trade policy due to firm-delocation effects. This point has also recently been made by Etro (2011) and Bagwell and Staiger (2012a, b) in the contexts of strategic trade policy and trade agreements respectively. We do not necessarily view (b) as a shortcoming. Depending on the industry characteristics, such as industry-specific fixed costs or stability of demand, some industries fit an exogenous market structure description better, while for some other industries with fluid entry and volatile demand, an endogenous market structure is more apt.

Proposition 4.1(ii) suggests that the relationship between t and β depends crucially on how the aggregate output, $\hat{Q}$, varies with β and also on the curvatures of inverse demand and its slope. As $\hat{Q}$ varies non-monotonically with β the relationship between t and β is non-monotone.

To see the non-monotonicity clearly, consider once again $P(Q) = a - Q^d$, $d > 0$. Suppose $K_H = K_F$, and $s(m,n) = \frac{mn}{m+n}$ which implies $S(z) = \frac{z}{1+z}$. Then, according to Proposition 4.1,

for concave demand ($d > 1$), the optimal tariff $t(\beta)$ initially declines as β increases, becoming lowest at equal bargaining power (i.e., $\hat{\beta} = \frac{1}{2}$), and then increases as β increases further. For convex demand ($d < 1$), the relationship between β and $t(\beta)$ is exactly the opposite. If the demand is linear, i.e., $d = 1$, $t(\beta) = 0$ for $\beta \in [0, 1]$. Figure 4 illustrates the relationship between β and $t(\beta)$. We conclude by noting that the non-monotonicity of $t(\beta)$ is in fact more general.

Proposition 4.2 *Consider all inverse demand functions $P(Q)$ that satisfy Assumptions 1 and 2, and are either concave for all $Q \geq 0$ or convex for all $Q \geq 0$. The relationship between t and β is U-shaped (inverted U-shaped) if the demand function is strictly concave (convex). If the demand function is linear, free trade is optimal (i.e., optimal tariff is zero) irrespective of the bargaining power.*

5 Endogenous Bargaining Power

Up until now, we have assumed that the bargaining power of Home firms, β, is exogenously given. It has yielded general results while allowing us to be agnostic about the particular source of bargaining power. Exogenous β is particularly apt for section 3 where m and n are fixed. In section 4, however, m, n and $s = s(m, n)$ are all endogenously determined. In such an environment, the assumption of exogenous β might not be suitable. While a range of factors might affect bargaining power, the number of Home and Foreign firms, i.e., m and n, and in particular the relative thickness (of one side) seems to be a natural candidate. The challenge is to endogenize bargaining power as well as to perform comparative statics with respect to bargaining power. To do so, we introduce the following specification:

$$\beta = \beta(z, b),$$

where $z = \frac{n}{m}$, b is a shift parameter, and $\beta(z, b) \in [0, 1]$ for all permissible values of (z, b).

We assume that $\beta_z(z, b) = \frac{\partial \beta(z,b)}{\partial z} > 0$. The assumption is easy to understand once we cast the transaction in intermediate inputs in terms of buyers and sellers. Home firms are buyers of intermediate input and Foreign firms are sellers of intermediate input. The assumption $\beta_z(z, b) > 0$ effectively states that each buyer's bargaining power increases as the number of sellers increases and it decreases as the number of buyers increases.[13] The assumption also tells us what happens when both the number of sellers, n, and the number of buyers, m, change. If n and m changes in the same proportion, β does not change. Else, β increases as the upstream sector becomes relatively thicker (i.e., $\frac{n}{m}$ increases) while it decreases as the downstream sector becomes relatively thicker (i.e., $\frac{n}{m}$ decreases).

Regarding the relationship between $\beta(z, b)$ and b, we assume that $\beta_b(z, b) = \frac{\partial \beta(z,b)}{\partial b} > 0$.

---

[13]This is indeed a well-established result in the literature. See, for example, Wolinsky (1987) who shows that the acquiring of alternative sellers is a way of improving a buyer's bargaining position.
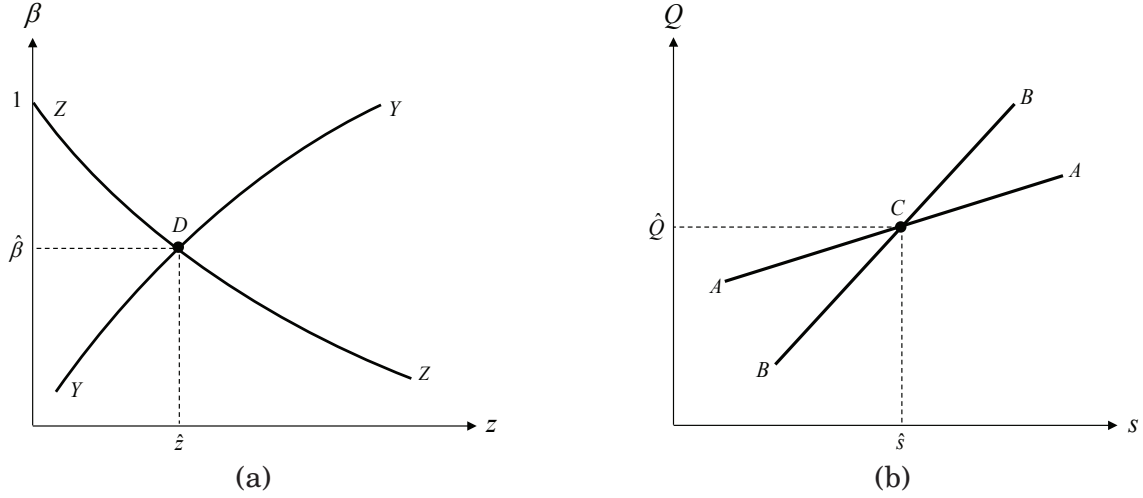
FIGURE 5 – Equilibrium outcomes

Furthermore, b lies in an open interval $(\underline{b}, \overline{b})$ where

$$\lim_{b \to \underline{b}} \beta(z, b) = 0, \quad \lim_{b \to \overline{b}} \beta(z, b) = 1.$$

for all $z > 0$. Consider for example $\beta(z, b) = \frac{bz}{1+bz}$ where $b \in (0, \infty)$. Rewriting $\frac{bz}{1+bz}$ as $\frac{z}{\frac{1}{b}+z}$ and recognizing that $\underline{b} = 0, \overline{b} = \infty$, it immediately follows that $\lim_{b \to \underline{b}} \beta(z, b) = 0$ and $\lim_{b \to \overline{b}} \beta(z, b) = 1$. The conditions on limit allow for all possible values of β to appear in equilibrium. This in turn makes it easier to compare this case with the exogenous β case in section 4 where β could take any value between zero and unity.

Equilibrium: In this model's setup with entry, matching and endogenous bargaining power, an equilibrium is a vector $(Q, s, z, \beta) = (\hat{Q}, \hat{s}, \hat{z}, \hat{\beta})$ which solves the following system of equations:

$$sP(Q) + QP'(Q) = s(c + t), \tag{5.1}$$

$$\frac{\beta S(z)(P(Q) - c - t)Q}{s} = K_H, \tag{5.2}$$

$$z = \left(\frac{1 - \beta}{\beta}\right)\left(\frac{K_H}{K_F}\right), \tag{5.3}$$

$$\beta = \beta(z, b). \tag{5.4}$$

The first three equations are the same as (4.9), (4.10) and (4.11) respectively, while the last one is the newly introduced equation in this section which postulates that bargaining power of each Home firm increases as the number of Foreign firms increases.

The equilibrium outcomes characterized by $(5.1) - (5.4)$ are illustrated in Figure 5. Note first that $\hat{\beta}$ and $\hat{z}$ are determined by (5.3) and (5.4) alone. In Figure 5(a), YY depicts (5.4) and ZZ depicts (5.3) in (z, β) space. The positive slope of YY reflects the assumption $\beta_z(z, b) > 0$,
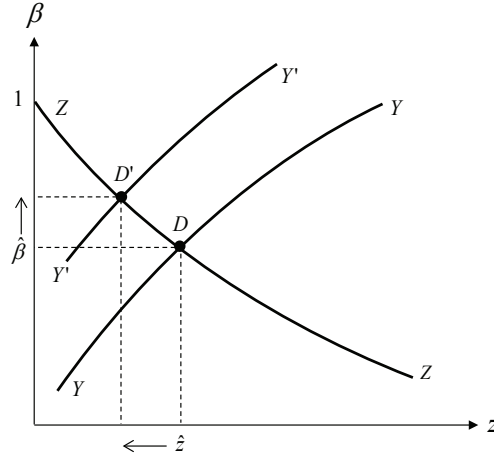
26

FIGURE 6 – Effect of an increase in b

while the negative slope of ZZ is the same as before. Point D, the intersection of YY and ZZ, endogenously determines $\hat{z}$ and $\hat{\beta}$. Taking $(\hat{z}, \hat{\beta})$ as given, Figure 5(b) depicts (5.1) and (5.2) which are given by AA and BB. As in Figure 4(b), Point C separately determines $\hat{Q}$ and $\hat{s}$.

Comparative statics: The effect of a change in tariff is the same as before. From (5.3) and (5.4), a change in t does not affect YY or ZZ and it does not affect $\hat{\beta}$. Thus, the effect of tariff on $\hat{s}$, $\hat{Q}$, $\hat{P}$ and $\hat{r}$ with endogenous β is the same as the ones with exogenous β.

What about the effect of a change in the bargaining power parameter b? Since $\beta_b(z, b) > 0$, an increase in b shifts YY upwards, whereas ZZ does not change. As a result $\hat{\beta}$ increases (see Figure 6). Given $\frac{d\hat{\beta}}{db} > 0$, $\lim_{b \to \underline{b}} \beta(z, b) = 0$ and $\lim_{b \to \overline{b}} \beta(z, b) = 1$, it follows that each $\hat{\beta} \in (0, 1)$ can be traced to a unique $b \in (\underline{b}, \overline{b})$. In particular, there exists a unique $b = \tilde{b}$ that induces $\hat{\beta} = \tilde{\beta}$ and for all $b < (>)\tilde{b}$, $\hat{\beta} < (>)\tilde{\beta}$. The following non-monotonicity results, i.e., Lemmas 5.1(i) and 5.1(ii), then immediately follow from Lemmas 4.2 and 4.3 by replacing β with b and replacing $\tilde{\beta}$ with $\tilde{b}$ respectively.

Lemma 5.1

(i) *There exists $\tilde{b} \in (0, \infty)$ such that*

$$\frac{\partial \hat{\beta} S(\hat{z})}{\partial b} \gtreqless 0 \iff b \lesseqgtr \tilde{b}.$$

(ii) *The relationship between aggregate output $\hat{Q}$ and b is non-monotone. In particular, there exists $\tilde{b} \in (0, \infty)$ such that*

$$\frac{\partial \hat{Q}}{\partial b} \gtreqless 0 \iff b \lesseqgtr \tilde{b}.$$

A similar non-monotone relationship holds between the number of matched pairs $\hat{s}$ and b.

The logic as well as the derivation of optimal tariff are very similar to the ones described in section 4.3 and hence omitted here. Note that three features are crucial in characterizing optimal tariff in section 4: (i) $\hat{z}$ is independent of t; (ii) bargaining power is independent of t (as β was exogenous); and (iii) there is a non-monotone relationship between $\hat{Q}$ and β. All three features remain intact in this section. As a result, the characterization of optimal tariff under endogenous β remains effectively the same as the characterization under exogenous β (i.e., as in Proposition 4.1) by replacing β with b.

Proposition 5.1 *Let $t(b)$ denote the optimal tariff. At $t = t(b)$ the following holds:*

$$t(b) = \left(\frac{-P'(\hat{Q}(t(b), b))\hat{q}(t(b), b)}{2} \right) \cdot \hat{\epsilon}. \tag{5.5}$$

$\hat{Q}(t, b)$ and $\hat{\epsilon}$ respectively are the aggregate output and elasticity of slope evaluated at $t = t(b)$. Furthermore,

(i) *$t(b)$ is strictly positive (negative) if the inverse demand is strictly concave (convex). Note that $t(b) = 0$ for linear demand.*

(ii) *Suppose Assumptions 1 and 2 hold. Unless the inverse demand function is linear ($\epsilon = 0$), $t(b)$ changes non-monotonically with b. More specifically, the following holds:*

$$\operatorname{sgn} \frac{\mathrm{d}t}{\mathrm{d}b} = \operatorname{sgn} P''(\hat{Q}(t(b), b)) \frac{\partial \hat{Q}}{\partial b}.$$

We conclude the section with an example. Consider once again $P(Q) = a - Q^d$ and $s(m, n) = \frac{mn}{m+n}$ which respectively yields constant elasticity of slope $\epsilon = d - 1$ and normalized matching function $S(z) = \frac{z}{1+z}$ where $z = \frac{n}{m}$. Assume $\beta(z, b) = \frac{bz}{1+bz}$ where $b \in (0, \infty)$. Furthermore, assume that $K_H = K_F$, i.e., $k = 1$. Using these assumptions and specific functional forms for inverse demand and matching functions we get:

$$\hat{z} = \frac{1}{\sqrt{b}}, \quad \hat{\beta} = \frac{\sqrt{b}}{1 + \sqrt{b}}, \quad \hat{\beta}S(\hat{z}) = \frac{\sqrt{b}}{(1 + \sqrt{b})^2}.$$

Note that

$$\frac{\partial \hat{\beta}S(\hat{z})}{\partial b} \equiv \frac{1 - \sqrt{b}}{2(1 + \sqrt{b})^3 \sqrt{b}} \gtreqless 0 \quad \Leftrightarrow \quad b \lesseqgtr 1,$$

which implies

$$\frac{\partial \hat{Q}}{\partial b} \gtreqless 0 \quad \Leftrightarrow \quad b \lesseqgtr 1.$$

Then, according to Proposition 5.1, for concave demand ($d > 1$), the optimal tariff $t(b)$ initially declines as b increases, becoming lowest at $b = 1$ (or $\beta = \frac{1}{2}$) and then increases as b increases further. For convex demand ($d < 1$), the relationship between b and $t(b)$ is exactly the opposite.
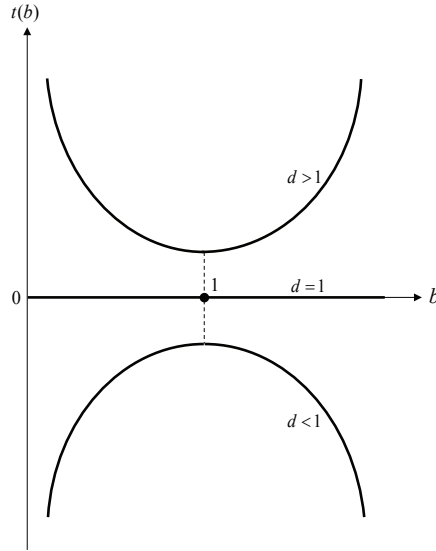
FIGURE 7 – Non-monotonicity of $t(b)$

If the demand is linear, i.e., $d = 1$, $t(b) = 0$ for $b \in (0, \infty)$. Figure 7 illustrates the relationship between b and $t(b)$.

6 Concluding Remarks

With reductions in trade costs, firms from various countries are increasingly specializing in different but complementary stages of production. In such environments of vertical specialization, under what conditions might a welfare maximizing government impose a tariff? We show that weak bargaining power of its firms might prompt a country's government to impose a tariff on Foreign producers. This negative relationship between bargaining power and tariff is fairly robust to a variety of alternative specifications of the model, including the presence of Foreign consumers, strategic interactions between governments, ad valorem tariffs and an alternative bargaining mechanism among others. Surprisingly, we find that an increase in Home firms' bargaining power not only benefits Home producers but it can also benefit Foreign producers by lowering tariff rates. The inverse monotone relationship between bargaining power and tariff breaks down in the long run where a change in bargaining power affects the market structure through matching and entry. Both when bargaining power is exogenously given and endogenously determined optimal Home tariff is non-monotone in bargaining power unless the demand function is linear. For linear demand, free trade is optimal irrespective of the bargaining strength.

Throughout the paper, we have focused on oligopolistic competition and complete contracts to highlight the novel interaction between bargaining power and trade policy. Conceptually, a similar analysis could be applied to models of monopolistic competition and incomplete con-

29

tracts – two features that have been extensively used in the recent literature on outsourcing and vertical specialization (see, e.g., Grossman and Helpman (2002)). Typically, these models have relationship-specific investments. Although our model abstracts from such investments, entry plays a similar role in our framework. In a model with relationship-specific investments, an increase in the bargaining power of downstream firms would reduce upstream investments. In a similar spirit, we also find that an increase in the bargaining power of downstream firms discourages entry by upstream firms. Given the similarity in effects of entry and investment, we conjecture that a non-monotone relationship between bargaining power and tariff would arise in models with relationship-specific investments.

Finally, we conclude with a remark on cost heterogeneity. While firms with heterogeneous costs have become an integral part of new trade models with monopolistic competition since the seminal work of Melitz (2003), the implications of cost heterogeneity on trade policy have not been adequately explored. In the context of partial equilibrium oligopoly models, Lahiri and Ono (2007, Chapter 4) and Long and Soubreyan (1997), for example, have shown that trade and industrial policies arising from asymmetric oligopoly can be significantly different from those arising from symmetric oligopoly. In future work, we plan to incorporate cost asymmetry in oligopoly models with vertical relationships and examine its implication for trade policy.

Appendix

A Proofs for Section 3

A.1 Equivalence between Assumptions 1 and 1'

The assumption $Q(P)$ is logconcave implies

$$\frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{\mathrm{d}\ln Q(P)}{\mathrm{d}P}\right] = \frac{\mathrm{d}}{\mathrm{d}P}\left[\frac{Q'(P)}{Q(P)}\right]$$
$$= \frac{Q(P)\cdot Q''(P) - [Q'(P)]^2}{[Q(P)]^2} \leq 0,$$

which can be expressed as

$$\frac{Q(P)Q''(P)}{[Q'(P)]^2} \leq 1. \tag{A.1}$$

Differentiating $P = P(Q(P))$ with respect to P, we get

$$1 = P'(Q(P))Q'(P).$$

Differentiating this once again with respect to P gives

$$0 = P''[Q'(P)]^2 + P'Q''(P).$$

Rewriting this equation, we get

$$\frac{Q''(P)}{[Q'(P)]^2} = -\frac{P''}{P'}.$$

Substituting this relationship into (A.1), we find that

$$-\frac{QP''(Q)}{P'(Q)} \leq 1,$$

which implies $P'(Q) + QP''(Q) \leq 0$. $\qquad\square$

A.2 Proof of Lemma 3.1

(i) It immediately follows from noting that $\hat{Q}$, i.e., the value of Q that solves (3.3), does not depend on β.
(ii) Totally differentiating (3.2) and (3.3) yields

$$\frac{\mathrm{d}\hat{Q}}{\mathrm{d}t} = \frac{s/P'(\hat{Q})}{s+1+\epsilon(\hat{Q})},$$

$$\frac{\mathrm{d}\hat{P}}{\mathrm{d}t} = P'(\hat{Q})\frac{\mathrm{d}\hat{Q}}{\mathrm{d}t} = \frac{s}{s+1+\epsilon(\hat{Q})},$$

$$\frac{\mathrm{d}\hat{r}}{\mathrm{d}t} = (1-\beta)\frac{\mathrm{d}\hat{P}}{\mathrm{d}t} + \beta = \frac{s+\beta(1+\epsilon(\hat{Q}))}{s+1+\epsilon(\hat{Q})},$$

where

$$\epsilon(Q) \equiv \frac{P''(Q)Q}{P'(Q)}$$

represents the elasticity of the slope of the demand. From Assumption 1' it follows that $\epsilon(Q) \geq -1$, which in turn implies that $s+1+\epsilon(\hat{Q}) > 0$. The claim directly follows from noting that $P'(\hat{Q}) < 0$ and $s+\beta(1+\epsilon(\hat{Q})) > 0$.
(iii) We have that

$$\frac{\mathrm{d}r^*}{\mathrm{d}t} = \frac{\mathrm{d}\hat{r}}{\mathrm{d}t} - 1 = (1-\beta)\left(\frac{\mathrm{d}\hat{P}}{\mathrm{d}t} - 1\right) = -(1-\beta)\frac{1+\epsilon(\hat{Q})}{s+1+\epsilon(\hat{Q})}.$$

The claim follows from observing that $1 - \beta > 0$ and $s + 1 + \epsilon(\hat{Q}) > 0$. $\qquad\square$

A.3 Proof of Proposition 3.1

Substitute the expressions of $\frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t}$ and $\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t}$ from the proof of Lemma 3.1 in the right-hand side of (3.4). Subsequently setting $\frac{\mathrm{d}W_H}{\mathrm{d}t} = 0$ and solving for t gives (3.6). Concerning the properties of $t(\beta)$, consider (ii) first. Differentiating $\frac{\mathrm{d}W_H}{\mathrm{d}t}\big|_{t=t(\beta)} = 0$ with respect to β gives:

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = -\frac{\frac{\partial^2 W_H}{\partial\beta\partial t}}{\frac{\partial^2 W_H}{\partial t^2}}.$$

Assuming the second-order condition is satisfied, i.e. $\frac{\partial^2 W_H}{\partial t^2} < 0$, it follows that

$$\mathrm{sgn}\frac{\mathrm{d}t}{\mathrm{d}\beta} = \mathrm{sgn}\frac{\partial^2 W_H}{\partial\beta\partial t}. \tag{A.2}$$

Differentiating $\frac{\partial W_H}{\partial t}$ in (3.4) with respect to β gives

$$\frac{\partial^2 W_H}{\partial\beta\partial t} = (\hat{P}(t) - c - t)\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} - \left(1 - \frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t}\right)\hat{Q}(t) < 0,$$

where the inequality follows from noting that $\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}t} < 0$ and $1 - \frac{\mathrm{d}\hat{P}(t)}{\mathrm{d}t} > 0$. Since $\frac{\partial^2 W_H}{\partial\beta\partial t} < 0$, equation (A.2) implies that $\frac{\mathrm{d}t}{\mathrm{d}\beta} < 0$. From $\Pi_H(t) = \beta(\hat{P}(t) - c - t)\hat{Q}(t)$, it also follows that $\frac{\mathrm{d}\Pi_H(t)}{\mathrm{d}t} = \beta\frac{\partial^2 W_H}{\partial\beta\partial t}$ and hence $\mathrm{sgn}\frac{\mathrm{d}\Pi_H(t)}{\mathrm{d}t} = \mathrm{sgn}\frac{\mathrm{d}t(\beta)}{\mathrm{d}\beta}$. The proof of part (i) follows from combining part (ii) and the following implication of (3.6): $t(0) > 0$ and $t(1) < 0$. $\qquad\square$

A.4 Proof of Proposition 3.2

First, we show the derivation of $\frac{\mathrm{d}\pi}{\mathrm{d}t}$ for the class of inverse demand functions with constant ϵ. The optimal tariff $t = t(\beta)$ is given by

$$t = \left\{-P'(\hat{Q}(t))\hat{Q}(t)\cdot(2+\hat{\epsilon})\right\}(\hat{\beta} - \beta),$$

where $\hat{\beta} = (1 + \epsilon)/(2 + \epsilon)$. If ϵ is constant, $\hat{\epsilon} = \epsilon(\hat{Q}) = \epsilon$. Differentiating t with respect to β gives

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = \left\{-P'(\hat{Q}(t))\hat{Q}(t)(2+\epsilon)\right\}(-1) - (\hat{\beta} - \beta)(2+\epsilon)\left[P''(\hat{Q})\frac{\hat{Q}(t)}{s} + \frac{P'(\hat{Q}(t))}{s}\right]\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}\beta}, \tag{A.3}$$

where $\frac{\mathrm{d}\hat{Q}(t)}{\mathrm{d}\beta} = \frac{s/P'(\hat{Q}(t))}{s+1+\epsilon}\cdot\frac{\mathrm{d}t}{\mathrm{d}\beta}$ from (3.3). Substituting this into (A.3) yields

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = -(2+\epsilon)\left[(\hat{\beta} - \beta)\frac{\mathrm{d}t}{\mathrm{d}\beta}\left(\frac{1+\epsilon}{s+1+\epsilon}\right) - P'(\hat{Q}(t))\hat{Q}(t)\right],$$

which gives the following upon rearrangement:

$$\frac{\mathrm{d}t}{\mathrm{d}\beta} = \frac{P'(\hat{Q}(t))\hat{Q}(t)\cdot(2+\epsilon)}{1 + [(1+\epsilon) - \beta(2+\epsilon)]\left(\frac{1+\epsilon}{s+1+\epsilon}\right)} < 0.$$

Next, we show Proposition 3.2. Since $\Pi_F = s\pi_F$ and s is fixed, we have $\frac{\mathrm{d}\Pi_F}{\mathrm{d}\beta} > 0 \Leftrightarrow \frac{\mathrm{d}\pi_F}{\mathrm{d}\beta} > 0$. We have that

$$\frac{\mathrm{d}\pi_F}{\mathrm{d}\beta} = -\pi + (1 - \beta)\frac{\partial\pi}{\partial t}\cdot\frac{\mathrm{d}t}{\mathrm{d}\beta}.$$

Differentiating $\pi = [P(\hat{Q}) - c - t]\hat{q} = -P'(\hat{Q})\hat{q}^2$ with respect to t gives $\frac{\mathrm{d}\pi}{\mathrm{d}t} = -\hat{q}\cdot\left(\frac{2+\epsilon}{s+1+\epsilon}\right) < 0$. Substituting $\frac{\mathrm{d}\pi}{\mathrm{d}t}$ and $\frac{\mathrm{d}t}{\mathrm{d}\beta}$ derived

above into the above equation, we get

$$\frac{\mathrm{d}\pi_F}{\mathrm{d}\beta} = \left(\frac{(2+\epsilon)\pi}{s+(1-\beta)(\epsilon+1)(\epsilon+2)}\right)\left(\frac{2+\epsilon-s}{2+\epsilon}-\beta\right).$$

The result follows from noting that the value in the first parentheses is strictly positive. $\square$

B Proofs for Section 4

B.1 Proof of Lemma 4.1

Rewrite the Home firm's free entry condition (4.10) as

$$-\frac{P'(\hat{Q})\hat{Q}^2}{\hat{s}^2} = \frac{K_H}{\beta S(\hat{z})}. \tag{B.1}$$

(i) Differentiating (B.1) with respect to t gives

$$\frac{\partial \hat{Q}}{\partial t} = \frac{2\hat{q}}{2+\hat{\epsilon}} \cdot \frac{\partial \hat{s}}{\partial t}. \tag{B.2}$$

Furthermore, totally differentiating (4.9) with respect to t and rearranging, we get

$$\frac{\partial \hat{Q}}{\partial t} = \frac{\hat{s}/P'(\hat{Q})}{\hat{s}+1+\hat{\epsilon}} + \frac{\hat{q}}{\hat{s}+1+\hat{\epsilon}}\frac{\partial \hat{s}}{\partial t}. \tag{B.3}$$

Solving (B.2) and (B.3) for $\frac{\partial \hat{s}}{\partial t}$ and $\frac{\partial \hat{Q}}{\partial t}$ yields

$$\frac{\partial \hat{s}}{\partial t} = \left(\frac{2+\hat{\epsilon}}{2\hat{s}+\hat{\epsilon}}\right)\left(\frac{\hat{s}}{P'(\hat{Q})\hat{q}}\right), \quad \frac{\partial \hat{Q}}{\partial t} = \frac{2\hat{s}}{(2\hat{s}+\hat{\epsilon})P'(\hat{Q})},$$

which in turn gives

$$\frac{\partial \hat{P}}{\partial t} = P'(\hat{Q})\frac{\partial \hat{Q}}{\partial t} = \frac{2\hat{s}}{2\hat{s}+\hat{\epsilon}}, \quad \frac{\partial \hat{r}}{\partial t} = (1-\beta)\frac{\partial \hat{P}}{\partial t} + \beta = \frac{2\hat{s}+\beta\hat{\epsilon}}{2\hat{s}+\hat{\epsilon}}.$$

The claim follows from observing that $2\hat{s}+\hat{\epsilon} > 0$, $2\hat{s}+\beta\hat{s} > 0$, and $P'(\hat{Q}) < 0$.

(ii) Differentiating $r^* \equiv \hat{r} - t = (1-\beta)(\hat{P}-c-t)+\beta c$ with respect to t gives

$$\frac{\partial r^*}{\partial t} = (1-\beta)\left(\frac{\partial \hat{P}}{\partial t}-1\right) = -\frac{(1-\beta)\hat{\epsilon}}{2\hat{s}+\hat{\epsilon}}.$$

The claim follows from observing that $1-\beta > 0$ and $2\hat{s}+\hat{\epsilon} > 0$. $\square$

B.2 Proof of Lemma 4.2

Defining $k = \frac{K_H}{K_F}$ we rewrite (4.11) as $z = (\frac{1}{\beta}-1)k$. Thus,

$$\beta S(\hat{z}) = \beta S\left(\left(\frac{1}{\beta}-1\right)k\right).$$

Since both $\beta \in (0,1)$ and $S(\hat{z}) \in (0,1)$,

$$0 \le \beta S\left(\left(\frac{1}{\beta}-1\right)k\right) \le \min\left\{\beta, S\left(\left(\frac{1}{\beta}-1\right)k\right)\right\}.$$

We have that $\lim_{\beta \to 0} \beta = 0$ and $\lim_{\beta \to 1} S(\hat{z}) = 0$ (by applying (4.7)) which together with the fact that $\beta \leq 1$ and $S(z) \leq 1$ implies

$$\lim_{\beta \to 0} \beta S(\hat{z}) = 0, \quad \lim_{\beta \to 1} \beta S(\hat{z}) = 0.$$

These limit values imply that $\beta S(\hat{z})$ is non-monotone in β. The particular nature of non-monotonicity then follows from noting that $\frac{\partial \beta S((\frac{1}{\beta}-1)k)}{\partial \beta} = \frac{k^2}{\beta^3} S''(\hat{z}) < 0$, i.e., $\beta S(\hat{z})$ is strictly concave in β. $\qquad\square$

B.3   Proof of Lemma 4.3

The proof is immediate from Lemma 4.2 and the discussion preceding Lemma 4.3. Here we just derive $\frac{\partial \hat{Q}}{\partial \beta}$ and $\frac{\partial \hat{s}}{\partial \beta}$ for future reference. Differentiating the first-order condition (4.9) with respect to β, we get

$$\frac{\partial \hat{Q}}{\partial \beta} = \frac{\hat{q}}{\hat{s} + 1 + \hat{\epsilon}} \frac{\partial \hat{s}}{\partial \beta},$$

where $\hat{\epsilon} \equiv \epsilon(\hat{Q}(t(\beta), \beta))$. Similarly, differentiating the Home firm's free-entry condition (B.1) with respect to β, we get

$$\frac{\sigma(\beta)}{1 - \beta} - 1 = \beta \left(\frac{2 + \hat{\epsilon}}{\hat{Q}} \frac{\partial \hat{Q}}{\partial \beta} - \frac{2}{\hat{s}} \frac{\partial \hat{s}}{\partial \beta} \right),$$

where $\sigma(\beta) \equiv \frac{\hat{z}(\beta) S'(\hat{z}(\beta))}{S(\hat{z}(\beta))}$. Note from (4.8) that $\sigma(\beta) \in (0, 1)$. Solving the above two equations for $\frac{\partial \hat{Q}}{\partial \beta}$ and $\frac{\partial \hat{s}}{\partial \beta}$ gives

$$\frac{\partial \hat{Q}}{\partial \beta} = \frac{\hat{Q}}{\beta(2\hat{s} + \hat{\epsilon})} \left(1 - \frac{\sigma(\beta)}{1 - \beta} \right), \quad \frac{\partial \hat{s}}{\partial \beta} = \frac{\hat{s}(\hat{s} + 1 + \hat{\epsilon})}{\beta(2\hat{s} + \hat{\epsilon})} \left(1 - \frac{\sigma(\beta)}{1 - \beta} \right).$$

The result follows from observing that $\lim_{\beta \to 0} \frac{\partial \hat{Q}}{\partial \beta} > 0$, $\lim_{\beta \to 1} \frac{\partial \hat{Q}}{\partial \beta} < 0$, $\lim_{\beta \to 0} \frac{\partial \hat{s}}{\partial \beta} > 0$, $\lim_{\beta \to 1} \frac{\partial \hat{s}}{\partial \beta} < 0$ and $P'(Q) < 0$ for all $Q \geq 0$. $\qquad\square$

B.4   Proof of Proposition 4.1

Substitute the expressions for $\frac{d\hat{P}}{dt}$ and $\frac{d\hat{Q}}{dt}$ from proof of Lemma 4.1 in the right-hand side of (4.12). Subsequently setting $\frac{dW_H}{dt} = 0$ and solving for t gives (4.13). The proof of part (i) is immediate from observing (4.13). To see (ii), differentiate $\frac{dW_H}{dt}\big|_{t=t(\beta)} = 0$ with respect to β which gives:

$$\frac{dt}{d\beta} = -\frac{\frac{\partial^2 W_H}{\partial \beta \partial t}}{\frac{\partial^2 W_H}{\partial t^2}}.$$

Since $\frac{\partial^2 W_H}{\partial t^2} < 0$ (second-order condition), $\text{sgn}\frac{dt}{d\beta} = \text{sgn}\frac{\partial^2 W_H}{\partial \beta \partial t}$. Differentiating W_H with respect to t and simplifying, we get:

$$\frac{dW_H}{dt} = \frac{2\hat{s}t + P'(\hat{Q})\hat{Q}\hat{\epsilon}}{(2\hat{s} + \hat{\epsilon})P'(\hat{Q})}.$$

Note from (4.13) that $2\hat{s}t + P'(\hat{Q})\hat{Q}\hat{\epsilon} = 0$ at $t = t(\beta)$. Differentiating the above equation with respect to β yields:

$$\begin{aligned}
\frac{\partial^2 W_H}{\partial \beta \partial t}\Big|_{t=t(\beta)} &= \frac{2t\frac{\partial \hat{s}}{\partial \beta} + \left[\hat{Q}^2 P'''(\hat{Q}) + 2\hat{Q}P''(\hat{Q}) \right] \frac{\partial \hat{Q}}{\partial \beta}}{(2\hat{s} + \hat{\epsilon})P'(\hat{Q})} &&\text{(using } \tfrac{dt}{d\beta} = 0 \text{ at } t = t(\beta)) \\
&= \frac{\left[2t\frac{\hat{s}+1+\hat{\epsilon}}{\hat{q}} + \hat{Q}P''(\hat{Q})(2 + \hat{\alpha}) \right]}{(2\hat{s} + \hat{\epsilon})P'(\hat{Q})} \frac{\partial \hat{Q}}{\partial \beta} &&\text{(using } \tfrac{\partial \hat{s}}{\partial \beta} = \tfrac{\hat{s}+1+\hat{\epsilon}}{\hat{q}} \tfrac{\partial \hat{Q}}{\partial \beta} \text{ from Lemma 4.3)} \\
&= \frac{\left[-\hat{Q}P''(\hat{Q})(\hat{s} + 1 + \hat{\epsilon}) + \hat{Q}P''(\hat{Q})(2 + \hat{\alpha}) \right]}{(2\hat{s} + \hat{\epsilon})P'(\hat{Q})} \frac{\partial \hat{Q}}{\partial \beta} &&\text{(using (4.13))} \\
&= -\frac{\hat{\epsilon}(\hat{s} - 1 + \hat{\epsilon} - \hat{\alpha})}{2\hat{s} + \hat{\epsilon}} \frac{\partial \hat{Q}}{\partial \beta}.
\end{aligned}$$

where $\alpha(Q) \equiv \frac{QP'''(Q)}{P''(Q)}$. Then, the result follows from noting that $\operatorname{sgn} \epsilon(\hat{Q}) = -\operatorname{sgn} P''(\hat{Q})$ (definition of $\epsilon(Q)$), $2\hat{s} + \hat{\epsilon} > 0$ (Assumption 1), $\hat{s} - 1 + \hat{\epsilon} - \hat{\alpha} > 0$ (Assumption 2), and non-monotonicity of $\partial \hat{Q}/\partial \beta$ (Lemma 4.3). $\square$

B.4   Proof of Proposition 4.2

The proof is immediate from applying Proposition 4.1 and Lemma 4.3. $\square$

References

[1] ANDERSON, S. P. AND RENAULT, R. (2003): "Efficiency and Surplus Bounds in Cournot Competition." *Journal of Economic Theory*, 113(2), 253-64.

[2] ANTRÀS, P. AND ROSSI-HANSBERG, E. (2009): "Organizations and Trade." *Annual Review of Economics*, 1, 43-64.

[3] ——— AND STAIGER, R. W. (2012a): "Offshoring and the Role of Trade Agreements." *American Economic Review*, 102(7), 3140-83.

[4] ——— AND ——— (2012b): "Trade Agreements and the Nature of Price Determination." *American Economic Review Papers and Proceedings*, 102(3), 470-76.

[5] BAGWELL, K. AND STAIGER, R. W. (2012a): "The Economics of Trade Agreements in the Linear Cournot Delocation Model." *Journal of International Economics,* 88(1), 32-46.

[6] ——— AND ——— (2012b): "Profit Shifting and Trade Agreements in Imperfectly Competitive Markets." *International Economic Review,* 53(4), 1067-104.

[7] BLANCHARD, E. (2007): "Foreign Direct Investment, Endogenous Tariffs, and Preferential Trade Agreements." *The B.E. Journal of Economic Analysis & Policy*, 7(1), [Advances].

[8] ——— (2010): "Reevaluating the Role of Trade Agreements: Does Investment Globalization Make the WTO Obsolete?" *Journal of International Economics*, 82(1), 63-72.

[9] ——— AND MATSCHKE, X. (forthcoming): "U.S. Multinationals and Preferential Market Access." *Review of Economics and Statistics*.

[10] BRANDER, J. A. AND SPENCER, B. J. (1984a): "Trade and Welfare: Tariffs and Cartels." *Journal of International Economics,* 16(3-4), 227-42.

[11] ——— AND ——— (1984b): "Tariff Protection and Imperfect Competition." In *Monopolistic Competition and Product Differentiation and International Trade,* edited by Kierzkowski, H., Oxford: Oxford University Press, 1984, 194-207.

[12] ——— AND ——— (1985): "Export Subsidies and International Market Share Rivalry." *Journal of International Economics,* 18, 83-100.

[13] CHEN, Y., ISHIKAWA, J. AND YU, Z. (2004): "Trade Liberalization and Strategic Outsourcing." *Journal of International Economics,* 63(2), 419-36.

[14] COLE, M. T. AND DAVIES, R. B. (2011): "Strategic Tariffs, Tariff Jumping, and Heterogeneous Firms." *European Economic Review*, 55(4), 480-96.

[15] COSTINOT, A. (2009): "An Elementary Theory of Comparative Advantage." *Econometrica*, 77(4), 1165-92.

[16] COWAN, S. (2007): "The Welfare Effects of Third-Degree Price Discrimination with Nonlinear Demand Functions." *RAND Journal of Economics,* 38(2), 419-28.

[17] DENECKERE, R. AND KOVENOCK, D. (1999): "Direct Demand-Based Cournot Existence and Uniqueness Conditions." Mimeo.

[18] ETRO, F. (2011): "Endogenous Market Structures and Strategic Trade Policy." *International Economic Review,* 52(1), 63-84.

[19] GROSSMAN, G. M. AND HELPMAN, E. (2002): "Integration Versus Outsourcing in Industry Equilibrium." *Quarterly Journal of Economics,* 117(1), 85-120.

[20] HANSON, G. H., MATALONI, R. J., AND SLAUGHTER, M. J. (2005): "Vertical Production Networks in Multinational Firms." *Review of Economics and Statistics*, 87(4), 664-78.

[21] HASAN, R., MITRA, D., AND RAMASWAMY, K.V. (2007): "Trade Reforms, Labor Regulations, and Labor-Demand Elasticities: Empirical Evidence from India." *Review of Economics and Statistics*, 89(3), 466-81.

[22] HELPMAN, E. AND KRUGMAN, P. (1989): *Trade Policy and Market Structure.* Cambridge, MA: MIT Press.

[23] HORSTMANN, I. J. AND MARKUSEN, J. R. (1986): "Up the Average Cost Curve: Inefficient Entry and the New Protectionism." *Journal of International Economics,* 20(3-4), 225-47.

[24] HUMMELS, D., ISHII, J. AND YI, K.-M. (2001): "The Nature and Growth of Vertical Specialization in World Trade." *Journal of International Economics,* 54(1), 75-96.

[25] ISHIKAWA, J. AND LEE, K.-D. (1997): "Backfiring Tariffs in Vertically Related Markets." *Journal of International Economics,* 42(3-4), 395-423.

[26] ——— AND SPENCER, B. (1999): "Rent-Shifting Export Subsidies with an Imported Intermediate Product." *Journal of International Economics,* 48(2), 199-232.

[27] JONES, R., KIERZKOWSKI, H. AND LURONG, C. (2005): "What Does Evidence Tell Us about Fragmentation and Outsourcing?" *International Review of Economics and Finance,* 14(3), 305-16.

[28] KIMURA, F. AND ANDO, M. (2005): "Two-Dimensional Fragmentation in East Asia: Conceptual Framework and Empirics." *International Review of Economics and Finance,* 14(3), 317-48.

[29] LAHIRI, S. AND ONO, Y. (2007): *Trade and Industrial Policy under International Oligopoly.* Cambridge, UK: Cambridge University Press.

[30] LIMÃO, N. (2006): "Preferential Trade Agreements as Stumbling Blocks for Multilateral Trade Liberalization: Evidence for the United States." *American Economic Review*, 96(3), 896-914.

[31] LONG, N. V. AND SOUBREYAN, A. (1997) "Cost Heterogeneity, Industry Concentration and Strategic Trade Policies." *Journal of International Economics*, 43(1-2), 207-20.

[32] MELITZ, M. J. (2003): "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity." *Econometrica,* 71(6), 1695-725.

[33] NOCKE, V. AND WHINSTON, M. D. (2010): "Dynamic Merger Review." *Journal of Political Economy*, 118(6), 1201-51.

[34] OLARREAGA, M. AND ÖZDEN, Ç. (2005): "AGOA and Apparel: Who Captures the Tariff Rent in the Presence of Preferential Market Access?" *World Economy*, 28(1), 63-77.

[35] ORNELAS, E. AND TURNER, J. L. (2008): "Trade Liberalization, Outsourcing, and the Hold-Up Problem." *Journal of International Economics*, 74(1), 225-41.

[36] ——— AND ——— (2012): "Protection and International Sourcing." *Economic Journal*, 112(559), 26-63.

[37] QIU, L. D. AND SPENCER, B. J. (2002): "Keiretsu and Relationship-Specific Investment: Implications for Market-Opening Trade Policy." *Journal of International Economics,* 58(1), 49-79.

[38] SHIMER, R., AND SMITH, L. (2000): "Assortative Matching and Search." *Econometrica*, 68(2), 343-69.

[39] SPENCER, B. J. (2005): "International Outsourcing and Incomplete Contracts." *Canadian Journal of Economics*, 38(4), 1107-135.

[40] ——— AND QIU, L. D. (2001): "Keiretsu and Relationship-Specific Investment: A Barrier to Trade?" *International Economic Review,* 42(4), 871-901.

[41] VENABLES, A. (1985): "Trade and Trade Policy with Imperfect Competition: The Case of Identical Products and Free Entry." *Journal of International Economics*, 19(1-2), 1-19.

[42] WOLINSKY, A. (1987): "Matching, Search and Bargaining." *Journal of Economc Theory*, 42(2), 311-33.

[43] YEATS, A. J. (2001): "Just How Big Is Global Production Sharing?" In *Fragmentation: New Production Patterns in the World Economy,* edited by Arndt, S. W. and Kierzkowski, H., New York and Oxford: Oxford University Press, 2001, 108-43.

[44] YI, K.-M. (2003): "Can Vertical Specialization Explain the Growth of World Trade?" *Journal of Political Economy*, 111(1), 52-102.