Trade with Search Frictions: Identifying New Gains from Trade

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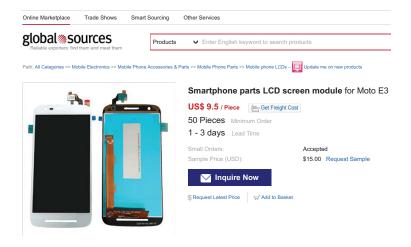
- Firms often search for suppliers to procure specialized inputs:
  - While a few core inputs are made in-house, other non-core inputs are largely purchased from the outside
  - Transactions of such outsourced inputs involve a substantial investment in customizing inputs for the needs of firms
  - Recent advances of information technology make it easier to search for suppliers not only within borders but also across borders
  - $\Rightarrow$  Consider Apple's sourcing strategy

### Example: smartphones



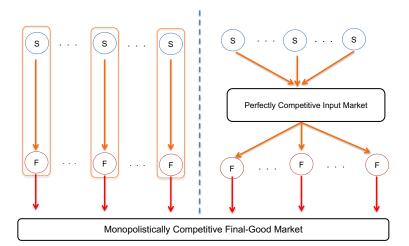
"Non-market" transaction

## Example: smartphones (cont.)



"Market" transaction

### Two types of input transactions



- Sources of welfare gains:
  - Resource reallocation between firms (Melitz, 2003)
  - Matching improvement of firms
- New impact of trade:
  - The number of suppliers rises relative to the number of firms, thereby thickening the market of intermediate inputs
  - $\bullet\,$  Gains from trade  $\Rightarrow$  0.9% without search but 3.1% with search

- Search frictions and trade:
  - Grossman and Helpman (2002, 2005), Antràs and Costinot (2011), Felbermayr et al. (2011)
  - This paper ⇒ Economic integration in goods/matching markets in monopolistic competition
- Ontractual frictions and offshoring:
  - Antràs (2003), Antràs and Helpman (2004), Ornelas and Turner (2008, 2012)
  - This paper  $\Rightarrow$  Possibility of welfare losses associated with offshoring



• Consumer preferences:

$$U = \left(\int_{0}^{N^{F}} y(\omega)^{rac{\sigma-1}{\sigma}} d\omega
ight)^{rac{\sigma}{\sigma-1}}, \quad \sigma>1$$

where  $N^F$  is the number (measure) of varieties in the industry

• Demand and expenditure for variety  $\omega$ :

$$y(\omega) = Ap(\omega)^{-\sigma}$$
  
 $r(\omega) = Ap(\omega)^{1-\sigma}$ 

where A is the index of industry demand

# Setup (cont.)

#### • Firm technology:

$$y(\alpha) = \alpha \left(\frac{x^{\mathsf{F}}}{\eta}\right)^{\eta} \left(\frac{x^{\mathsf{S}}}{1-\eta}\right)^{1-\eta}, \quad 0 < \eta < 1$$

where  $\boldsymbol{\alpha}$  is the industry-specific parameter

	Unmatched	Matched
Input transaction	Market	Non-market
Input type	Generic	Customized
Input quality	lpha = 1	$\alpha > 1$
Variable profit	$r(1)/\sigma$	$r(lpha)/\sigma$
Firm profit	$r(1)/\sigma$	$r^{F}(lpha)/\sigma$
Supplier profit	0	$r^{S}(lpha)/\sigma$

• Equilibrium output and revenue:

$$y(\alpha) = A\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \alpha^{\sigma}$$
$$r(\alpha) = A\left(\frac{\sigma-1}{\sigma}\right)^{\sigma-1} \alpha^{\sigma-1}$$

• The ratio of equilibrium revenue of matched firms to unmatched firms:

$$\frac{r(\alpha)}{r} = \alpha^{\sigma-1}$$

where  $r \equiv r(1)$ 

• Number of matches:

$$m(u^F, u^S)$$

which satisfies CRS in matching

• Probability of matches:

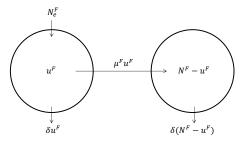
$$\mu^{F} \equiv m(u^{F}, u^{S})/u^{F} = m(1, \theta)$$
  
$$\mu^{S} \equiv m(u^{F}, u^{S})/u^{S} = m(1/\theta, 1) = \mu^{F}/\theta$$

where  $\theta \equiv u^S/u^F$ 

 $\bullet\,$  Probability of a bad shock:  $\delta\,$ 

# Setup (cont.)

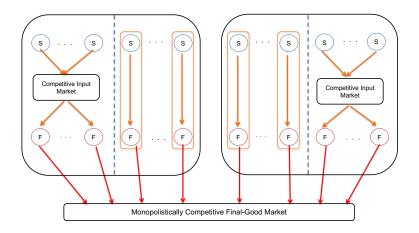
• Search process for firms:



• The law of motion:

$$\begin{split} \dot{N}^F &= \delta N^F - N^F_e \\ \dot{N}^F_e &= (\delta + \mu^F) u^F - N^F_e \\ \dot{u}^F &= \delta (N^F - u^F) - \mu^F u^F \end{split}$$

#### Costly trade: X-integration



• When only matched firms export, the Bellman equations are given by

$$\gamma V^{F} = \frac{r}{\sigma} + \mu^{F} \left( V^{F}(\alpha) - F_{x} - V^{F} \right) - \delta V^{F} + \dot{V}^{F}$$
$$\gamma V^{F}(\alpha) = \frac{r^{F}(\alpha)}{\sigma} - \delta V^{F}(\alpha) + \dot{V}^{F}(\alpha)$$
$$\gamma V^{S} = \mu^{S} \left( V^{S}(\alpha) - F_{d} - V^{S} \right) - \delta V^{S} + \dot{V}^{S}$$
$$\gamma V^{S}(\alpha) = \frac{r^{S}(\alpha)}{\sigma} - \delta V^{S}(\alpha) + \dot{V}^{S}(\alpha)$$

where  $F_d$  and  $F_x$  are a one-time investment cost

• Assuming that  $\gamma = 0$  and setting  $\dot{V}^F = \dot{V}^F(\alpha) = 0$ :

$$V^{F} = \frac{r}{\delta\sigma} + \left(\frac{\mu^{F}}{\delta + \mu^{F}}\right) \left(\frac{r^{F}(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_{x}\right)$$
$$V^{F}(\alpha) = \frac{r^{F}(\alpha)}{\delta\sigma}$$

where the probability  $\delta$  introduces an effect similar to time discounting

• Similarly, setting  $\dot{N}^F = \dot{N}_e^F = \dot{u}^F = 0$ :  $n = \left(\frac{\mu^F}{\delta + \mu^F}\right) N^F$ 

where  $n \equiv N^F - u^F$ 

• Bargaining within matched agents:

$$\max_{\frac{r^{F}(\alpha)}{\sigma}, \frac{r^{S}(\alpha)}{\sigma}} \left( V^{F}(\alpha) - F_{x} - V^{F} \right) \left( V^{S}(\alpha) - F_{d} - V^{S} \right)$$

subject to  $r^{F}(\alpha)/\sigma + r^{S}(\alpha)/\sigma = r(\alpha)/\sigma$ 

• Optimal sharing rule:

$$\frac{r^{F}(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_{x} = \beta \left(\frac{r(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_{d} - F_{x}\right)$$
$$\frac{r^{S}(\alpha)}{\delta\sigma} - F_{d} = (1 - \beta) \left(\frac{r(\alpha)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_{d} - F_{x}\right)$$
$$= (\delta + \mu^{F})/(2\delta + \mu^{F} + \mu^{S})$$

where  $\beta \equiv (\delta + \mu^F)/(2\delta + \mu^F + \mu^S)$ 

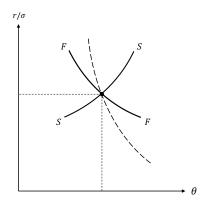
• FE conditions:

$$V_e^F \equiv V^F - F_e^F = 0$$
$$V_e^S \equiv V^S - F_e^S = 0$$

• From the steady-state relationships, this can be written as

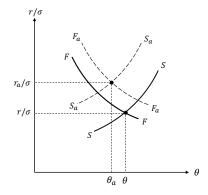
$$\frac{r}{\sigma} + \frac{n}{N^F} \beta \left( \frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d - f_x \right) - f_e^F = 0$$
$$\frac{n}{N^S} (1 - \beta) \left( \frac{r(\alpha)}{\sigma} - \frac{r}{\sigma} - f_d - f_x \right) - f_e^S = 0$$

where  $f_d \equiv \delta F_d$ ,  $f_x \equiv \delta F_x$ ,  $f_e^F \equiv \delta F_e^F$  and  $f_e^S \equiv \delta F_e^S$ 



$$\theta = u^S / u^F = (N^S - n) / (N^F - n)$$

- *FF* curve  $\theta \uparrow \Rightarrow \mu^F \uparrow \Rightarrow r/\sigma \downarrow$
- SS curve  $\theta \uparrow \Rightarrow \mu^{S} \downarrow \Rightarrow r/\sigma \uparrow$
- $\theta$  and  $r/\sigma$  are consistent with free entry in X-integration equilibrium



• Impact of X-integration

 $r/\sigma < r_a/\sigma$  $heta > heta_a$ 

 Matched firms get a larger rent by reductions in trade costs (*τ<sub>x</sub>*, *f<sub>x</sub>* ↓)

$$rac{r(lpha)}{\sigma} - rac{r}{\sigma} - f_d - f_x \uparrow$$

which induces new entry of agents

- Gains from trade (GFT) in X-integration:
  - r/σ < r<sub>a</sub>/σ ⇒ Resources are reallocated from (less efficient) unmatched firms to (more efficient) matched firms
  - **2**  $\theta > \theta_a \implies$  Firms have the higher probability to meet suppliers  $(n/N^F > n_a/N_a^F)$ , enhancing overall production efficiency of the industry

#### • GFT are expressed as

$$\frac{W}{W_a} = \left[ \left( \frac{N_a^F + (\alpha^{\sigma-1} - 1)n_a}{N^F + (\alpha^{\sigma-1} - 1)n} \right) \lambda \right]^{-\frac{1}{\sigma-1}}$$

where  $\lambda$  is the expenditure share on domestic goods

- In Krugman (1980) where  $N^F = N^F_a$  and  $n = n_a = 0$ , this ratio is simply given as  $W/W_a = \lambda^{-1/(\sigma-1)}$  (Arkolakis et al., 2012)
- **2** In our model where  $n/N^F > n_a/N_a^F$ , the values in the brackets (endogenous firm matches) matter for welfare
- 0 Numerical solutions  $\Longrightarrow$  GFT are 0.9% without search but 3.1% with search

- Main findings:
  - Search in standard workhorse models of trade can amplify welfare gains
  - Such gains are important not only qualitatively but also quantitatively
- Extensions:
  - Economic integration in matching markets
  - Trade liberalization facilitating this integration may cause welfare losses from trade