Online Appendix

The Online Appendix is organized as follows. Online Appendix A derives the Bellman equations and bargaining outcomes in the closed and open economies. Online Appendix B analyzes export selection under X-integration. Online Appendix C shows extensions and robustness checks for M-integration. Online Appendix D reports details of the quantitative analysis.

A Bellman Equations and Surplus Sharing

This appendix presents the Bellman equations underlying the value functions used in the main text and derives the corresponding bargaining outcomes and surplus-splitting rules in both the closed and open economies.

A.1 Closed Economy

Let γ denote a common discount rate. Then, the value functions must satisfy the following Bellman equations that characterize the expected profits of the different types of agents:

$$\begin{aligned}
\gamma V^F(\varphi) &= \pi^F(\varphi) - \delta V^F(\varphi) + \dot{V}^F(\varphi),\\
\gamma V^F &= \pi + \mu^F\Big(V^F(\varphi) - V^F\Big) - \delta V^F + \dot{V}^F,\\
\gamma V^S(\varphi) &= \pi^S(\varphi) - \delta V^S(\varphi) + \dot{V}^S(\varphi),\\
\gamma V^S &= \mu^S\Big(V^S(\varphi) - V^S\Big) - \delta V^S + \dot{V}^S.
\end{aligned} \tag{A.1}$$

Since we focus only on a stationary equilibrium, the value functions satisfy $\dot{V}^F(\varphi) = \dot{V}^F = \dot{V}^S(\varphi) = \dot{V}^S = 0$. We also assume a zero discount rate. The exogenous exit rate δ limits agents' expected lifetimes, introducing a force analogous to time discounting, while a positive discount rate would generally break the equality between aggregate profits and aggregate entry costs in a stationary equilibrium. As noted in Melitz (2003, footnote 16), the absence of time discounting restores this equality, which greatly simplifies the general-equilibrium analysis. Setting $\gamma = 0$ in (A.1) and rearranging yields the steady-state value functions reported in Section 3.1.

A.2 Open Economy

Let μ_d^F and μ_m^F denote the rates at which an unmatched firm meets a domestic and a foreign supplier in the open economy, respectively. Similarly, let μ_d^S and μ_m^S denote the corresponding rates for suppliers. Since cross-border matches constitute a constant fraction κ of domestic matches, we have $\mu_m^F/\mu_d^F = \mu_m^S/\mu_d^S = \kappa$, implying

$$\mu_d^F = \frac{\mu^F}{1+\kappa}, \qquad \mu_m^F = \frac{\kappa\mu^F}{1+\kappa}, \qquad \mu_d^S = \frac{\mu^S}{1+\kappa}, \qquad \mu_m^S = \frac{\kappa\mu^S}{1+\kappa}, \tag{A.2}$$

where $\mu^F = \mu_d^F + \mu_m^F$ and $\mu^S = \mu_d^S + \mu_m^S$ denote the total matching rates of firms and suppliers, respectively.

Using the matching rates in (A.2), the Bellman equations for firms in the open economy are expressed as

$$\begin{aligned}
\gamma V^F(\varphi) &= \pi^F(\varphi) - \delta V^F(\varphi) + \dot{V}^F(\varphi),\\
\gamma V^F(\varphi_m) &= \pi^F(\varphi_m) - \delta V^F(\varphi_m) + \dot{V}^F(\varphi_m),\\
\gamma V^F &= \pi + \mu_d^F\Big(V^F(\varphi) - V^F\Big) + \mu_m^F\Big(V^F(\varphi_m) - V^F\Big) - \delta V^F + \dot{V}^F.
\end{aligned} \tag{A.3}$$

Likewise, the Bellman equations for suppliers are expressed as

$$\begin{aligned}
\gamma V^S(\varphi) &= \pi^S(\varphi) - \delta V^S(\varphi) + \dot{V}^S(\varphi), \\
\gamma V^S(\varphi_m) &= \pi^S(\varphi_m) - \delta V^S(\varphi_m) + \dot{V}^S(\varphi_m), \\
\gamma V^S &= \mu_d^S\Big(V^S(\varphi) - V^S\Big) + \mu_m^S\Big(V^S(\varphi_m) - V^S\Big) - \delta V^S + \dot{V}^S.
\end{aligned} \tag{A.4}$$

Unlike the closed economy, unmatched agents may transition into either a domestic or a foreign relationship. Accordingly, the unmatched values V^F and V^S incorporate the option value of obtaining either type of future match and therefore serve as common outside options in all subsequent bargaining problems.

Setting $\gamma = 0$ and $\dot{V}^F = \dot{V}^F(\varphi) = \dot{V}^F(\varphi_m) = \dot{V}^S = \dot{V}^S(\varphi) = \dot{V}^S(\varphi_m) = 0$ in (A.3)–(A.4) and rearranging, the value functions of matched agents with domestic and cross-border matches are given by

$$\begin{aligned}
V^F(\varphi) &= \frac{\pi^F(\varphi)}{\delta}, \qquad V^F(\varphi_m) = \frac{\pi^F(\varphi_m)}{\delta}, \\
V^S(\varphi) &= \frac{\pi^S(\varphi)}{\delta}, \qquad V^S(\varphi_m) = \frac{\pi^S(\varphi_m)}{\delta}.
\end{aligned}$$

By contrast, the value functions of unmatched agents in the open economy are given by

$$\begin{aligned}
V^F &= \frac{\pi}{\delta} + \left(\frac{\mu_d^F}{\delta + \mu^F}\right)\left(\frac{\pi^F(\varphi)}{\delta} - \frac{\pi}{\delta}\right) + \left(\frac{\mu_m^F}{\delta + \mu^F}\right)\left(\frac{\pi^F(\varphi_m)}{\delta} - \frac{\pi}{\delta}\right), \\
V^S &= \left(\frac{\mu_d^S}{\delta + \mu^S}\right)\frac{\pi^S(\varphi)}{\delta} + \left(\frac{\mu_m^S}{\delta + \mu^S}\right)\frac{\pi^S(\varphi_m)}{\delta}.
\end{aligned} \tag{A.5}$$

To isolate the effects of heterogeneous match surpluses on congestion and welfare in the open economy while abstracting from additional heterogeneity in bargaining outcomes, we use a simplifying tractability assumption. Specifically, we maintain the same effective surplus-sharing coefficient β given by equation (5) and assume that it governs the division of flow surplus for all matches. We therefore apply the same effective surplus-sharing rule directly to both domestic and cross-border matches:

$$\begin{aligned}
\frac{\pi^F(\varphi)}{\delta} - \frac{\pi}{\delta} &= \beta\left(\frac{\pi(\varphi)}{\delta} - \frac{\pi}{\delta}\right), \qquad & \frac{\pi^F(\varphi_m)}{\delta} - \frac{\pi}{\delta} &= \beta\left(\frac{\pi(\varphi_m)}{\delta} - \frac{\pi}{\delta}\right), \\
\frac{\pi^S(\varphi)}{\delta} &= (1-\beta)\left(\frac{\pi(\varphi)}{\delta} - \frac{\pi}{\delta}\right), \qquad & \frac{\pi^S(\varphi_m)}{\delta} &= (1-\beta)\left(\frac{\pi(\varphi_m)}{\delta} - \frac{\pi}{\delta}\right).
\end{aligned} \tag{A.6}$$

The steady-state masses are given by (6), except that $n = n_d + n_m$ denotes the total mass of matched pairs in the open economy. Together with the matching rates in (A.2), the steady-state relationships imply

$$\frac{\mu_d^F}{\delta + \mu^F} = \frac{n_d}{N^F}, \qquad \frac{\mu_m^F}{\delta + \mu^F} = \frac{n_m}{N^F}, \qquad \frac{\mu_d^S}{\delta + \mu^S} = \frac{n_d}{N^S}, \qquad \frac{\mu_m^S}{\delta + \mu^S} = \frac{n_m}{N^S}. \tag{A.7}$$

In addition, the matching structure implies $n_m = \kappa n_d$.

As in the closed economy, free entry requires that the net value of entry be zero in stationary equilibrium. Since agents enter the matching market as unmatched and firms incur the fixed export cost F_x irrespective of matching status, free entry implies $V^F = F_e^F + F_x$ for firms and $V^S = F_e^S$ for suppliers. Combining the value functions in (A.5) with the match-specific surplus-sharing rules in (A.6) and the steady-state match ratios in (A.7) yields open-economy free-entry conditions (16) presented in Section 4.2.

B Export Selection under X-Integration

This appendix characterizes the equilibrium of X-integration when only matched firms export, showing that the key results continue to hold both theoretically and quantitatively under export selection.

B.1 Equilibrium Characterization

Modified Free-Entry Conditions When export selection is present, unmatched firms earn only domestic revenue r_d, whereas matched firms earn both domestic and export revenues:

$$r_d(\varphi) = \varphi^{\sigma-1} r_d, \qquad r_x(\varphi) = \left(\frac{\varphi}{\tau_x}\right)^{\sigma-1} r_d,$$

so that total revenue is $r(\varphi) = (1 + \tau_x^{1-\sigma})\varphi^{\sigma-1} r_d$. The revenue ratio between matched and unmatched firms is therefore larger than in autarky (see (1)):

$$\frac{r(\varphi)}{r_d} = (1 + \tau_x^{1-\sigma})\varphi^{\sigma-1}.$$

Unmatched firms earn domestic profits $\pi_d = r_d/\sigma (= \pi)$, while matched firms incur both matching costs f and export entry costs f_x. Total profits of matched agents are therefore

$$\pi(\varphi) = (1 + \tau_x^{1-\sigma})\varphi^{\sigma-1}\pi_d - f - f_x.$$

The free-entry conditions in X-integration with export selection are

$$\begin{aligned} \pi + \frac{n}{N^F}\beta\big(\pi(\varphi) - \pi\big) - f_e^F &= 0, \\ \frac{n}{N^S}(1-\beta)\big(\pi(\varphi) - \pi\big) - f_e^S &= 0. \end{aligned} \tag{B.1}$$

While (B.1) resembles (16) in Section 4.2, the export cost f_x does not appear directly in (B.1). Unlike the baseline model, only matched firms export under export selection, so unmatched firms do not incur the fixed export cost. Accordingly, the outside option remains π rather than $\pi - f_x$. The export cost instead affects entry through the surplus generated by a successful match, captured by $\pi(\varphi)$.

Using $\pi_d = \pi$, the domestic profits of unmatched firms are

$$\begin{aligned} \pi_d &= \frac{f_e^F + (f + f_x)\phi^F}{1 + \big((1 + \tau_x^{1-\sigma})\varphi^{\sigma-1} - 1\big)\,\phi^F}, \\ \pi_d &= \frac{f_e^S + (f + f_x)\phi^S}{\big((1 + \tau_x^{1-\sigma})\varphi^{\sigma-1} - 1\big)\,\phi^S}. \end{aligned} \tag{B.2}$$

Equation (B.2) is the counterpart of (17), with the key difference being that unmatched firms do not export in equilibrium. Nevertheless, entry decisions are forward-looking: unmatched agents anticipate that they may form a match in the future, at which point they begin exporting. The fixed export cost f_x enters as an additional fixed cost of exporting, whereas the iceberg trade cost τ_x lowers export profitability by reducing export revenues through the term $(1 + \tau_x^{1-\sigma})\varphi^{\sigma-1}$. Consequently, both trade costs affect expected entry profits through the profitability of future export relationships, as summarized by ϕ^F and ϕ^S.

Impact of X-Integration To examine how (B.2) responds to X-integration, recall the assumption that only matched firms export final goods. This outcome requires that exporting is profitable for matched firms but not for unmatched firms. Holding the domestic profit fixed at π_d, these conditions are

$$(1+\tau_x^{1-\sigma})\varphi^{\sigma-1}\pi_d - f - f_x > \varphi^{\sigma-1}\pi_d - f,$$
$$(1+\tau_x^{1-\sigma})\pi_d - f_x < \pi_d.$$

These inequalities imply that trade costs must lie in the intermediate range:

$$\pi_d < \tau_x^{\sigma-1} f_x < \varphi^{\sigma-1}\pi_d. \tag{B.3}$$

Comparing (8) and (B.2) under (B.3), the two curves in X-integration lie *below* those in autarky, reducing the unmatched profits π_d. Moreover, cancelling the common term in (B.1) shows that the relationship between $\pi_d(=\pi)$ and θ in (11) continues to hold in X-integration. Hence a decline in π_d increases θ, as illustrated in Panel (a) of Figure 3.

Intuitively, the downward shift reflects the larger joint surplus in X-integration relative to autarky:

$$\left((1+\tau_x^{1-\sigma})\varphi^{\sigma-1} - 1\right)\pi_d - f - f_x > \left(\varphi^{\sigma-1} - 1\right)\pi_d - f.$$

Higher surplus raises *ex ante* expected profits, inducing additional entry and thereby reducing *ex post* profitability in X-integration. Note that this inequality reduces to the second inequality in (B.3), which requires that matched firms export. Hence whether unmatched firms export is irrelevant for the downward shift of the two curves and the resulting equilibrium outcomes.

Equation (B.2) also implies that the two curves shift downward by different amounts. Although X-integration raises the joint match surplus for both firms and suppliers, it affects their expected profits differently because their outside options differ. Equation (B.1) shows that X-integration affects the free-entry condition of firms through two opposing channels. On the one hand, foreign competition lowers the outside option of firms (π), reducing their *ex post* profitability. On the other hand, access to export markets raises the joint surplus. These opposing effects dampen firms' entry incentives. For suppliers, however, the outside option remains zero, so they are unaffected by the competitive effect. As a result, X-integration increases the *ex ante* expected profits of suppliers relative to firms, leading to stronger *ex post* competition among suppliers. Since the mass of suppliers rises relative to firms, the ratio $\theta = u^S/u^F$ increases with X-integration.

Although the above discussion compares equilibria between autarky and X-integration, equation (B.2) also implies that trade liberalization within X-integration—through a reduction in iceberg trade costs τ_x—shifts the supplier-side curve downward by more than the firm-side curve, thereby affecting the key endogenous variables in a similar manner. This contrasts with the baseline analysis without export selection, where a reduction in τ_x proportionally shifts the two curves and therefore leaves θ unchanged within X-integration.

Once the two endogenous variables (θ, π_d) are determined, the remaining equilibrium variables can be written as a function of them. In particular, the labor-market-clearing condition implies $R = L$ in stationary equilibrium. The masses of firms, suppliers, and matches are therefore given by (9). Under export selection, however, matched firms earn both domestic and export revenue, whereas unmatched firms earn only domestic revenue. Accordingly, (9) continues to hold with $\Xi = \sigma\pi_d\big(1+(1+\tau_x^{1-\sigma})\varphi^{\sigma-1}\mu^F/\delta\big)$. Welfare is given by (10), except that π is replaced by π_d, which is determined by the right-hand side of (B.2). This completes the characterization of the unique stationary equilibrium of the open economy with export selection, where only matched firms export.

B.2 Welfare-Relevant Statistics

Given the impact of X-integration described above, we now examine the gains from trade under export selection. In particular, we show that the key welfare-relevant statistics in the model—the unmatched expenditure share s and the mass of unmatched firms u^F—continue to characterize welfare.

Welfare Decomposition As the unmatched profits π_d fall in X-integration relative to autarky, the welfare expression in (10) implies that this integration always generates gains from trade in equilibrium.

To characterize the gains from trade in this extension, we follow ACR and define the partial trade elasticity as $\varepsilon = -\partial \ln(R_x/R_d)/\partial \ln \tau_x$, where the partial derivative holds matching outcomes fixed. Under export selection $R_d = (N^F - n)r + nr_d(\varphi)$ and $R_x = nr_x(\varphi)$. From (6), the export share satisfies $R_x/R_d = \tau_x^{1-\sigma}\Lambda$, where

$$\Lambda = \frac{\varphi^{\sigma-1}\mu^F}{\delta + \varphi^{\sigma-1}\mu^F} < 1$$

represents the market share of exporters. This term captures the selection effect and introduces an additional adjustment margin through entry and exit. Taking the *total* derivative of R_x/R_d with respect to τ_x yields

$$-\frac{d\ln(R_x/R_d)}{d\ln \tau_x} = (\sigma - 1) - \frac{\partial \ln \Lambda}{\partial \ln \mu^F}\frac{d\ln \mu^F}{d\ln \tau_x}.$$

The first and second terms correspond to the intensive and extensive margins, respectively. Export selection therefore introduces an additional trade margin through endogenous matching. However, the welfare formulas in (20) and (21) are expressed in terms of the *partial* trade elasticity, as in ACR. Holding matching outcomes fixed eliminates the second term, implying that the partial trade elasticity remains $\varepsilon = \sigma - 1$ as in the baseline. Intuitively, a reduction in τ_x lowers unmatched profit π_d through a decline in the price index, which raises the matching rate μ^F. This indirect adjustment disappears when only the direct effect of τ_x is considered.

In contrast, the domestic expenditure share is defined as $\lambda = R_d/R = [(N^F - n)r + nr_d(\varphi)]/R$, which is closely related to the unmatched expenditure share $s = (N^F - n)r_d/R$. Without firm-to-firm matching ($n = 0$), these shares coincide ($\lambda = s$). With such matching, however, the latter is smaller than the former ($s < \lambda$). Under export selection, these expenditure shares are given by

$$\lambda = \frac{1 + \varphi^{\sigma-1}\mu^F/\delta}{1 + (1 + \tau_x^{1-\sigma})\varphi^{\sigma-1}\mu^F/\delta}, \qquad s = \frac{1}{1 + (1 + \tau_x^{1-\sigma})\varphi^{\sigma-1}\mu^F/\delta}.$$

Although these expressions differ from those in Section 4.3, the welfare change in (20) and the welfare ratio in (21) remain unchanged because the welfare characterization in (10) continues to apply. Moreover, in contrast to the baseline model, a reduction in τ_x increases θ under export selection. The expenditure shares therefore satisfy $d\ln s < d\ln \lambda$ within X-integration, while the global comparison with autarky continues to imply $s/s_a < \lambda$, as in the baseline model. Hence, if changes in the mass of unmatched firms are limited, both the welfare change and the welfare ratio are larger with firm-to-firm matching than without it.

Takeaway Taken together, the unmatched expenditure share continues to play a central role in welfare under export selection. Although export selection modifies the mapping from equilibrium outcomes to welfare, the welfare effects of firm-to-firm matching continue to operate through changes in matching outcomes, preserving the qualitative mechanism highlighted in Section 4.3.

C Extensions of M-Integration

This appendix examines extensions of the baseline M-integration setting in Section 4.4. Specifically, we relax the following assumptions: (i) identical elasticities of substitution between final goods and intermediate inputs; (ii) symmetric countries with identical sourcing costs; and (iii) the productivity ordering of matched firms $\varphi > \varphi_m$.

C.1 Different Goods Elasticities

This appendix considers the case in which the elasticities of substitution differ between final and intermediate goods and examines how this generalization affects the equilibrium analysis.

Suppose that the elasticity of substitution between final goods in the utility function remains σ, while the elasticity between intermediate inputs in production is $\rho > 1$. The production function is therefore

$$y = \left((x^F)^{\frac{\rho-1}{\rho}} + (x^S)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}},$$

where the variety index ω is omitted for notational simplicity. Cost minimization yields the same unit cost of final-good production, $c = wa^F/\varphi$, but the productivity of matched firms is now given by

$$\varphi = \left(1 + \left(\frac{a^F}{a^S} \right)^{\rho-1} \right)^{\frac{1}{\rho-1}},$$

which reflects the elasticity ρ between intermediate inputs.

Matched agents choose two inputs x^F and x^S to maximize profits $\pi = py - wa^F x^F - wa^S x^S - w(f^F + f^S)$, which yields the input demands

$$x^F(\varphi) = A \left(\frac{\sigma - 1}{\sigma} \frac{1}{wa^F} \right)^{\sigma} \varphi^{\sigma-\rho}, \qquad x^S(\varphi) = \left(\frac{a^F}{a^S} \right)^{\rho} x^F(\varphi). \tag{C.1}$$

(C.1) shows that when the elasticity is higher for final goods than for intermediate inputs ($\sigma \geq \rho$), matched firms with $\varphi > 1$ demand more inputs than unmatched firms with $\varphi = 1$. Empirical estimates in production networks suggest moderate elasticities across intermediate inputs (Antràs et al. 2017), typically below elasticities across final goods in the trade literature, making the case $\sigma \geq \rho$ plausible. By contrast, when elasticities are identical ($\sigma = \rho$) as in the baseline, productivity drops out of (C.1). Consequently, matched and unmatched firms have identical core-input demands, although matched firms continue to use the non-core input $x^S(\varphi)$.

Using (C.1), profit maximization yields the following equilibrium price, output, and revenue:

$$p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{wa^F}{\varphi}, \qquad y(\varphi) = A \left(\frac{\sigma - 1}{\sigma} \frac{\varphi}{wa^F} \right)^{\sigma}, \qquad r(\varphi) = A \left(\frac{\sigma - 1}{\sigma} \frac{\varphi}{wa^F} \right)^{\sigma-1}.$$

These expressions depend only on the final-good elasticity σ, as in the baseline model; the input elasticity ρ affects equilibrium outcomes only through productivity φ. Consequently, the equilibrium ratios in (1) and profits in (2) remain unchanged. Intuitively, while the input elasticity ρ affects the input demands $x^F(\varphi)$ and $x^S(\varphi)$, it does not affect total input expenditure $e(\varphi) = wa^F x^F(\varphi) + wa^S x^S(\varphi)$, because (C.1) implies

$$e(\varphi) = A \left(\frac{\sigma - 1}{\sigma} \right)^{\sigma} \left(\frac{\varphi}{wa^F} \right)^{\sigma-1}. \tag{C.2}$$

Since (C.2) holds regardless of whether $\sigma = \rho$, profits $\pi(\varphi) = r(\varphi) - e(\varphi) - wf$ depend only on σ, implying that profit maximization yields the same revenue ratio $r(\varphi)/r$ in (1) and profits $\pi(\varphi)$ in (2). Moreover, with labor as the numéraire ($w = 1$), matches generate the joint surplus $\pi(\varphi) - \pi = (\varphi^{\sigma-1} - 1)\pi - f$, as in the baseline. Hence, the equilibrium characterization remains unchanged, although the equilibrium values of θ and π may vary with ρ through their dependence on φ.

Turning to the equilibrium analysis of trade integration, productivity of firms matched across borders is

$$\varphi_m = \left(1 + \left(\frac{1}{\tau_m}\right)^{\rho-1}\right)^{\frac{1}{\rho-1}},$$

where ρ is the elasticity of substitution between inputs. Using φ_m, input demands are

$$x^F(\varphi_m) = A\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \varphi_m^{\sigma-\rho}, \qquad x^S(\varphi_m) = \left(\frac{1}{\tau_m}\right)^{\rho} x^F(\varphi_m).$$

This implies the equilibrium price, output, and revenue of firms matched abroad:

$$p(\varphi_m) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_m}, \qquad y(\varphi_m) = A\left(\frac{\sigma-1}{\sigma}\varphi_m\right)^{\sigma}, \qquad r(\varphi_m) = A\left(\frac{\sigma-1}{\sigma}\varphi_m\right)^{\sigma-1}.$$

The input expenditure of cross-border matches $e(\varphi_m) = x^F(\varphi_m) + \tau_m x^S(\varphi_m)$ is

$$e(\varphi_m) = A\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \varphi_m^{\sigma-1},$$

which also holds regardless of whether $\sigma = \rho$. Consequently, cross-border matches generate profits and the joint surplus given by (15) and $\pi(\varphi_m) - \pi = (\varphi_m^{\sigma-1} - 1)\pi - f_m$, respectively, as in the baseline model.

We now examine the effects of trade. Under X-integration, setting $\tau_m = \infty$ implies $\varphi_m = 1$, so a cross-border relationship would generate negative surplus, $\pi(\varphi_m) - \pi = -f_m < 0$. Since no such relationships are sustained in equilibrium, the free-entry conditions remain the same as in the baseline model. Likewise, the equilibrium characterization remains unchanged in M-integration. The key difference is that the condition under which cross-border matches generate higher surplus than remaining unmatched is no longer written as (23), because

$$\varphi_m^{\sigma-1} = \left(1 + \left(\frac{1}{\tau_m}\right)^{\rho-1}\right)^{\frac{\sigma-1}{\rho-1}}.$$

Consequently, the profitability of cross-border sourcing now depends on both the final-good elasticity σ and the input elasticity ρ, and cannot be compared as directly with (18) as in the baseline model. Nevertheless, allowing $\rho \neq \sigma$ affects only the productivity terms φ and φ_m, although the equilibrium ratios and profit expressions retain the same functional form. Conditional on cross-border supplier relationships remaining profitable and active in equilibrium, the matching and welfare mechanisms continue to operate. Therefore, the qualitative mechanisms through which M-integration affects entry, matching, and welfare are preserved whenever the corresponding condition is satisfied.

C.2 Asymmetric Countries

This appendix studies an extension with asymmetric countries to examine how trade integration affects welfare in this setting.

Setting Suppose two countries, North and South, differ in unit labor requirements $a^F, a^S, a^{F^*}, a^{S^*}$, where asterisks denote Southern variables. Assume North has a comparative advantage in producing non-core inputs:

$$\frac{a^S}{a^F} < \frac{a^{S^*}}{a^{F^*}} \qquad \Longleftrightarrow \qquad \frac{a^F}{a^S} > \frac{a^{F^*}}{a^{S^*}}. \tag{C.3}$$

We also assume $a^F < a^{F^*}$ so that North has an absolute advantage in core inputs. To keep the analysis tractable, all other structural parameters remain symmetric across countries, including market size and trade costs. In this setting, the welfare implications differ from those in the baseline model: *M-integration can generate welfare gains in North but may cause welfare losses in South.*

This extension also helps clarify the welfare implications of offshoring in Antràs and Helpman (2004), who study firms' sourcing decisions in a North–South setting but do not analyze the welfare effects on each country. Interpreting the offshoring equilibrium as that of M-integration, the model shows that search and matching can generate a wage difference (i.e., country productivity) between North and South. Moreover, when this wage gap is sufficiently large, offshoring can generate welfare gains in North while causing welfare losses in South.

Autarky We begin by describing the autarky equilibrium. Using (C.3), productivity of matched firms is

$$\varphi = \left(1 + \left(\frac{a^F}{a^S}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}, \qquad \varphi^* = \left(1 + \left(\frac{a^{F^*}}{a^{S^*}}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}.$$

Since North has a comparative advantage in non-core inputs, productivity is higher in North ($\varphi > \varphi^*$), reflecting more efficient input expansion under CES technology. This productivity gap leads to the following differences.

First, wages are no longer equalized across countries. The labor-market clearing conditions are $R = wL$ and $R^* = w^*L^*$. To compare the two countries, we measure nominal variables in a common unit of account. Throughout this extension, we focus on an equilibrium with a sufficiently large equilibrium wage gap such that $w > w^*$, consistent with North's higher productivity. Since $L = L^*$, it then follows that $R > R^*$. We normalize the Southern wage to unity by choosing Southern labor as the numéraire.

Second, the two key endogenous variables may differ across countries. Since wages are no longer equalized, profit must be measured relative to wage, π/w and π^*/w^*. Here we focus on an equilibrium in which North has lower normalized unmatched profit than South, $\pi/w < \pi^*/w^*$, which implies $\theta > \theta^*$ from (11):

$$\frac{\pi}{w} = f_e^F - \frac{\tilde{\beta}}{1-\tilde{\beta}} f_e^S \theta, \qquad \frac{\pi^*}{w^*} = f_e^F - \frac{\tilde{\beta}}{1-\tilde{\beta}} f_e^S \theta^*.$$

Thus, North's higher unmatched ratio is associated with lower unmatched profit. Intuitively, higher productivity increases the surplus from successful matches, encouraging supplier participation in North. This, in turn, raises firms' matching rates, so free entry drives down the equilibrium unmatched profit.

Third, welfare is higher in North than in South ($W > W^*$) because the welfare expression in (10) becomes

$$W = \frac{\sigma-1}{\sigma}\frac{1}{a^F}\left(\frac{L}{\sigma(\pi/w)}\right)^{\frac{1}{\sigma-1}}, \qquad W^* = \frac{\sigma-1}{\sigma}\frac{1}{a^{F^*}}\left(\frac{L^*}{\sigma(\pi^*/w^*)}\right)^{\frac{1}{\sigma-1}}.$$

The result follows from noting $L = L^*$, $a^F < a^{F^*}$, and $\pi/w < \pi^*/w^*$. Intuitively, Northern consumers spend a smaller share on goods produced by unmatched firms ($s < s^*$), reflecting a greater allocation of resources to matched firms and a larger share of goods produced by them in North.

M-Integration Under M-integration, a Northern firm matched with a Southern supplier continues to source core inputs domestically, while non-core inputs are obtained from the South and incur the iceberg trade cost τ_m. Since the delivered unit cost of Southern non-core inputs is $\tau_m w^* a^{S^*}$, whereas the unit cost of Northern core inputs is wa^F, productivity becomes

$$\varphi_m = \left(1 + \left(\frac{wa^F}{\tau_m w^* a^{S^*}}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}.$$

This reflects that cross-border sourcing affects productivity through both technological differences (a^F, a^{S^*}) and wage differences (w, w^*). Therefore, if the equilibrium wage differential ($w > w^*$) persists under M-integration, cross-border matches need not be of intermediate efficiency, in contrast to the baseline model. In particular, $\varphi_m > \varphi$ if and only if $wa^S > \tau_m w^* a^{S^*}$, which is *more* likely as the relative wage w/w^* is larger. Intuitively, Northern firms sourcing from Southern suppliers benefit from lower Southern wages, which may outweigh the transport cost τ_m.

Similarly, productivity of Southern firms matched with Northern suppliers is

$$\varphi_m^* = \left(1 + \left(\frac{w^* a^{F^*}}{\tau_m wa^S}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}.$$

In this case, we obtain $\varphi_m^* > \varphi^*$ if and only if $w^* a^{S^*} > \tau_m wa^S$, which is *less* likely as w/w^* is larger, because a larger North–South wage gap raises the cost of importing Northern inputs, making input sourcing by Southern firms less attractive. Hence, a sufficiently large wage gap can generate the productivity ordering:

$$\varphi_m > \varphi > \varphi^* > \varphi_m^*. \tag{C.4}$$

To determine whether M-integration raises welfare, it suffices to check whether condition (24) holds in each country. Assuming that the fixed matching costs f and f_m are measured in units of labor in the firm's country and dividing the profit comparison by the local wage, the condition now becomes

$$(\varphi_m^{\sigma-1} - \varphi^{\sigma-1})\frac{\pi_d}{w} > f_m - f,$$
$$(\varphi_m^{*\sigma-1} - \varphi^{*\sigma-1})\frac{\pi_d^*}{w^*} > f_m - f.$$

If the productivity ordering (C.4) holds in equilibrium, this condition can be satisfied in North but not in South. Consequently, M-integration can generate welfare gains in North while causing welfare losses in South.

The intuition behind the contrasting welfare implications between North and South is as follows. In the symmetric-country model, M-integration lowers welfare by reallocating resources from the most efficient firms to moderately efficient ones, reducing the productivity of matched firms and thereby raising both the price index and the unmatched expenditure share. In the asymmetric-country model, this compositional effect can be mitigated in North because firms matched abroad become the most efficient among operating firms. Even in this case, however, welfare losses may still arise in South.

One broad implication of this extension is that a less advanced country (with lower wages and productivity) is more likely to suffer welfare losses from the integration of matching markets. This result suggests that the welfare effects of offshoring may differ systematically between developed and less developed countries.

C.3 Nested Foreign Sourcing

This appendix extends the M-integration environment by allowing a fraction of matched pairs to source an additional foreign input, making them more productive than purely domestic pairs.

Environment The economy features the key matching structure of the baseline model. In this extension, firms and suppliers first form domestic relationships through the matching process. Then, conditional on such a relationship being formed, a fraction $\alpha \in (0, 1)$ of matched pairs procure an additional foreign input from the foreign market by paying the input cost τ_m, which raises productivity through the CES input aggregation. Because the imported input is purchased through the market, it does not involve another matching process, thereby preserving the one-to-one matching structure. Accordingly, the share α is treated as exogenous below. Thus, this extension does not introduce an endogenous compositional effect through firms' sourcing decisions; instead, it isolates how the compositional effect depends on the average match surplus associated with a given sourcing structure. We therefore examine whether the welfare mechanism of M-integration continues to depend on matching-market interactions when foreign sourcing raises productivity conditional on matching.

Nested Foreign Sourcing and Productivity In Section 4.4, a matched firm–supplier pair combines the firm's in-house core input with one outsourced intermediate. Let a^F denote the unit labor requirement for the in-house input and a^S that of the domestic supplier. As in the baseline model, effective productivity is

$$\varphi = \left(1 + \left(\frac{a^F}{a^S}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}.$$

In this extension, however, a matched firm–supplier pair may additionally engage in nested foreign sourcing. Imported inputs incur iceberg trade costs $\tau_m > 1$, while country symmetry implies that foreign suppliers also produce inputs with the unit labor requirement a^S. Combining the firm's input, the domestic supplier's input, and the imported input through the CES technology yields productivity of this firm:

$$\tilde{\varphi} = \left(1 + \left(\frac{a^F}{a^S}\right)^{\sigma-1} + \left(\frac{a^F}{\tau_m a^S}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}.$$

Nested foreign sourcing therefore further increases productivity, $\tilde{\varphi} > \varphi$, because CES aggregation exhibits gains from input variety. Without a loss of generality, we normalize these unit labor requirements to one, $a^F = a^S = 1$. However, nested foreign sourcing requires an additional fixed cost associated with the foreign input purchase. Hence, the total fixed cost of a nested foreign sourcing relationship is $\tilde{f} = f + f_m$, where f denotes the fixed cost of the domestic supplier relationship and f_m denotes the additional fixed cost of nested foreign sourcing. Relative to domestic-only matches, nested foreign sourcing therefore lowers marginal costs but raises fixed costs, creating a trade-off analogous to that between domestic matching and remaining unmatched.

Modified Free-Entry Conditions When nested foreign sourcing is available, supplier relationships differ in the surplus they generate. The operating profits of unmatched firms are $\pi = \pi_d$, while those of matched firms are $\pi(\varphi) = \varphi^{\sigma-1}\pi_d - f$ and $\pi(\tilde{\varphi}) = \tilde{\varphi}^{\sigma-1}\pi_d - \tilde{f}$ for domestic sourcing and nested foreign sourcing, respectively. These relationship surpluses are therefore expressed as

$$\pi(\varphi) - \pi = (\varphi^{\sigma-1} - 1)\pi_d - f, \qquad \pi(\tilde{\varphi}) - \pi = (\tilde{\varphi}^{\sigma-1} - 1)\pi_d - \tilde{f}.$$

Since a fraction α of matched firm–supplier pairs engage in nested foreign sourcing, while the remaining fraction $1-\alpha$ engage only in domestic sourcing, the expected surplus generated by matching is

$$\begin{aligned}\bar{S} &= (1-\alpha)\big(\pi(\varphi) - \pi\big) + \alpha\big(\pi(\tilde{\varphi}) - \pi\big)\\ &= \bar{A}\pi_d - \bar{f},\end{aligned}$$

where $\bar{A} = (1-\alpha)(\varphi^{\sigma-1} - 1) + \alpha(\tilde{\varphi}^{\sigma-1} - 1)$ is the expected productivity component, while $\bar{f} = (1-\alpha)f + \alpha\tilde{f}$ is the expected fixed cost. When $\alpha = 0$, this expected surplus reduces to its autarky counterpart.

The free-entry conditions remain identical in form to those in the baseline model: $\pi_d + \phi^F\bar{S} = f_e^F$ for firms and $\phi^S\bar{S} = f_e^S$ for suppliers. Substituting $\bar{S}$ and solving for π_d yields

$$\begin{aligned}\pi_d &= \frac{f_e^F + \bar{f}\,\phi^F}{1 + \bar{A}\,\phi^F},\\ \pi_d &= \frac{f_e^S + \bar{f}\,\phi^S}{\bar{A}\,\phi^S}.\end{aligned} \tag{C.5}$$

As in the baseline model, free-entry conditions (C.5) jointly determine the key equilibrium values of π_d and θ. In this extension, however, expected match surplus now depends on the composition of supplier relationships through α, thereby affecting entry incentives and equilibrium matching outcomes. Because $\bar{A} = \varphi^{\sigma-1} - 1$ and $\bar{f} = f$ at $\alpha = 0$, free-entry conditions (C.5) reduce to the autarky counterparts in (8).

Effects of Nested Foreign Sourcing Before examining these equilibrium effects, consider the profitability of nested foreign sourcing at the relationship level. Relative to domestic-only matching, nested foreign sourcing raises productivity, $\tilde{\varphi} > \varphi$, while it also raises the fixed cost, $\tilde{f} > f$. From the operating profits derived above, nested foreign sourcing is more profitable than domestic-only matching, $\pi(\tilde{\varphi}) > \pi(\varphi)$, if and only if

$$(\tilde{\varphi}^{\sigma-1} - \varphi^{\sigma-1})\pi_d > f_m,$$

which we maintain throughout the analysis; otherwise the matched pair is better off remaining a domestic-only match and does not procure the additional foreign input. This condition serves only as a profitability benchmark between the two types of matches, rather than characterizing an endogenous sourcing decision. Because the share α is exogenous, the extension instead studies how the incidence of nested sourcing affects entry, matching, and welfare through the matching market while allowing for productivity gains.

Nested foreign sourcing increases both productivity and fixed costs relative to domestic-only matching, with the corresponding differences given by $\Delta A = \tilde{\varphi}^{\sigma-1} - \varphi^{\sigma-1}$ and $\Delta f = \tilde{f} - f$. These opposing forces jointly determine how the free-entry equilibrium responds to an increase in the incidence of nested foreign sourcing. From (8) and (C.5), the firm and supplier free-entry loci lie above their autarky counterparts, respectively, if

$$\Delta f(1 + (\varphi^{\sigma-1} - 1)\phi^F) > \Delta A(f_e^F + f\phi^F),$$
$$(\varphi^{\sigma-1} - 1)\Delta f\, \phi^S > \Delta A(f_e^S + f\phi^S).$$

Thus, a sufficiently large increase in fixed costs shifts each locus upward, whereas a sufficiently large productivity improvement shifts it downward. The incidence of nested foreign sourcing, α, determines the magnitude of the shift away from autarky but not its direction. Moreover, because the two conditions differ, one free-entry locus may shift upward while the other shifts downward.

Compositional Effects and Welfare Since the productivity and fixed-cost forces of nested sourcing can shift the free-entry loci in either direction, its effects on equilibrium entry and matching are ambiguous. Intuitively, nested foreign sourcing increases productivity conditional on matching, but it also changes the composition of firm–supplier relationships. Because matching is random, entry by one agent affects the matching rates faced by other agents, while the coexistence of domestic-only and nested-sourcing relationships creates a compositional margin that individual entrants take as given. The interaction between this compositional margin and matching congestion is similar in spirit to Acemoglu's (2001) "good jobs versus bad jobs" mechanism, although we treat the incidence of nested foreign sourcing as exogenous.

These effects feed into welfare through the allocation of expenditure between matched and unmatched firms. In particular, the unmatched expenditure share becomes

$$s = \frac{1}{1 + \Big((1-\alpha)\varphi^{\sigma-1} + \alpha\tilde{\varphi}^{\sigma-1}\Big)\mu^F/\delta}.$$

Relative to the baseline model, the productivity term is replaced by a weighted average reflecting the coexistence of domestic-only and nested-sourcing relationships. A higher incidence of nested foreign sourcing therefore lowers the expenditure share on goods produced by unmatched firms directly through higher productivity. At the same time, however, changes in entry incentives alter the matching rate μ^F, which can reinforce or offset this direct productivity effect. Entry also changes the mass of unmatched firms, which constitutes the other matching-market statistic entering welfare. As a result, welfare continues to depend on equilibrium matching outcomes rather than on productivity differences alone.

In sum, our welfare mechanism under M-integration does not require foreign sourcing to be less productive than domestic sourcing, provided that nested foreign sourcing also entails higher fixed costs. Even when nested foreign sourcing is more productive than domestic-only sourcing, the additional fixed cost can alter expected match surplus and thereby affect entry and matching outcomes. The interaction between compositional effects and matching congestion therefore remains central to the welfare consequences of foreign sourcing.

D Quantitative Analysis

This appendix describes (i) the calibration strategy, (ii) parameter values, (iii) robustness to export selection, and (iv) the effect of trade liberalization for the quantitative analysis in Section 5.

D.1 Calibration Strategy

We calibrate the model so that the unmatched ratio satisfies $\theta = 1.52$ in the autarky equilibrium. Combining the two free-entry conditions in (8) yields

$$(\varphi^{\sigma-1} - 1)(\phi^S f_e^F - \phi^F f_e^S) = f_e^S + f\phi^S. \tag{D.1}$$

Since ϕ^F and ϕ^S depend on the unmatched ratio θ, this condition implicitly determines the equilibrium value of θ under the CES matching function (25). Thus, we choose exogenous parameter values so that (D.1) holds at the calibrated benchmark $\theta = 1.52$. Similar calibration strategies apply under X- and M-integration, although the equilibrium values of (θ, π_d) differ across trade regimes, as shown in Figure 3.

We compute the two key variables using nonlinear fixed-point iteration. Starting from an initial guess for θ, we update the matching rates and effective surplus-sharing coefficients implied by the CES matching function and the surplus-sharing rules, and then solve the free-entry conditions for the implied equilibrium allocation. The procedure is iterated until the supremum norm of successive updates falls below 10^{-8}. Trade liberalization experiments are conducted by varying τ_x or τ_m holding other parameters fixed at their benchmark values.

Once (θ, π_d) are determined, the remaining equilibrium variables follow directly. We recover s/s_a from the regime-specific expenditure-share equations and u^F/u_a^F from the stationary mass equations. We then compute welfare by substituting these objects into the decomposition in (21). Under X-integration, changes in τ_x affect s/s_a through the domestic expenditure component, whereas u^F/u_a^F remains unchanged because θ is unaffected. Under M-integration, changes in τ_m alter φ_m, thereby changing θ, which in turn determines s/s_a and u^F/u_a^F.

D.2 Parameter Values

Production Parameters We set the elasticity of substitution between varieties to $\sigma = 4$, following Heise (2024). Normalizing unit labor requirements to one ($a^F = a^S = 1$), the productivity of domestic matches is set to $\varphi = 1.26$, implying that matched firms are 26 percent more productive than unmatched firms ($\varphi = 1$). This value generates an exporter revenue premium that is broadly consistent with the evidence reported by Bernard et al. (2007a). Because observed exporter premia reflect several mechanisms beyond relationship-specific productivity, we use their estimate as a plausibility check rather than as an exact calibration target. Under constant returns to scale in matching, market size does not affect the key endogenous variables (θ, π_d). We therefore set $L = 168$ million to match the U.S. labor force in 2024.

Matching and Depreciation Parameters The exit rate δ enters (D.1) through $f_e^F = \delta F_e^F$ and $f_e^S = \delta F_e^S$. In much of the literature, this parameter is calibrated to match job destruction rates, which are typically low (e.g., $\delta = 0.025$ in Ghironi and Melitz 2005). However, given the calibrated matching rate $\mu^F = 0.26$, equation (12) implies a very small unmatched expenditure share ($s = 0.04$). Following Heise (2024), we instead interpret δ as the depreciation rate of relationship capital and set $\delta = 0.05$, close to the estimate $\delta = 0.063$, under which equation (12) implies $s = 0.09$. We set the firm's primitive bargaining weight to $\tilde{\beta} = 0.6$, motivated by evidence that buyers possess substantial bargaining power in firm-to-firm trade (Alviarez et al. 2025).

Fixed Costs We normalize the matching cost to $f = 0.1$, following Bernard et al. (2007b). We also choose the firm and supplier entry costs to be $F_e^F = 18$ and $F_e^S = 2.55$, respectively. Using $\delta = 0.05$, these values imply amortized entry costs of $f_e^F = 0.9$ and $f_e^S = 0.127$. The firm entry cost is calibrated so that its flow-equivalent magnitude is in line with the benchmark specification of Felbermayr et al. (2011), while the supplier entry cost is chosen so that the equilibrium unmatched ratio matches the value reported by Heise (2024) in (D.1).

Export-Side Parameters To solve for the open-economy equilibrium, we specify additional parameter values for trade costs while maintaining the same parameter values as in autarky. We set $\tau_x = 1.6$, close to the estimate $\tau_x = 1.7$ reported by Anderson and van Wincoop (2004). This implies a domestic expenditure share of $\lambda = \frac{1}{1+\tau_x^{1-\sigma}} = 0.8$. We also set $f_x = 0.13$ (and hence $F_x = f_x/\delta = 2.6$) to satisfy condition (18). This value is similar but is different from that used by Felbermayr et al. (2011), because we abstract from export selection.

Import-Side Parameters Tariffs on final goods tend to exceed those on inputs, a pattern known as *tariff escalation* (Antràs et al. 2024). Motivated by this evidence, we set $\tau_m = 1.2 < \tau_x = 1.6$. Then, $\varphi_m = 1.16$, implying that firms with cross-border matches are 16 percent more productive than unmatched firms ($\varphi = 1$), but less productive than firms matched with domestic suppliers ($\varphi = 1.26$). When we keep $f_m = 0.13\,(= f_x)$, condition (23) holds whereas condition (24) does not, as assumed in Section 4.4. We set $\kappa = 0.4$, which implies that the fraction of matched firms engaged in cross-border sourcing is $\kappa/(1+\kappa) = 0.29$. Since κ is a reduced-form parameter and direct evidence on the prevalence of cross-border supplier relationships is limited, we choose this value to broadly reflect the prevalence of importing among exporters documented by Bernard et al. (2007a).

Calibration Targets Table D.1 summarizes the calibrated parameter values in the quantitative analysis. Several parameters are taken directly from the empirical literature, whereas others are chosen to match key equilibrium moments, including the unmatched ratio and the relative magnitudes of entry and matching costs.

D.3 Robustness to Export Selection

A natural concern is whether the quantitative mechanisms identified in Section 4.3 continue to operate when trade induces export selection in the spirit of Melitz (2003). To examine this, we raise the fixed export cost from $f_x = 0.13$ in the benchmark calibration to $f_x = 0.17$, while keeping all other parameters at their benchmark values reported in Table D.1. Under this parameterization, we solve the free-entry conditions (B.2) for (θ, π_d). The resulting equilibrium satisfies condition (B.3), implying that only matched firms export.

Table D.2 shows the main quantitative results under export selection. Although higher export costs reduce aggregate welfare relative to the benchmark calibration in Section 5.2, firm-to-firm matching continues to amplify the gains from trade. In the matching economy, productivity differences between matched and unmatched firms generate export selection: productive matched firms export, whereas unmatched firms serve only the domestic market. By contrast, firms are homogeneous in the benchmark economy without firm-to-firm matching, so there is no corresponding selection margin and all firms export whenever exporting is profitable.

Quantitatively, the gains from trade are 2.8 percent in the matching economy and 1.5 percent in the no-matching benchmark. The larger gains in the matching economy reflect the additional adjustment associated with firm-to-firm matching: export selection reallocates expenditure toward productive matched firms and lowers the unmatched expenditure share, while endogenous matching outcomes also respond to trade integration. Thus, the qualitative role of firm-to-firm matching in amplifying the gains from trade remains robust when X-integration induces export selection.

Table D.1: Calibrated parameter values

Parameter	Interpretation	Value	Source/Target
Technology and demand			
L	Labor endowment	168 million	U.S. labor force (2024)
a^F, a^S	Unit labor requirements	1	Normalization
φ	Productivity of domestic matches	1.26	Bernard et al. (2007a)
σ	Elasticity of substitution	4	Heise (2024)
Matching market			
ι	Matching elasticity	0.45	Heise (2024)
δ	Depreciation rate	0.05	Heise (2024)
κ	Relative difficulty of cross-border matches	0.4	Bernard et al. (2007a)
$\tilde{\beta}$	Firm's primitive bargaining weight	0.6	Alviarez et al. (2025)
Entry and matching costs			
F_e^F	One-time firm entry cost	18	Felbermayr et al. (2011)
F_e^S	One-time supplier entry cost	2.55	Target: $\theta = 1.52$
f	Domestic flow matching cost	0.1	Bernard et al. (2007b)
f_e^F	Per-period firm entry cost	0.9	$f_e^F = \delta F_e^F$
f_e^S	Per-period supplier entry cost	0.127	$f_e^S = \delta F_e^S$
Trade costs			
τ_x	Iceberg export cost	1.6	Anderson and van Wincoop (2004)
τ_m	Iceberg import cost	1.2	Grossman et al. (2024)
F_x	One-time export entry cost	2.6	Felbermayr et al. (2011)
f_x	Per-period export entry cost	0.13	$f_x = \delta F_x$
f_m	Foreign flow matching cost	0.13	Assumption ($f_m = f_x$)

Notes: The table reports the benchmark parameter values used in the quantitative analysis. Several parameters are taken directly from the empirical literature, while others are calibrated to match equilibrium moments, including the unmatched ratio and the scale of entry and matching costs. The wage rate is normalized to one by choosing labor as the numéraire. Details of the calibration procedure are provided in the text.

Table D.2: Quantitative effects of trade integration under export selection

	θ	$\beta - \eta^F$	s/s_a	u^F/u_a^F	W/W_a
Autarky	1.52	0.05	1.00	1.00	1.000
X-integration (with matching)	1.77	0.03	0.77	0.83	1.028
X-integration (without matching)	N.A.	N.A.	0.80	N.A.	1.015

Notes: The table reports equilibrium outcomes under export selection with the fixed export cost $f_x = 0.17$, while all remaining parameter values are held at those reported in Table D.1. Welfare and matching outcomes are normalized relative to autarky. The first row under X-integration reports the equilibrium with firm-to-firm matching, in which only matched firms export. The second row reports the homogeneous benchmark economy without firm-to-firm matching. Since there is no corresponding export-selection margin in this case, all firms export whenever exporting is profitable. Accordingly, the welfare ratios with and without firm-to-firm matching are given by (21) and $W^{NM}/W_a^{NM} = \left(\lambda\left((f_e^F + f_x)/f_e^F\right)\right)^{-1/\varepsilon}$, with and without firm-to-firm matching, respectively.

This comparison parallels the distinction between heterogeneous- and homogeneous-firm models emphasized by Melitz and Redding (2015): endogenous heterogeneity introduces an additional selection margin that is absent from the homogeneous-firm benchmark. Our setting differs from theirs in that productivity heterogeneity arises endogenously through firm-to-firm matching and interacts with congestion externalities. The unmatched expenditure share and the mass of unmatched firms therefore remain key welfare-relevant matching outcomes when X-integration induces export selection.

D.4 Quantitative Effects of Trade Liberalization

In Section 5.2, we examined the quantitative effect of trade integration relative to autarky. In this section, we examine the quantitative effects of trade liberalization under X- and M-integration. Specifically, we allow the iceberg costs to vary across the ranges $\tau_x, \tau_m \in [1, 1.6]$, while holding all other parameters—including κ under M-integration—fixed at their benchmark values. While Table 1 reports discrete welfare comparisons between autarky and the two integration regimes, the present analysis examines comparative statics within each regime.

Effects of X-Integration The comparison between autarky and X-integration is not the same as that across different levels of τ_x within X-integration. As noted in Section 4.3, relative to autarky, X-integration increases the equilibrium unmatched ratio ($\theta > \theta_a$), which in turn improves firms' matching rates. By contrast, conditional on being in the X-integration regime, the unmatched ratio θ remains unchanged because changes in τ_x scale profits proportionally without affecting the free-entry conditions in terms of total profits (see (19)).

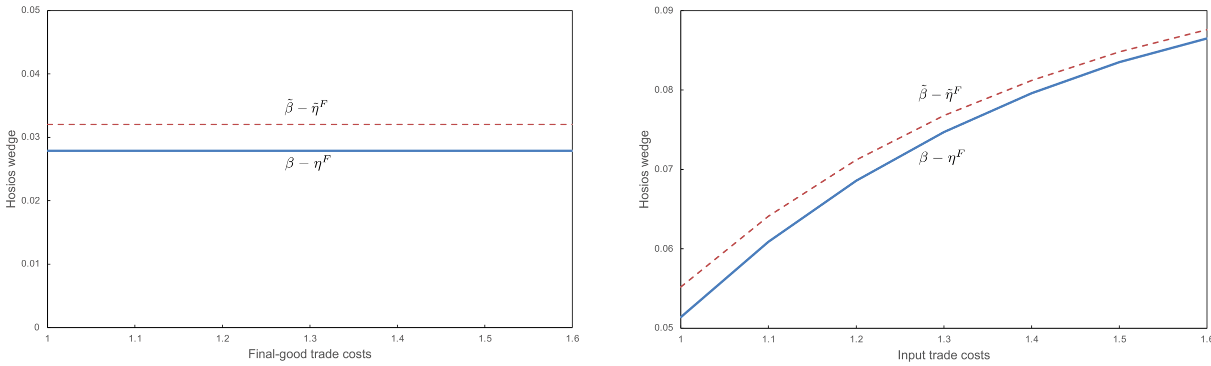
Since lower τ_x leaves θ unchanged over the range considered, both the dynamic and primitive Hosios wedges —$\beta - \eta^F$ and $\tilde{\beta} - \tilde{\eta}^F$, respectively—remain constant at around 0.03. The mass of unmatched firms u^F/u_a^F also remains constant because θ is invariant to τ_x. A reduction in τ_x raises the export revenues of matched and unmatched firms proportionally and therefore does not reallocate expenditure toward matched firms. Instead, it reduces the unmatched expenditure share s/s_a through the direct trade-cost and price-index margin.

Welfare rises as τ_x falls because the decline in s/s_a lowers the price index, whereas the congestion channel is inactive. Under the calibrated parameter values, the gains from trade with firm-to-firm matching, W/W_a, exceed those without matching, W^{NM}/W_a^{NM}, as implied by the normalized welfare components in equation (21). These comparative statics are illustrated in the left panels of Figure D.1.

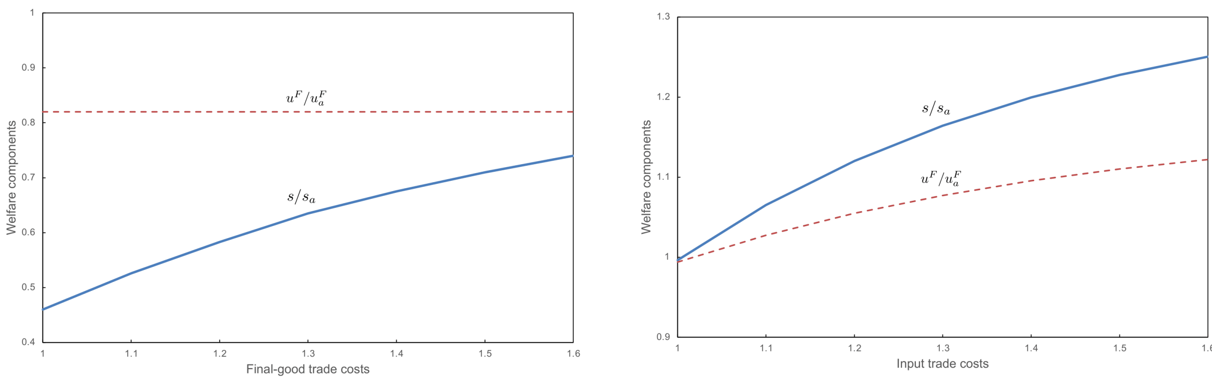
Effects of M-Integration A reduction in τ_m raises the equilibrium unmatched ratio θ by making cross-border matches more attractive (see (22)). The resulting increase in θ raises both the effective bargaining share β and the dynamic social matching share η^F, with the latter rising more strongly, so that $\beta - \eta^F$ narrows as τ_m falls. The primitive Hosios wedge $\tilde{\beta} - \tilde{\eta}^F$ exhibits the same qualitative pattern. Because both wedges remain positive throughout the range, the increase in θ alleviates firm-side congestion. At the same time, both the unmatched expenditure share s/s_a and the mass of unmatched firms u^F/u_a^F decline steadily as τ_m falls, reflecting improved matching outcomes and the higher productivity of cross-border supplier relationships.

Lower input trade costs improve welfare by making foreign supplier relationships more productive and by narrowing the Hosios wedge. As a result, the welfare ratio W/W_a rises as τ_m falls. The magnitude of this welfare response is larger when κ is higher because foreign supplier relationships receive greater weight in average match productivity. Consequently, changes in τ_m have a stronger direct effect on match surplus and the unmatched expenditure share, while endogenous adjustments in matching outcomes provide an additional welfare channel. These comparative statics are illustrated in the right panels of Figure D.1.

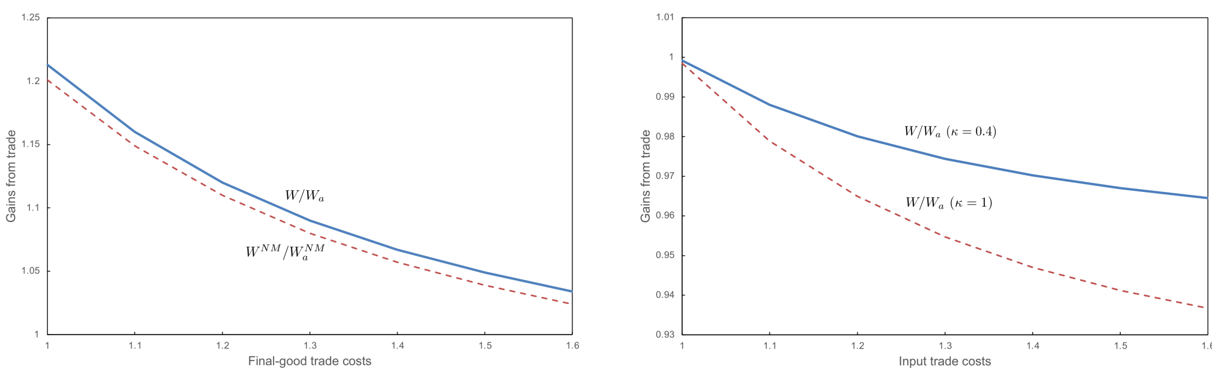
X-integration M-integration



(a) Hosios wedges



(b) Welfare components



(c) Gains from trade

Figure D.1: Comparative statics with respect to variable trade costs under X- and M-integration