Trade with Search Frictions: Identifying New Gains from Trade*

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Abstract

This paper develops a general-equilibrium trade model to examine the effect of search frictions on welfare. We consider a search-theoretic setting with firms and suppliers. Each firm seeks a supplier to get a specialized input from outside, but search is costly and does not always end in success. An unmatched firm uses only a single input provided by itself, while a matched firm uses multiple inputs provided by itself and a matched supplier. In equilibrium, the number of all types of agents is endogenously determined. We use this model to contrast the welfare implications of two forms of economic integration: integration of final-good markets that allows firms to ship varieties to another market and integration of matching markets that allows firms to seek suppliers from another market. We show that the former form of integration amplifies welfare gains by improving firms' matching probability and reallocating resources from unmatched firms to matched firms. In contrast, the latter may cause welfare losses by hindering the resource-reallocation process and worsening the matching probability of firms.

Keywords: Search, matching, gains from trade, firm heterogeneity, economic integration
JEL Classification Numbers: F12, F15

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1 Introduction

Firms frequently seek suppliers to procure specialized inputs in manufacturing processes. Many anecdotes have documented that although a few core inputs to final production are made in-house, other non-core inputs are largely purchased from outside in order to take advantage of suppliers' specialization in arm's-length dealing. It is argued that advances in information technology reduce search frictions and make it easier to seek suppliers not only within borders but also across borders. Furthermore, the range of specialized inputs is associated with production technology of firms, in the sense that a greater range of inputs directly leads to higher productivity. Apple's sourcing strategy known as "Designed by Apple in California Assembled in China" is often portrayed as a symbol of this economic phenomenon; it clearly indicates that successful matching with compatible suppliers spread across the globe is essential for firms to maintain productive exchanges of inputs and thus is considered a main source of firm productivity.[1] As globalization effectively expands firms' activities by integrating markets, the circumstance might significantly influence firm productivity through buyer–seller linkages across countries, thereby creating less-known welfare gains or losses from trade.

How does firms' search alter aggregate welfare implications of economic integration via international trade? To address the question, this paper develops a general-equilibrium trade model to examine the effect of search frictions on welfare. We consider a search-theoretic setting with two types of agents, namely, firms and suppliers. Each firm seeks a supplier to get a specialized input from outside, but search is costly and does not always end in success. Thus the status of firms and suppliers is either unmatched or matched. When an agent fails to find a partner, an unmatched supplier has no choice to sell its input, while an unmatched firm uses only a single input provided by itself. When an agent finds a partner and agrees on provision of a specialized input, in contrast, a matched supplier sells a specialized input for its partner, while a matched firm uses multiple inputs provided by itself and a matched supplier. As a result, a matched firm can produce with better efficiency by exploiting a love-of-variety effect from input expansion (e.g., Ethier, 1982). In this way, successful matching serves as a central vehicle to enhance firm productivity, which proves useful in shedding light on the role of search frictions in characterizing aggregate welfare.

We first consider an autarky version of the model to rationalize a distinct welfare channel of search frictions. Like a canonical search model (Pissarides, 2000), two types of agents randomly match and bargain over surplus generated by matches. We embed this framework into a monopolistic competition model in general equilibrium (Krugman, 1980) by assuming that firms undergo a costly search process in order to seek potential suppliers. This implies that, if search frictions are prohibitively large, our model collapses to the Krugman (1980) model. If search frictions are not so large that firms can search for suppliers, however, firms have a chance to improve efficiency. Exploiting our model's property that the number of all types of agents is endogenously determined, we find that a decrease in search frictions (from a prohibitively large level) increases the equilibrium ratio of suppliers to firms and hence improves the matching probability for firms. This industry structure gives rise to the mechanism through which search amplifies welfare, both because more varieties are produced by matched firms than unmatched firms and because more resources are allocated to matched firms than unmatched firms. Operating through these separate but interacted channels, we show that search always raises aggregate welfare relative to that in a standard monopolistic competition model.

[1] This kind of anecdotal stories has been corroborated by rigorous empirical work. For instance, using global input-output tables, Johnson and Noguera (2017) find that the value of imported inputs embodied in exports has been steadily rising during 1970–2009, which is explained by decreased frictions on imported inputs. As for the relationship between imported inputs and firm productivity, using Hungarian firm-level data, Halpern et al. (2015) find that firm productivity increases with the number of imported inputs: increasing the fraction of imported inputs by a firm from 0 to 100 percent would increase revenue productivity by 22 percent.

We use this model to contrast the welfare implications of two forms of economic integration: integration of final-good markets that allows firms to ship varieties to another market and integration of matching markets that allows firms to seek suppliers from another market. This paper refers to the two different forms of integration as X-integration and M-integration, respectively.[2] As in Antràs and Costinot (2011), the former form of integration aims to derive the implications of convergence of goods price indices across countries, while the latter seeks to capture the consequence of entry of foreign agents into domestic matching markets. Despite the fact that both forms of economic integration help internationalize trading opportunities, we find that X-integration generates welfare gains but M-integration causes welfare losses for each of trading countries so long as they are symmetric. More importantly, however, our model uncovers a new source of welfare gains or losses from trade in the presence of search frictions: X-integration can amplify welfare gains from trade by improving firms' matching probability that is associated with resource reallocations from unmatched firms to matched firms. Conversely, welfare losses from trade in M-integration arise from hindering the resource-reallocation process of firms as well as worsening the matching probability of firms.

Intuition behind X-integration comes from our model structure which reduces to the Krugman (1980) model with prohibitively large search frictions. In that case, welfare gains are due solely to increased product variety, which can be captured solely by the trade elasticity and the domestic expenditure share (Arkolakis et al., 2012). If search frictions are not so large, a decrease in price indices (by costly trade) increases the equilibrium ratio of suppliers to firms and hence improves the matching probability of firms, just like a decrease in search frictions. This industry restructuring lays out the mechanism through which search amplifies welfare gains.[3] In contrast, intuition of M-integration comes from the aspect that matched firms are split into firms matched with domestic and foreign suppliers. Relative to unmatched firms, cross-border matched firms are efficient because they enjoy productivity gains from imported inputs; however, relative to firms matched with domestic suppliers, they are inefficient because of transport costs to source inputs from foreign suppliers that are, by assumption, symmetric with domestic suppliers. Thus, M-integration reallocates resources to moderately efficient firms which increases price indices. From this force for price indices opposite to X-integration, it follows that M-integration decreases the equilibrium ratio of suppliers to firms and hence worsens the matching probability of firms.

At this point, it is worth discussing what X- and M-integration actually mean in the model and in practice. In a symmetric-country trade setting with differentiated goods, X-integration corresponds to "intra-industry" trade, i.e., two-way trade of similar final products between similar countries, as examined by Krugman (1980). Then our welfare result in X-integration means that search *within* borders can amplify welfare gains from trade. On the other hand, given that M-integration allows firms to source inputs from another market, this integration corresponds to "offshoring" of similar intermediate inputs between similar countries. Then our welfare result in M-integration means that search *across* borders may reduce welfare gains from trade. We emphasize that not only is intra-industry trade but also offshoring is widely observed between similar countries in the real world. For example, Antràs et al. (2017, Table 1) report that, among the top ten source countries for US manufacturers in 2007, the eight countries are OECD member countries where Canada ranks number one in terms of both the number of US importers and total import value. Although we focus primarily on a symmetric-country setting to highlight the contrasting welfare implications most sharply, the welfare losses in M-integration can survive even in an asymmetric-country setting (see Section 6).

[2] These two forms of integration are first introduced by Antràs and Costinot (2011) into a perfectly competitive Ricardian model. As they study Walrasian markets where homogenous goods are transacted, the former is referred to as W-integration in their paper. This paper refers to it differently because we study monopolistically competitive markets where differentiated goods are transacted.

[3] Our result is similar to that in Melitz and Redding (2015) in the sense that firm heterogeneity matters for welfare gains beyond two sufficient statistics. The key difference is that such heterogeneity is *endogenously* driven by search and matching in this paper.

We contribute to a growing literature that explores the role of search and matching in international trade. By explicitly introducing frictions associated with search and matching among agents in production processes, the literature has found interesting insights in otherwise standard models. For example, Grossman and Helpman (2002, 2005) study firms' organizational and locational choices in a monopolistically competitive environment, Antràs and Costinot (2011) investigate economic integration with intermediation in a standard Ricardian model, and Felbermayr et al. (2011) analyze the selection effect on labor markets in a model with heterogenous firms. Among existing work, the analysis in our paper is most closely related to that in Antràs and Costinot (2011). As described above, we consider, like them, the welfare consequences of two different forms of economic integration. There are, however, several key differences in the analysis. First, we develop a monopolistic competition model which is treated as a strict generalization of Krugman (1980). Second, intermediation plays no role in our model where firms have direct access to markets where varieties are exchanged; instead, firms seek suppliers to procure specialized inputs whose production can be offshored. Finally, our model's picture of X- and M-integration fits well with the empirical literature documenting the effect of search and matching on recent trade flows.[4]

Our approach to search frictions closely follows that in Allen (2014) and Krolikowski and McCallum (2021). Incorporating search frictions into perfect or monopolistic competition models, these papers assume that firms incur a fixed cost of search when looking for suppliers, though the label of agents differs. Allen (2014) assumes that farmers undergo a costly search process to learn about the prices elsewhere in perfectly competitive markets, which is formalized by a fixed cost that formers pay; however, the effect of search frictions on aggregate welfare is left unanswered. Our focus on aggregate welfare implications in the presence of search frictions is closer to Krolikowski and McCallum (2021). Using the Pissarides (2000) search model in the Chaney (2008) trade model, they find that when producers face search frictions to find potential retailers, the presence of such frictions alters the response of welfare to trade shocks. While our welfare result in X-integration has a similar flavor to theirs, this paper is based on Krugman (1980) where firm heterogeneity stems only from matching status of firms. As the number of all types of agents is endogenously determined, our model structure offers a different mechanism through which search amplifies welfare gains from trade, i.e., trade-induced industry restructuring. In addition, our welfare result in M-integration is not addressed by their work.

Our findings on the effect of search frictions are related to some of existing work that examines the effect of contractual frictions. Papers in this strand of the literature can be broadly categorized into the following two classes. First is to model the effect of contractual frictions on firms' choices between integration and outsourcing in procuring specialized inputs (e.g., Antràs, 2003; Antràs and Helpman, 2004; Ornelas and Turner, 2008, 2012). Second is to investigate the effect of contractual frictions on country productivity and comparative advantage (e.g., Acemoglu et al., 2007; Costinot, 2009; Levchenko, 2007; Nunn, 2007). Our contribution to this literature is to show that the model of search frictions may enable us to obtain a better understanding of aggregate welfare. For example, our model helps to appreciate welfare implications of offshoring à la Antràs and Helpman (2004) who study firms' decisions on where to procure specialized inputs in a setting of North-South trade. Interpreting offshoring equilibrium as M-integration, we show that search and matching can account for a wage difference (i.e., country productivity) between North and South. Our model also reveals that, if a wage difference is large, offshoring can generate welfare gains for North, whereas it may simultaneously cause welfare losses for South in M-integration.

[4] The recent literature on production networks in international trade is particularly relevant to the final point. See, for example, Chaney (2014), Bernard et al. (2022), Eaton et al. (2022) and Sugita et al. (2023). Motivated by facts on buyer–seller relationships, all of these papers show that the variation in firm sales is largely accounted for by the way firms search for and match with suppliers. As a result, changes in search frictions can have an impact as great as those in trade frictions on the sales variation. Their analysis, however, remains silent about the welfare consequences of two different forms of economic integration.

2 Setup

2.1 Demand

Consider an industry that is populated by L units of identical agents who consume a continuum of varieties. The preferences of a representative agent are given by a CES utility function with an elasticity $\sigma > 1$:

$$U = \left(\int_{\omega} y(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}},$$

where $y(\omega)$ is consumption of variety ω. Utility maximization subject to budget constraint yields the optimal level of consumption and expenditure for variety ω:

$$\begin{aligned} y(\omega) &= Ap(\omega)^{-\sigma}, \\ r(\omega) &= Ap(\omega)^{1-\sigma}, \end{aligned}$$

where $p(\omega)$ is the price of variety ω and A is the index of industry demand. Using $P = \left(\int p(\omega)^{1-\sigma} d\omega\right)^{1/(1-\sigma)}$ and $R = \int r(\omega) d\omega$ to denote the price index and aggregate expenditure (or revenue), the utility function implies $A = RP^{\sigma-1}$ which is treated as a constant by an individual firm in the industry.

2.2 Production

The production of variety ω requires specialized inputs, $x^F(\omega)$, $x^S(\omega)$, each provided by a firm and a supplier. These inputs are combined by a CES production function with an elasticity of $\sigma > 1$:

$$y(\omega) = \left((x^F(\omega))^{\frac{\sigma-1}{\sigma}} + \mathbb{1}(\omega)(x^S(\omega))^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},$$

where $\mathbb{1}(\omega)$ is an indicator function which takes the value of one if firm ω uses $x^S(\omega)$ obtained from a supplier in final-good production and zero otherwise. We think of $x^F(\omega)$ as a "core" input that must be developed at a firm's own expense, and $x^S(\omega)$ as a "non-core" input that can be sourced from an outside independent supplier. Hereafter superscripts F and S are attached to variables relevant to firms and suppliers, respectively.

Following the literature on input-output linkages (e.g., Antràs et al., 2017), we make a simplifying assumption that while final goods are monopolistically produced with a markup over marginal cost, intermediate inputs are competitively produced at marginal cost. Specifically, $x^F(\omega)$ requires a^F units of labor in a country in which a firm locates, and $x^S(\omega)$ requires a^S units of labor in a country in which a supplier locates, where a^F and a^S are common for each variety and thus the variety index ω is unattached on them. If firms and suppliers operate in the same country so that the two inputs are produced within borders (which we will assume until Section 4), cost minimization yields the unit cost of final-good production:

$$c(\omega) = \left((wa^F)^{1-\sigma} + \mathbb{1}(\omega)(wa^S)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = \frac{wa^F}{\varphi(\omega)},$$

where w is the common wage rate and $\varphi(\omega)$ is the parameter measuring the unit cost difference across firms:

$$\varphi(\omega) \equiv \left(1 + \mathbb{1}(\omega) \left(\frac{a^F}{a^S} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}}.$$

Each firm needs to find a potential supplier to obtain a non-core input but the search does not always end in success. This leads to the following differences among firms and suppliers, depending on their matching status. An unmatched supplier has no opportunity to sell a non-core input to any firm and earns zero profit. In turn, an unmatched firm is unable to use a non-core input and produces variety ω according to the technology with $\mathbb{1}(\omega) = 0$ and thus $\varphi(\omega) = 1$; however, monopolistic competition among firms makes it possible to earn profit. In contrast, a matched supplier has a chance to sell a non-core input to a matched firm and earns profit. In turn, a matched firm is able to use both core and non-core inputs and produces variety ω according to the technology with $\mathbb{1}(\omega) = 1$ and thus $\varphi(\omega) > 1$. This difference in $\varphi(\omega)$ captures a realistic aspect that successful matching with a compatible supplier enables a matched firm to produce with better efficiency than an unmatched firm. We refer to $\varphi(\omega)$ as firm productivity in this paper.

Next, consider the optimal behavior of agents. From consumer demand and firm technology, we can express firm ω's revenue, $r(\omega)$, as a function of inputs. Further, an unmatched firm chooses only a core input, $x^F(\omega)$, while a matched firm chooses both core and non-core inputs, $x^F(\omega), x^S(\omega)$. Using the indicator function $\mathbb{1}(\omega)$, then, firm ω's profit maximization problem is given by $\max\ r(\omega) - wa^F x^F(\omega) - \mathbb{1}(\omega)wa^S x^S(\omega)$.[5] As will be explained in Section 2.4, after solving the profit maximization problem, the equilibrium profit is distributed to a matched firm and a matched supplier through Nash bargaining. Regardless of the matching status of agents, the first-order conditions for profit maximization yields the following equilibrium price:

$$p(\varphi) = \frac{\sigma}{\sigma - 1}\frac{wa^F}{\varphi},$$

where the variety index ω is dropped hereafter. The equilibrium output and revenue are respectively given by

$$y(\varphi) = A\left(\frac{\sigma - 1}{\sigma}\frac{\varphi}{wa^F}\right)^{\sigma},$$

$$r(\varphi) = A\left(\frac{\sigma - 1}{\sigma}\frac{\varphi}{wa^F}\right)^{\sigma-1},$$

and the equilibrium profit (excluding any fixed cost) is given by $r(\varphi)/\sigma$ with CES preferences. Hence a matched firm with $\varphi > 1$ charges a lower price and earns a larger revenue and profit than an unmatched firm with $\varphi = 1$. Let $r \equiv r(1)$ denote the equilibrium revenue of an unmatched firm. Then the ratio of revenue of a matched firm to an unmatched firms depends only on firm productivity φ:

$$\frac{r(\varphi)}{r} = \varphi^{\sigma-1}. \tag{1}$$

Thus higher productivity implies larger differences in revenue and profit between matched and unmatched firms.

A few points are in order for our measure of firm productivity φ. First, φ captures the love-of-variety effect from input expansion on final-good production. This follows from observing that matched firms combine core and non-core inputs in a CES fashion, whereas unmatched firms use only a core input in final-good production. In that sense, firm productivity is endogenously determined by matching in the model. Second, φ is increasing in the relative unit labor requirement a^F/a^S, as the smaller is the ratio, the greater is the love-of-variety effect that firms can enjoy from matching. In reality, the unit labor requirements may be different between countries due to comparative advantage. We examine how the difference leads to gains or losses from trade in Section 6.

[5] This model setting implicitly assumes that contracts between a matched firm and a matched supplier are perfectly enforceable, which again follows from the literature on input-output linkages in international trade (e.g., Antràs et al., 2017).

In the baseline model, however, we focus on country symmetry and thus the difference between countries plays a minor role. From this reason, the unit labor requirements of two inputs are normalized to one ($a^F = a^S = 1$); for the same reason, the common wage rate is normalized to one ($w = 1$) by choosing labor as the numéraire. Third, the definition of φ indicates that the productivity level is different only in terms of the matching status of firms and hence all matched firms have the same productivity level. This differs from most of existing work in the heterogeneous firm literature where productivity continuously differs across firms.

2.3 Search and Matching

Upon incurring a fixed entry cost, new firms and new suppliers first enter the industry with being unmatched. Then these entrants undergo a search process to find potential partners and learn about their matching status. The search process involves one-to-one random matching, where directed search is not possible for any agents. After meeting suppliers, matched firms incur a fixed cost of starting up relationships with matched suppliers. We denote by N_e^F, N_e^S the number of newly entered agents, N^F, N^S the number of all (matched and unmatched) agents, and u^F, u^S the number of unmatched agents per unit of time. Thus $N^F - u^F, N^S - u^S$ are the number of matched agents per unit of time. The number of matches per unit of time is allocated by a matching function $m(u^F, u^S)$, which is increasing, concave, homogeneous of degree one, and satisfies standard Inada conditions. As a result, there are constant returns to scale in matching, e.g., a doubling in the number of unmatched agents results in a doubling in the number of matched agents.[6]

Using the matching function, we define the rate at which matches randomly occur across unmatched agents. The rate at which unmatched firms meet unmatched suppliers is equal to $\mu^F \equiv m(u^F, u^S)/u^F = m(1, \theta)$ where $\theta \equiv u^S/u^F$ is the ratio of unmatched suppliers to unmatched firms in the industry. Similarly, the rate at which unmatched suppliers meet unmatched firms is equal to $\mu^S \equiv m(u^F, u^S)/u^S = m(1/\theta, 1) = \mu^F/\theta$. Obviously, these contact rates are equivalent with the matching probabilities, where μ^F increases with θ but μ^S decreases with θ (from the property of $m(u^F, u^S)$). Later we impose a free entry condition so that any entrant ends up with zero expected profit; hence the total number of agents N^F, N^S is endogenously determined by free entry. Furthermore, as the matching function allocates the number of matches, both the number of matched agents $N^F - u^F, N^S - u^S$ and the ratio of unmatched agents θ are endogenously determined by search technology (referenced by $m(u^F, u^S)$). Finally, regardless of matching status, all agents face an exogenous probability δ of a bad shock that forces them to exit the industry at every point in time.

This paper considers a dynamic industry model in which matching and exiting simultaneously occur among unmatched and matched agents. Figure 1 shows the search process of firms that arises at every point in time. As described above, the number N_e^F of new firms first enters the unmatched pool. Then, among the number u^F of existing unmatched firms, a fraction of them randomly finds their partners at the rate of μ^F, and hence the number $\mu^F u^F$ of firms enters the matched pool at every point in time. At the same time, however, among the number u^F of existing unmatched firms and the number $N^F - u^F$ of existing matched firms, a fraction of them is hit by a bad shock at the rate of δ. As the number δu^F of unmatched firms and the number $\delta(N^F - u^F)$ of matched firms are hit by this shock, in total, the number δN^F of firms exits the industry at every point in time. Note that matched agents hit by a bad shock immediately exit the industry without moving from the matched pool to the unmatched pool. This reflects that a bad shock δ is modeled as a "death shock" like Melitz (2003), but not as a "separation shock" like Antràs and Costinot (2011). The search process of suppliers similarly arises at every point in time.

[6] As emphasized by Krolikowski and McCallum (2021), these assumptions on search and matching—constant returns to search and one-to-one matching—are largely supported by recent empirical work.

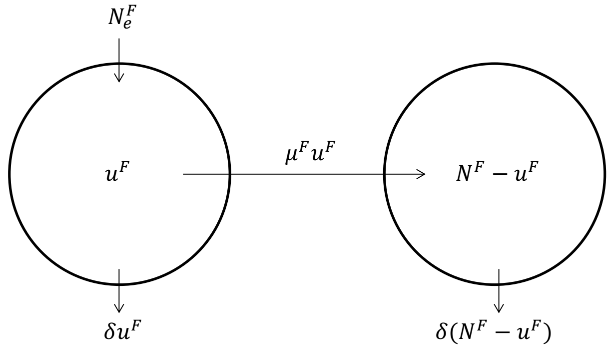

Figure 1: Search process of firms

2.4 Bargaining

Matched agents negotiate the division of surplus generated by matches. Firms and suppliers of matched status have complete information that makes bargaining bilaterally efficient. In the current dynamic industry model, they need to consider future profit flows in the negotiation as they may be hit by a bad shock in the future, in which case the outside option obtained when being unmatched and the fixed startup cost required to incur when being matched again must be reflected by the future probability of matches. Thus, the surplus is divided between firms and suppliers to maximize the *net* value of matches.

Let $V^F(\varphi), V^S(\varphi)$ denote the value function of matched agents, and let V^F, V^S denote the value function of unmatched agents. As will be shown in the next section, the value functions include not only the probabilities of matches and a bad shock that may occur in the future, but also the profits obtained at every point in time. Let $r^F(\varphi)/\sigma, r^S(\varphi)/\sigma$ denote the variable profit of matched agents at every point in time. These variable profits sum to $r(\varphi)/\sigma$ in (1) which are included in the value functions of matched agents $V^F(\varphi), V^S(\varphi)$, respectively. We formalize the bargaining within matched agents as symmetric Nash bargaining in which matched firms and matched suppliers capture an equal share of the ex-post net surplus generated from bilateral relationships:

$$\max_{\frac{r^F(\varphi)}{\sigma}, \frac{r^S(\varphi)}{\sigma}} \quad \Big(V^F(\varphi) - V^F - F_d\Big)\Big(V^S(\varphi) - V^S\Big),$$

subject to $r^F(\varphi)/\sigma + r^S(\varphi)/\sigma = r(\varphi)/\sigma$. F_d is a one-time fixed cost (measured in units of labor) that matched firms incur to start up the relationships with matched suppliers. This fixed cost captures various costly activities to facilitate production such as search, monitoring and communication, which we simply refer to as search cost. The bilateral input price is implicitly determined at this profit sharing.

2.5 Timing

The timing of events at every point in time is divided into four periods. In the first period, new agents enter the industry as either firms or suppliers with being unmatched at which point entrants incur the entry cost. In the second period, existing unmatched agents seek potential partners and learn about their matching status at which point matched firms incur the search cost. In the third period, matched suppliers produce and sell non-core inputs to matched firms at which point matched agents bargain over the surplus. In the last period, both matched and unmatched firms produce and sell final goods to consumers at which point they obtain their variable profit.

3 Autarky

3.1 Equilibrium Conditions

Denoting a common discount rate by γ, the value functions must satisfy the following Bellman equations that characterize the expected profit of any type of agents:

$$\begin{aligned}\gamma V^F(\varphi) &= \frac{r^F(\varphi)}{\sigma} - \delta V^F(\varphi) + \dot{V}^F(\varphi),\\ \gamma V^F &= \frac{r}{\sigma} + \mu^F\Big(V^F(\varphi) - V^F - F_d\Big) - \delta V^F + \dot{V}^F,\\ \gamma V^S(\varphi) &= \frac{r^S(\varphi)}{\sigma} - \delta V^S(\varphi) + \dot{V}^S(\varphi),\\ \gamma V^S &= \mu^S\Big(V^S(\varphi) - V^S\Big) - \delta V^S + \dot{V}^S.\end{aligned} \tag{2}$$

The first equation shows that matched firms obtain a gain $r^F(\varphi)/\sigma$, but become inactive at the rate δ at which point they suffer a loss $V^F(\varphi)$ from exiting the industry (as δ is modeled as a death shock), with a potential gain or loss $\dot{V}^F(\varphi)$ from remaining matched. On the other hand, the second equation shows that unmatched firms obtain a gain r/σ and become matched at the rate μ^F at which point they obtain a gain $V^F(\varphi) - V^F - F_d$, but become inactive at the rate δ at which point they suffer a loss V^F from exiting the industry, with a potential gain or loss $\dot{V}^F$ from remaining unmatched. A similar interpretation applies to the Bellman equations of suppliers, but the last equation shows that suppliers obtain no gain when being unmatched (as they earn zero profit).

We can describe how Nash bargaining between firms and suppliers affects the division of surplus. As formally derived in Appendix A.1, Nash bargaining imposes the following condition at any point in time:

$$V^F(\varphi) - V^F - F_d = \frac{1}{2}\Big(V^F(\varphi) - V^F - F_d + V^S(\varphi) - V^S\Big), \tag{3}$$

which states that, under symmetric bargaining power and complete information between two types of agents, matched firms obtain half of the net surplus at the bargaining stage.

The number of agents evolves according to the following law of motion in the matched pool:

$$\begin{aligned}\dot{N}^F - \dot{u}^F &= \delta(N^F - u^F) - \mu^F u^F,\\ \dot{N}^S - \dot{u}^S &= \delta(N^S - u^S) - \mu^S u^S,\end{aligned} \tag{4}$$

where the number of matched firms must be equal to the number of matched suppliers:

$$N^F - u^F = N^S - u^S \equiv n.$$

While we mainly use (4), the same applies to the unmatched pool $\dot{u}^F = (\delta + \mu^F)u^F - N_e^F, \dot{u}^S = (\delta + \mu^S)u^S - N_e^S$ as well as the industry as a whole $\dot{N}^F = \delta N^F - N_e^F, \dot{N}^S = \delta N^S - N_e^S$ (see Figure 1).

Free entry into any production activity ensures that the net value of entry must be zero at all points in time. Since any agents first enter the industry with being unmatched, the condition must hold for unmatched agents. Let $V_e^F \equiv V^F - F_e^F$, $V_e^S \equiv V^S - F_e^S$ denote the net value of entry for unmatched agents, where F_e^F, F_e^S are one-time fixed entry costs (measured in units of labor). Then we have

$$V_e^F = V_e^S = 0. \tag{5}$$

3.2 Equilibrium Characterization

In the last period of every point in time, firms and suppliers earn their profit until they are hit by a bad shock, which gives us the value function of each type of agents. This paper only considers a steady state equilibrium in which the aggregate variables remain constant over time, implying no potential gain or loss from any production activity and thus $\dot{V}^F(\varphi) = \dot{V}^F = \dot{V}^S(\varphi) = \dot{V}^S = 0$. Furthermore, the discount rate is assumed to zero because (i) the probability of a bad shock introduces an effect similar to time discounting; and (ii) the aggregate profit does not equal the aggregate investment cost at any point in time in equilibrium with a positive discount rate.[7] Setting $\gamma = 0$ and rearranging, we can express (2) as the steady-state value of each type of agents:

$$\begin{aligned} V^F(\varphi) &= \frac{r^F(\varphi)}{\delta\sigma}, \\ V^F &= \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{\delta + \mu^F}\right)\left(\frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right), \\ V^S(\varphi) &= \frac{r^S(\varphi)}{\delta\sigma}, \\ V^S &= \left(\frac{\mu^S}{\delta + \mu^S}\right)\frac{r^S(\varphi)}{\delta\sigma}. \end{aligned} \tag{6}$$

As the probability of a bad shock δ works as time discounting in our dynamic setting, the profit divided by δ represents the present value of profit flows. Then, (6) shows that the value of each type of agents consists of the present value of profit flows obtained from the current matching status plus that of net profit flows additionally obtained when being matched. To ensure an enough incentive for unmatched firms to incur the search cost F_d, we assume $r^F(\varphi)/\delta\sigma - r/\delta\sigma - F_d > 0$. Then (6) implies $V^F(\varphi) - V^F - F_d > 0$, i.e., the net value of matches is strictly positive for firms. In this model, the expectation of future profit flows augmented by matching is the only reason that unmatched firms consider sinking the search cost that is required to start up the relationships upon meeting potential suppliers.

In the third period, matched agents bargain over surplus. Solving (3) in light of (6), we get

$$\begin{aligned} \frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d &= \beta\left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right), \\ \frac{r^S(\varphi)}{\delta\sigma} &= (1-\beta)\left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right), \end{aligned} \tag{7}$$

where

$$\beta \equiv \frac{\delta + \mu^F}{2\delta + \mu^F + \mu^S} \tag{8}$$

is "effective" bargaining power of firms. This sharing rule indicates key implications when search and matching are present. On the one hand, (7) shows that while the search cost F_d is initially paid by firms, that cost is eventually shared with suppliers through the Nash bargaining; however, the extent to which the search cost is shared with firms and suppliers is determined by effective bargaining power β. On the other hand, (8) shows that while primitive bargaining power is symmetric between firms and suppliers, effective bargaining power is different and endogenously determined by the probability of a bad shock δ and that of matches μ^F, μ^S where the latter is affected by the number of firms and suppliers N^F, N^S in the industry. In particular, from the features of our matching function, it follows immediately that: (i) β is increasing in $\theta = (N^S - n)/(N^F - n)$; and (ii)

[7] These points are the same as those in Melitz (2003, footnote 16). See also Appendix A.2 for a formal proof in this paper.

β is greater than 1/2 if and only if N^F is smaller than N^S. As is well-known in the bargaining literature, thus, (8) says that each firm's bargaining power effectively increases with the number of suppliers but it effectively decreases with the number of firms: other things equal, the acquisition of alternative sellers improves a buyer's bargaining position but the invitation of competing buyers worsens its bargaining position.[8]

In the second period, firms and suppliers learn about their matching status. Setting $\dot{N}^F - \dot{u}^F = \dot{N}^S - \dot{u}^S = 0$ in (4) and using $\mu^F = \mu^S/\theta$, the steady-state number of matches satisfies

$$n = \left(\frac{\mu^F}{\delta + \mu^F}\right) N^F = \left(\frac{\mu^S}{\delta + \mu^S}\right) N^S. \tag{9}$$

(9) describes how the number of matches n is tied to the exogenous rate of a bad shock δ and the endogenous rate of matches, μ^F, μ^S among the total number of agents N^F, N^S in the industry. Though (9) is derived from the matched pool by focusing on (4), the same relationship is derived from the unmatched pool ($\dot{u}^F = \dot{u}^S = 0$) and the industry as a whole ($\dot{N}^F = \dot{N}^S = 0$) by rearranging them.

In the first period, firms and suppliers enter the industry until their net expected value is driven to zero. Using (6), (7), (8) and (9), the free entry condition in (5) can be written as

$$\begin{aligned} &\frac{r}{\sigma} + \frac{n}{N^F}\beta\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d\right) - f_e^F = 0, \\ &\frac{n}{N^S}(1-\beta)\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d\right) - f_e^S = 0, \end{aligned} \tag{10}$$

where $f_d \equiv \delta F_d$, $f_e^F \equiv \delta F_e^F$ and $f_e^S \equiv \delta F_e^S$ respectively denote the amortized per-period portion of the one-time fixed cost incurred by relevant agents at every point in time.

To understand this, consider the first equality in (10) which relates to the free entry condition of firms. When being unmatched, firms earn the variable profit r/σ at every point in time as their outside option. Moreover, the fraction n/N^F of firms finds suppliers and additionally earns the economic rent $r(\varphi)/\sigma - r/\sigma - f_d (> 0)$ weighted by effective bargaining power β. Hence the first two terms represent firms' expected profit. Let $\phi^F \equiv (n/N^F)\beta$ denote the expected share of economic rent that firms obtain in the Nash bargaining. From (8) and (9), this share is $\mu^F/(2\delta + \mu^F + \mu^S)$. Then the free entry condition of firms means that the expected profit at every point in time consists of the outside option r/σ plus the economic rent weighted by the expected share ϕ^F, which must equal the amortized per-period portion of the fixed entry cost f_e^F. A similar interpretation also applies to the free entry condition of suppliers given in the second equality of (10), in the sense that the expected profit at every point in time consists of the economic rent weighted by the expected share $\phi^S \equiv (n/N^S)(1-\beta) = \mu^S/(2\delta + \mu^F + \mu^S)$ where their outside option is zero as they earn zero profit when being unmatched.

(10) characterizes the model's equilibrium as follows. First, it pins down the profit of unmatched firms r/σ. Although both unmatched and matched firms earn profits, the profits of unmatched firms are exactly offset by the entry and search costs so that the net profits of unmatched firms reduce to zero. Second, together with the matching function, it determines the number of agents n, N^F, N^S, which simultaneously determines the ratio of unmatched agents $\theta = (N^S - n)/(N^F - n)$. Finally, by taking account of the labor market clearing condition, it ensures that aggregate revenue R equals the exogenously fixed index of market size L (see Appendix A.2). As we have chosen labor as the numéraire, $R(= L)$ also equals aggregate labor income.

[8]See Wolinsky (1987) in a closed-economy model. Applying this insight to an open-economy model, Ara and Ghosh (2016) show that when firms offshore some production processes to suppliers abroad, endogenous bargaining power might have a serious impact on optimal trade policy via the division of surplus between agents. While existing work mostly uses a partial-equilibrium setting, we extend to a general-equilibrium setting and uncover a new source of the gains from trade operating through the channel.

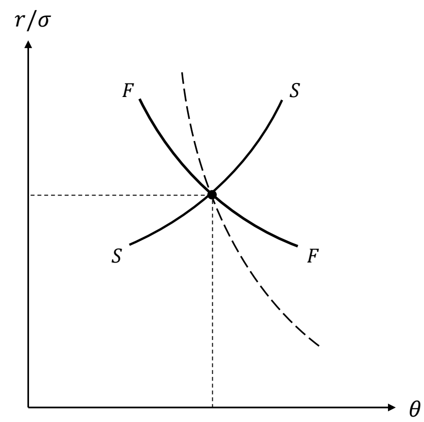

Figure 2: Autarky equilibrium

3.3 Existence and Uniqueness

The equilibrium revenue in (1) implies that the economic rent depends on productivity φ and search cost f_d:

$$\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d = \frac{\left(\varphi^{\sigma-1} - 1\right) r}{\sigma} - f_d.$$

Using this in (10), we can solve for the variable profit of unmatched firms given as the following equalities:

$$\begin{aligned} \frac{r}{\sigma} &= \frac{f_e^F + f_d \phi^F}{1 + (\varphi^{\sigma-1} - 1)\phi^F}, \\ \frac{r}{\sigma} &= \frac{f_e^S + f_d \phi^S}{(\varphi^{\sigma-1} - 1)\phi^S}, \end{aligned} \tag{11}$$

where the dependence of (11) on θ is understood from ϕ^F, ϕ^S. Noting that the search cost f_d is shared between matched firms and suppliers with the expected shares ϕ^F, ϕ^S at the bargaining stage, the right-hand side of (11) can be interpreted as the expected fixed cost of firms and suppliers under free entry. With this interpretation, (11) shows that any agents who enter the industry as firms or suppliers earn zero net expected profit.

The model's equilibrium can be characterized by (11) which is the system of two equations that endogenously pin down two unknowns, θ and r/σ. The solid curves in Figure 2 illustrate the relationship of (11), labeled as FF and SS, in the $(\theta, r/\sigma)$ space. Since μ^F is increasing in θ, the first equality of (11) is decreasing in θ and thus the FF curve is downward sloping.[9] Similarly, since μ^S is decreasing in θ, the second equality of (11) is increasing in θ and thus the SS curve is upward sloping. The economic intuition behind the downward FF curve and the upward SS curve is very simple. For firms, the higher θ, the higher the probability of matches and the higher the expected profit through the expected share ϕ^F. Under free entry, however, this *ex ante* expected profit induces further entry and drives down the *ex post* profitability for firms. It is clear to see that an increase in θ has an opposite effect on the expected profit for suppliers. These features of the FF and SS curves ensure the existence and uniqueness of autarky equilibrium. The intersection of two curves simultaneously determines the ratio of unmatched agents θ and the profit of unmatched firms r/σ that are both consistent with free entry, which is graphically represented in Figure 2.

[9] Differentiating the first equality of (11) with respect to θ, we also need $(\varphi^{\sigma-1} - 1)f_e^F - f_d > 0$; however, this is shown to hold by plugging the first equality into the condition under which $r(\varphi)/\sigma - r/\sigma - f_d > 0$.

Once these two equilibrium variables are determined, other endogenous variables are written as a function of them. The variable profit of matched firms is $r(\varphi)/\sigma = \varphi^{\sigma-1}r/\sigma$ from (1). This implies that their equilibrium net profit is strictly positive because free entry is imposed on unmatched agents, as given in (5). The number of agents is determined by both free entry and search technology in the present model where the former pins down N^F, N^S and the latter pins down n from the steady-state relationship in (9). Let f denote the expected fixed cost under free entry in autarky, given as the right-hand side of (11). Noting that free entry requires $r/\sigma = f$, the number of each type of agents is expressed as (see Appendix A.3)

$$n = \left(\frac{\mu^F}{\Xi}\right) L, \quad N^F = \left(\frac{\delta + \mu^F}{\Xi}\right) L, \quad N^S = \left(\frac{\delta\theta + \mu^F}{\Xi}\right) L, \tag{12}$$

where $\Xi \equiv \sigma f(\delta + \varphi^{\sigma-1}\mu^F)$. Using (12) in the price index, welfare per worker equivalent to the real wage is

$$W = \frac{\sigma - 1}{\sigma}\left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}}. \tag{13}$$

This completes the characterization of autarky equilibrium.

It is important to emphasize that market size L has no effect on the key equilibrium variables of the model, θ and r/σ. The result follows from inspecting that the free entry condition in (11) does not involve L, and hence the equilibrium represented by the intersection of FF and SS curves is not affected by market size. Intuitively, when market size increases, the number of firms and suppliers increases proportionately, and so does the number of matched agents under constant returns to scale in matching (see (12)). Since an increase in market size leads to a proportionate increase in all numbers of agents n, N^F, N^S in the industry, the ratio of unmatched agents $\theta = (N^S - n)/(N^F - n)$ remains fixed. The variable profit of unmatched firms r/σ remains fixed as well since an increase in the aggregate expenditure is exactly offset by a fall in the price index associated with an increase in the number of agents. Though market size does not affect the two key equilibrium variables, (13) shows that an increase in market size nonetheless leads to welfare gains due to increased product variety in the industry.

3.4 Search Frictions

Building on the equilibrium characterization, we study how the endogenous variables are affected by exogenous variables of the model. Before examining such comparative statics, we first need to show that if search frictions are prohibitively large, our model collapses to a standard monopolistic competition model of Krugman (1980). To see this point, imagine what would happen if firms were not able to search for suppliers. As the number of matches is zero ($n = 0$) in that case, the first equality of (10) is the only relevant free entry condition. Further, as the expected share of economic rent is zero ($\phi^F = 0$) in the first equality of (11), the number of firms in (12) reduces to $N^F = L/\sigma f$ (from $\mu^F = 0$), while welfare in (13) must be evaluated at $f = f_e^F$ (from $r/\sigma = f$). This equilibrium characterization is exactly the same as that derived by Krugman (1980). If search frictions are not so large that firms are able to search for suppliers, however, both aggregate variables are critically affected by the ratio of unmatched agents seeking partners θ, as seen in (12) and (13). We will show in Sections 4 and 5 that search opportunities matter for welfare gains or losses from trade by affecting the ratio of unmatched agents. The present model also features firm heterogeneity similar to Melitz (2003) in the sense that firms produce with different production efficiencies depending on their matching status. One of the key departures from his model, however, is that the firm distribution is *binomial* (i.e., firm status is only either unmatched or matched) which varies *endogenously* with exogenous shocks.

It is possible to make the point much clearer by examining the effect of search frictions on industry structure. Following the literature (e.g., Allen, 2014), we formalize search frictions as the search cost f_d and assume that the lager is the fixed cost, the more costly is the search for firms. Inspection of (11) reveals that a decrease in f_d shifts down the FF and SS curves in Figure 2 and thus decreases the variable profit of unmatched firms r/σ. Further, the downward shift in two curves always increases the ratio of unmatched agents seeking partners θ, since the following equilibrium relationship holds from cancelling out the common term from (10):

$$\frac{r}{\sigma} = f_e^F - f_e^S \theta. \tag{14}$$

The negative relationship between θ and r/σ derived from (14) is illustrated by the dashed curve in Figure 2, summarizing the locus of equilibria of different levels of search cost. Therefore, the smaller is the search cost f_d, the higher is the ratio of unmatched agents θ and the lower is the profit of unmatched firms r/σ. This implies, in a special case of prohibitively large search cost ($f_d = \infty$), that the ratio of unmatched agents approaches zero ($\theta = 0$) and thus the equilibrium characterization collapses to that described by Krugman (1980) as seen above. If search frictions are not so large, however, the model yields interesting comparative statics results regarding how search frictions affect industry structure. (9) shows that a decrease in f_d raises the ratio of matched firms n/N^F but lowers the ratio of matched suppliers n/N^S. Instead (12) shows that the number of agents n, N^F, N^S does not always rise or fall. Despite that ambiguity, (13) shows that welfare always rises, as the expected profit of unmatched firms $r/\sigma = f$ always falls by a decrease in f_d.

Intuition behind the above results is explained by the differential impact of search cost on firms and suppliers. An increase in economic rent (associated with a decrease in f_d) raises the *ex ante* expected profit, which induces entry of firms and suppliers and drives down the *ex post* profitability of agents under free entry. In Figure 2, this reflects that the two curves shift downwards by a decrease in f_d. The competitive pressure is weaker for firms than for suppliers, however, since further entry decreases the outside option of firms (r/σ) while it has no effect on the outside option of suppliers (zero), which dampens an entry incentive of firms relative to suppliers. In Figure 2, this implies that the downward shift is smaller in the FF curve than in the SS curve. As suppliers enter relatively more than firms, the matching probability improves for firms but worsens for suppliers, whereby the ratio of matched firms rises but the ratio of matched suppliers falls. In response to this shock, the number of agents takes a balance between the increased matching probability of firms and decreased *ex post* profitability. In (12), these opposing forces are reflected in increased μ^F and decreased f , which leads to an ambiguous effect on the number of agents in the industry.

As for welfare, we stress that search—even though it is costly—raises welfare relative to that in a standard monopolistic competition model. Let $\tilde{W}$ denote welfare when the search cost is so large that any firms cannot seek suppliers, while keeping W to denote welfare when such cost is small enough. From (13), the welfare ratio $W/\tilde{W}$ depends only on the (expected) fixed cost f, which in turn equals the variable profit r/σ under free entry. In light of (14), $\tilde{W}$ must be evaluated at $f = f_e^F$ (as $\theta = 0$ if the search cost is prohibitively large) and hence

$$\frac{W}{\tilde{W}} = \left(\frac{f_e^F}{f_e^F - f_e^S \theta} \right)^{\frac{1}{\sigma - 1}}.$$

The fact $W/\tilde{W} > 1$ follows from $\theta > 0$ in our model. Then the comparative statics results directly imply that the welfare ratio is higher, the smaller is the search cost that firms need to incur. Intuitively, a decrease in f_d raises the number of suppliers N^S relative to that of firms N^F in the industry (see (12)). This industry structure allows firms to find suppliers more easily (reflected by the higher ratio of unmatched agents $\theta = (N^S - n)/(N^F - n)$), and varieties are relatively more likely produced by matched firms than unmatched firms. At the same time, resources are relatively more allocated from unmatched firms to matched firms (reflected by the lower profit of unmatched firms r/σ). Operating through these separate but interacted channels, a decrease in search frictions has a significant impact on aggregate welfare in this model.

Our prediction for the effect of search frictions on welfare can be empirically investigated by computing the expenditure share on goods produced by unmatched firms. Let $s \equiv (N^F - n)r/R$ denote that expenditure share where aggregate expenditure R equals the exogenously fixed index of market size L (see the end of Section 3.2). Note that market size has no effect on the expenditure share s because the profit r/σ is independent of L while the number of firms n, N^F increases proportionately with $L(= R)$ in our model with constant returns to search. In fact, using (1) and (9) in $R = (N^F - n)r + nr(\varphi)$, the expenditure share is expressed as

$$s = \frac{\delta}{\delta + \varphi^{\sigma-1}\mu^F},$$

where μ^F is unaffected by market size (as θ is independent of L). In contrast, the expenditure share is affected by search cost f_d through a change in θ. From the comparative statics, it follows that a decrease in f_d increases θ and improves the matching probability of firms μ^F, which decreases the expenditure share s. This decrease contributes to aggregate welfare since consumers access more varieties produced by matched firms at lower cost. We are able to show this distinct welfare channel by totally differentiating (13) in light of $L = R$ and $f = r/\sigma$, which allows us to express welfare changes in terms of the expenditure share s:

$$d\ln W = \frac{1}{\sigma - 1}\left(d\ln\left(N^F - n\right) - d\ln s\right). \tag{15}$$

Hence, aggregate welfare is higher, the smaller the expenditure share on goods produced by unmatched firms.[10] Welfare changes in (15) are similar to those in Arkolakis et al. (2012), in the sense that such changes are captured by the observable moments and particularly the expenditure share is one of the sufficient statistics for welfare. One of the crucial differences from theirs, however, is that we consider the expenditure share on goods produced by unmatched firms, which must be distinguished from those produced by matched firms in evaluating welfare when search and matching play an important role in firm productivity.

The above finding can be alternatively seen in terms of the relative expenditure share of unmatched firms, $s/(1-s) = \delta/\varphi^{\sigma-1}\mu^F$, which is decomposed into the intensive margin ratio $r/r(\varphi) = 1/\varphi^{\sigma-1}$ and the extensive margin ratio $(N^F - n)/n = \delta/\mu^F$ from (1) and (12), respectively. It is clear that the relative expenditure share decreases with search cost only through the extensive margin ratio, suggesting that the smaller the search cost, the smaller the number of unmatched firms in the industry, and hence the lower the ratio of goods produced by unmatched firms to goods produced by matched firms. The result is consistent with the empirical evidence by Allen (2014) who reports that search frictions only affect firms' decision to export (i.e., extensive margin) in his structural estimation of regional trade. In this model, an increase in the extensive margin associated with a decrease in the search cost directly contributes to aggregate welfare.

Proposition 1: *A decrease in search frictions increases welfare by improving the matching probability of firms associated with resource allocations from unmatched firms to matched firms.*

[10] Observe that aggregate welfare is higher, the larger is the number of unmatched firms $N^F - n$ due to increased product variety. Unfortunately, the model cannot predict whether that number rises or falls without imposing strong restrictions on entry and search technology. Even when this number falls, a decrease in the expenditure share always outweighs a decrease in that product variety because welfare always increases with decreased search frictions.

4 Integration of Goods Markets

4.1 Assumptions

We turn to considering a world economy composed of two countries of the type described in Section 3. There is no difference in labor endowments and technology between these countries, so that $L = L^*$ and $a^F = a^{F^*} = a^S = a^{S^*} (= 1)$, where asterisks are attached to foreign variables. Further, they have the same matching function and bargaining power of agents. As a result, not only is the number of agents but also the number of unmatched and matched agents is the same between the two countries. Though the countries are assumed to be symmetric, they are not exactly identical as each firm produces a variety differentiated at home and abroad.

This section first studies economic integration of final-good markets referred to as *X-integration* in our paper, which allows firms to sell final goods across borders, maintaining the assumption that firms search for suppliers only within borders. In this integration, a firm wishing to export must incur an iceberg transport cost τ_x and a one-time fixed cost F_x (measured in units of labor) where the latter can be regarded as an investment that is required to enter the export market as modeled by Melitz (2003). Our assumptions of X-integration imply that the two countries have the same contact rates, given by μ^F and $\mu^S = \mu^F/\theta$, where θ is also the same across the countries. As formally shown in Appendix A.2, aggregate revenue continues to equal market size in this setting where the common wage rate is normalized to one ($w = w^* = 1$) as in autarky.

When X-integration takes place, there are three possible cases associated with different levels of trade costs: (i) no firm exports; (ii) only matched firms export; and (iii) both unmatched firms and matched firms export.[11] For expositional purposes, we restrict attention to the case where the level of trade costs is intermediate so that matched firms export but unmatched firms do not. As will be clear shortly, the level of trade costs is immaterial to our analysis, and the welfare result of X-integration is valid even in the case where the level of trade costs is so low that unmatched firms also export. In relation to this point, if there are no trade costs and all firms freely export, search frictions do not play any role in examining the impact of X-integration. In that case, the impact of X-integration is the same as that of market size in the sense that the gains from trade are solely associated with increased product variety without affecting the two key variables of the model, θ and r/σ. This impact is very similar to that described by Krugman (1980) and Melitz (2003), though they do not model search frictions. If firms incur trade costs, however, X-integration has a critical impact on these variables so as to reinforce the welfare gains by improving the matching probability of firms and reallocating resources from unmatched firms to matched firms. As equilibrium conditions in X-integration closely resemble those in autarky, we provide only important equations pertaining to this integration below and relegate others (including an equilibrium analysis where all firms export) to Appendix A.4.

4.2 Equilibrium Characterization

This section mainly characterizes and solves for X-integration equilibrium at home due to country symmetry. It is clear that the equilibrium domestic price, output, and revenue are expressed in the same way as those in autarky. In particular, the equilibrium domestic revenue of unmatched firms is denoted by $r = r(1)$ as before, while that of matched firms is $r_d(\varphi) = \varphi^{\sigma-1}r$. In addition, matched firms export in X-integration and set the higher equilibrium price in the export market due to the increased marginal cost τ_x:

$$p_x(\varphi) = \frac{\sigma}{\sigma - 1}\frac{\tau_x}{\varphi}.$$

[11] Since unmatched firms are less efficient than matched firms, there is no equilibrium where only unmatched firms export.

From the above optimal pricing, the equilibrium export revenue of matched firms is given by $r_x(\varphi) = \tau_x^{1-\sigma} r_d(\varphi)$. Similarly to (1) in autarky, the ratio of export revenue of matched firms to domestic revenue of unmatched firms must satisfy the following equality in X-integration:

$$\frac{r_x(\varphi)}{r} = \left(\frac{\varphi}{\tau_x}\right)^{\sigma-1}. \tag{16}$$

This in turn yields the equilibrium total revenue of matched firms, $r(\varphi) = r_d(\varphi) + r_x(\varphi) = (1 + \tau_x^{1-\sigma})\varphi^{\sigma-1} r$. Not surprisingly, thus, the ratio of total revenue (and profit) of matched firms to unmatched firms is enlarged in X-integration.

We next specify the equilibrium conditions of X-integration. If we assume that only matched firms export, the Bellman equations in X-integration entail the following differences from (2). First, unmatched firms become matched at the rate μ^F at which point they have to make a one-time investment F_x to enter the export market, and hence they obtain a gain $V^F(\varphi) - V^F - F_d - F_x$ in X-integration. Second, matched firms earn the variable profit from not only the domestic market $r_d^F(\varphi)/\sigma$ but also the export market $r_x^F(\varphi)/\sigma$, which sum to $r^F(\varphi)/\sigma$. As in autarky, this profit is included into the Bellman equation as a gain for matched firms in X-integration. While the Bellman conditions for suppliers are the same as (2), matched suppliers also obtain the variable profit $r^S(\varphi)/\sigma$ from the domestic market $r_d^S(\varphi)/\sigma$ and the export market $r_x^S(\varphi)/\sigma$.

Proceeding as in Section 3.2, we have the value functions of agents in stationary equilibrium of X-integration. Here we assume $r^F(\varphi)/\delta\sigma - r/\delta\sigma - F_d - F_x > 0$ in order to ensure that firms have an enough incentive to sink not only the fixed cost of search F_d but also the fixed cost of export F_x. Then, $V^F(\varphi) - V^F - F_d - F_x > 0$ so that the net value of matches is strictly positive for firms. This implies that the ex-post gains for firms in the Nash bargaining need to be replaced by $V^F(\varphi) - V^F - F_d - F_x$ in X-integration. As the constraint in (3) similarly holds in X-integration, however, the solution to the Nash bargaining problem subject to $r^F(\varphi)/\sigma + r^S(\varphi)/\sigma = r(\varphi)/\sigma$ gives us the following profit sharing rule like (7):

$$\frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d - F_x = \beta\left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d - F_x\right),$$
$$\frac{r^S(\varphi)}{\delta\sigma} = (1-\beta)\left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d - F_x\right),$$

where the effective bargaining power β is the same as (8) in autarky as the opportunities to search for partners are restricted only within borders in X-integration. For the same reason, the steady-state number of agents is the same as (9) in autarky.

Finally, using the steady-state value functions and the optimal profit sharing above, the free entry condition in (5) can be written as follows:

$$\begin{aligned} \frac{r}{\sigma} + \frac{n}{N^F}\beta\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d - f_x\right) - f_e^F &= 0, \\ \frac{n}{N^S}(1-\beta)\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d - f_x\right) - f_e^S &= 0, \end{aligned} \tag{17}$$

where $f_x \equiv \delta F_x$ denotes the amortized per-period portion of the one-time fixed export cost incurred at every point in time. We can interpret (17) in X-integration in a similar way to (10) in autarky. Further, this condition characterizes the model's equilibrium by simultaneously determining the ratio of unmatched agents θ and the profit of unmatched firms r/σ that are both consistent with free entry in X-integration.

Though the free entry condition (17) looks similar to that in autarky (10), the economic rent is earned from the domestic and export markets by matched firms in X-integration. It follows immediately from (16) that the economic rent of matched agents is expressed as

$$\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d - f_x = \frac{\left[\left(1+\tau_x^{1-\sigma}\right)\varphi^{\sigma-1} - 1\right] r}{\sigma} - f_d - f_x.$$

Hence the economic rent of matched agents critically depends on both variable and fixed export costs τ_x, f_x in X-integration. Substituting this relationship into (17), we can solve for the variable profit of unmatched firms, which satisfies the following equalities:

$$\begin{aligned} \frac{r}{\sigma} &= \frac{f_e^F + (f_d + f_x)\phi^F}{1 + \left[\left(1+\tau_x^{1-\sigma}\right)\varphi^{\sigma-1} - 1\right]\phi^F}, \\ \frac{r}{\sigma} &= \frac{f_e^S + (f_d + f_x)\phi^S}{\left[\left(1+\tau_x^{1-\sigma}\right)\varphi^{\sigma-1} - 1\right]\phi^S}, \end{aligned} \tag{18}$$

where the expected shares of matched agents ϕ^F, ϕ^S are the same as those in Section 3. Then, (18) shows that the variable profit equals the expected fixed cost of firms and suppliers in X-integration. It is important to see, however, that the expected fixed cost includes trade costs τ_x, f_x, even though we have derived the equilibrium where unmatched firms do not export in X-integration. This reflects the dynamics where unmatched agents may become matched in the future at which point they share trade costs at the bargaining stage with the expected shares ϕ^F, ϕ^S.

(18) can be illustrated in the $(\theta, r/\sigma)$ space. The argument similar to Figure 2 applies here to establish the existence and uniqueness of X-integration equilibrium, represented by the intersection of FF and SS curves. The questions that remain to ask are: (i) how the equilibrium variables are affected by X-integration; and (ii) how changes in the equilibrium variables are related to the gains from trade in X-integration. In what follows, we first address the impact of X-integration in Section 4.3 and next explore the gains from trade in X-integration in Section 4.4. Finally, we calibrate the model in order to assess the quantitative relevance in Section 4.5.

4.3 Impact of X-integration

To examine the impact of X-integration, we consider how the free entry condition (17) is affected by the level of trade costs associated with this integration. In equilibrium where only matched firms export, matched firms earn higher profit in X-integration than in autarky ($(1+\tau_x^{1-\sigma})\varphi^{\sigma-1}r/\sigma - f_d - f_x > \varphi^{\sigma-1}r/\sigma - f_d$), but unmatched firms do not ($(1+\tau_x^{1-\sigma})r/\sigma - f_x < r/\sigma$). Simplifying these inequalities, we find that trade costs must fall into the following intermediate level in such equilibrium:

$$\frac{r}{\sigma} < \tau_x^{\sigma-1} f_x < \frac{\varphi^{\sigma-1} r}{\sigma}. \tag{19}$$

Comparing (10) and (17) under (19) shows that the economic rent of matched agents is greater in X-integration ($\left[\left(1+\tau_x^{1-\sigma}\right)\varphi^{\sigma-1} - 1\right] r/\sigma - f_d - f_x$) than in autarky ($\left(\varphi^{\sigma-1} - 1\right) r/\sigma - f_d$) for given expected shares ϕ^F, ϕ^S. This increases the *ex ante* expected profit and induces further entry of agents under free entry, which decreases the *ex post* profit in X-integration relative to autarky. Indeed, comparing (11) and (18) under (19) shows that the FF and SS curves in X-integration are located below those in autarky for given θ, thereby decreasing r/σ. Moreover, the equilibrium relationship in (14) continues to hold in X-integration from rearrangement of (17), so that a decrease in r/σ always leads to an increase in θ which occurs only when the downward shift is smaller in the FF curve than in the SS curve. The changes in two curves imply that X-integration not only enlarges the profit differential between matched and unmatched firms, but also increases the ratio of unmatched agents which improves the matching probability of firms. Though we have focused on the impact of X-integration by comparing equilibria between autarky and this integration, simple inspection of (18) reveals that a decrease in trade costs (either variable τ_x or fixed f_x) induce a similar impact on the two curves and hence the two key equilibrium variables, θ and r/σ.

Intuitively, the downward shifts in FF and SS curves are triggered by an increase in the economic rent in X-integration, as seen above. Note, however, that this increase only requires the second inequality of (19), i.e., matched firms export in this integration. The extent to which the economic rent increases differs between firms and suppliers. From the free entry condition of firms given in the first equality of (17), X-integration has two effects on their expected profit. First, X-integration allows foreign firms to ship varieties to the domestic market, which drives down the *ex post* profitability of firms there and decreases their outside option (first term). Second, X-integration allows home firms to ship varieties to the export market, which gives them an additional opportunity to earn the export profit and increases the economic rent earned by matched firms (second term). While the latter dominates the former under (19), the opposing effects dampen the entry incentive of firms. As for the free entry condition of suppliers, in contrast, suppliers receive no negative effect on the expected profit from shipment of foreign firms to the domestic market because the outside option of being unmatched is zero. Thus X-integration raises the *ex ante* expected profit of suppliers relative to firms, which generates the *ex post* large competitive pressure on suppliers relative to firms. As the number of suppliers increases relative to firms, the ratio $\theta = (N^S - n)/(N^F - n)$ rises in X-integration relative to that in autarky.

The above intuition shows that, even when the level of trade costs is so low that unmatched firms also export, the impact of X-integration on the key equilibrium variables is similar. This can be confirmed by observing that the second inequality in (19) is only relevant to an increase in the expected profit that results in the downward shift in the two curves. When unmatched firms also export, the first inequality of (19) is reversed but matched firms nonetheless earn revenue and profit relatively more than unmatched firms in X-integration, which in turn raises the expected profit and induces the downward shift in two curves. In addition, the effect of X-integration on the expected profit is larger for suppliers than for firms as the outside option of suppliers is zero before and after X-integration even in this case. The differential effect induces further entry of suppliers relative to firms, leading to the greater downward shift for the SS curve than for the FF curve. Thus, unmatched firms' profit falls and the ratio of unmatched agents rises in this equilibrium. The only difference is that the decrease in profit applies to the *domestic* profit of unmatched firms (r_d/σ). In contrast, the *total* profit ($r/\sigma = (1 + \tau_x^{1-\sigma})r_d/\sigma$) increases due to the additional export profit, though matched firms earn relatively larger total profit.[12]

Once these two equilibrium variables are determined, other endogenous variables are written as a function of them. The number of agents and welfare per worker are expressed in a similar way to (12) and (13), respectively, by replacing f with the right-hand side of (18). The impact of X-integration leads to the following implications. First, from the fact that θ rises in X-integration, the ratio of matched firms is larger but the ratio of matched suppliers is smaller in X-integration than in autarky. Second, from the fact that $r/\sigma(= f)$ falls in X-integration, welfare is greater in X-integration than in autarky. Thus we have $n/N^F > n_a/N_a^F,\ n/N^S < n_a/N_a^S,\ W > W_a$ where the subscript a is attached to autarky variables. The impact on these aggregate variables is similar even in the case where both unmatched and matched firms export.

[12] See Appendix A.4 for a detailed analysis of equilibrium where unmatched firms also export. We show there that without fixed export costs ($f_x = 0$), any level of transport costs $\tau_x > 0$ does not affect the two key equilibrium variables in X-integration.

4.4 Gains from Trade in X-integration

The impact of X-integration on two equilibrium variables allows us to highlight a new mechanism through which economic integration of goods markets generates the gains from trade. We can use the same logic in Section 3.4 to make this point: if search frictions are prohibitively large so that firms are not able to search for suppliers, our model collapses to a standard monopolistic competition trade model of Krugman (1980) in X-integration. Since the number of matched agents is zero ($n = 0$) in that case, the free entry condition in (17) implies that the variable profit equals the fixed entry cost ($r/\sigma = f_e^F$) as in autarky.[13] Further, as the free entry condition pinning down the number of firms remains the same between autarky and X-integration, the number of varieties produced in each market N^F remains the same. In X-integration, however, firms export at cost of τ_x and hence the number of varieties available in each market increases to $(1+\tau_x^{1-\sigma})N^F$. This impact of trade on equilibrium variables is identical to that described by Krugman (1980): although costly trade does not affect the number of varieties *produced*, it increases the number of varieties *consumed*, which solely explains why aggregate welfare is higher in X-integration than in autarky.

If search frictions are not so large as in our model, however, costly trade does affect the number of firms as the expected fixed cost $f(= r/\sigma)$ falls from autarky to X-integration. This alters the impact of trade on welfare through a new mechanism arising from search, i.e., the *improved matching probability* of firms associated with resource reallocations between firms. On the one hand, the fact that unmatched firms' profit falls as a result of X-integration ($r/\sigma < r_a/\sigma$) implies that unmatched firms lose their market share (as $R = L$ in X-integration), reallocating resources from less efficient unmatched firms to more efficient matched firms within the industry. On the other hand, the fact that the ratio of unmatched agents rises as a result of this integration ($\theta > \theta_a$) implies that firms have the higher probability to meet suppliers, enhancing the overall production efficiency of the industry. These two features of our model jointly provide the mechanism through which countries can enjoy the gains from trade greater than those of a standard trade model.

To understand this mechanism of our model, we find it useful to compare the gains from trade with those of Arkolakis et al. (2012). They show that, in a large class of trade models, the gains from trade can be calculated by the only two sufficient statistics—the trade elasticity and the expenditure share on domestic goods. For our purpose, thus, we need to consider how these sufficient statistics are affected by the presence of search frictions. The first element is simply given as $\sigma - 1$, even though search frictions create selection into the export market in the present model. Following Arkolakis et al. (2012), define the trade elasticity as $\varepsilon \equiv -\partial \ln(R_x/R_d)/\partial \ln \tau_x$ where $R_d = (N^F - n)r + nr_d(\varphi)$ and $R_x = nr_x(\varphi)$ are aggregate expenditure on domestic and export goods, respectively. From (1), (9) and (16), we immediately obtain $R_x/R_d = \tau_x^{1-\sigma}\Lambda$ where $\Lambda \equiv \varphi^{\sigma-1}\mu^F/(\delta + \varphi^{\sigma-1}\mu^F)$ is the market share of exporters, capturing selection into the export market. Using this for the definition of ε,

$$\varepsilon = (\sigma - 1) - \frac{\partial \ln \Lambda}{\partial \ln \mu^F}\frac{\partial \ln \mu^F}{\partial \ln \tau_x},$$

where the first and second terms are the intensive and extensive margins of trade, respectively. This shows that relative to a standard model, the search opportunity can amplify the effect of iceberg trade cost on trade flows by improving the matching probability of firms μ^F. However, so long as our interest is in the *partial* elasticity, the iceberg trade cost has no direct effect on the extensive margin; here it has an indirect effect on that margin through the price index by lowering the profit of unmatched firms r/σ which in turn improves μ^F. Therefore, the trade elasticity remains the same as that in Krugman (1980).

[13]Strictly speaking, we have to assume the low level of trade costs so that all firms export in comparison with Krugman (1980). As shown in Appendix A.4, however, this statement holds true even in such equilibrium.

The domestic expenditure share is $\lambda \equiv R_d/R = [(N^F - n)r + nr_d(\varphi)]/R$. In contrast to the trade elasticity, this share is directly affected by search frictions through the number of agents (as f_d enters n and N^F from f). Further, that share has a close relationship with the expenditure share on goods produced by unmatched firms, $s = (N^F - n)r/R$, as in Section 3.4. In particular, if search frictions are prohibitively large so that n is zero and N^F remains the same after trade, these two expenditure shares coincide with one another, i.e., $s = \lambda$. Hence, if firms are unable to search for suppliers, the welfare changes in (15) reduce to those in Arkolakis et al. (2012). If search frictions are not so large, however, the expenditure shares are different; importantly, (15) shows that the share s plays a critical role in evaluating the gains from trade. From (9) and (16), this share is expressed as

$$s = \frac{\delta}{\delta + (1 + \tau_x^{1-\sigma})\varphi^{\sigma-1}\mu^F},$$

which decreases in X-integration. Together with $s < \lambda$, this captures the amplified gains from trade. Intuitively, consumers can access relatively more varieties produced by more efficient matched firms in X-integration due to reallocations. The mechanism has some similarity to that in Melitz (2003) but our novelty is that reallocations are associated with an increase in the ratio of unmatched agents seeking potential partners θ. This additionally increases the gains from trade via an improvement in the matching probability of firms μ^F.

We can show our new mechanism from a different angle. From (13), the welfare ratio between X-integration and autarky is expressed in terms of expected fixed costs: $W/W_a = (f/f_a)^{-1/(\sigma-1)}$. Using $f = r/\sigma,\ f_a = r_a/\sigma$,

$$\frac{W}{W_a} = \left[\left(\frac{N_a^F + (\varphi^{\sigma-1} - 1)n_a}{N^F + (\varphi^{\sigma-1} - 1)n}\right)\lambda\right]^{-\frac{1}{\sigma-1}}. \tag{20}$$

If search frictions are prohibitively large, the number of matches is zero ($n = n_a = 0$) and the number of firms is not affected by trade ($N^F = N_a^F$). Then, (20) reduces to $W/W_a = \lambda^{-1/(\sigma-1)}$ and the gains from trade can be captured only by the trade elasticity and domestic expenditure share (Arkolakis et al., 2012). If search frictions are not so large, however, the ratio of matched firms rises by X-integration ($n/N^F > n_a/N_a^F$). Then, (20) shows that the gains from trade cannot be calculated only by the two sufficient statistics. The result is along the lines of recent research showing that the gains from trade are greater in Melitz (2003) than in Krugman (1980) due to *endogenous firm selection* that is absent in the latter (Melitz and Redding, 2015). This argument applies to our paper by noticing that firm heterogeneity is driven by matching status. Hence, the gains from trade can be greater in our model than in Krugman (1980) due to *endogenous firm matches* that are absent in the latter.

Proposition 2: *X-integration increases welfare in both countries by improving the matching probability of firms associated with resource reallocations from unmatched firms to matched firms.*

4.5 Numerical Solutions

To get a sense of the magnitude involved in X-integration, we parameterize the model and solve it numerically. In both autarky and X-integration, we compare equilibrium with search as in this model to that without search as in Krugman (1980). The following conditions are imposed to render this comparison meaningful and sharp. First, we contrast the outcomes in these two cases by assuming that exporting firms incur both variable and fixed trade costs in X-integration, as there is no fixed export cost in Krugman (1980). Second, we restrict attention to X-integration equilibrium where only matched firms export. As stressed above, however, our results are similar in X-integration equilibrium where both unmatched and matched firms export.

Table 1: Quantitative impact of X-integration

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
	r/σ	$r_d(\varphi)/\sigma$	$r_x(\varphi)/\sigma$	θ	μ^F	μ^S	N^F	N^S	n	s	λ	W
Autarky	1.62	3.16	0	0.37	0.27	0.73	82	77	75	0.04	1	4.02
	2	0	0	0	0	0	125	0	0	0	1	3.75
X-integration	1.50	2.94	0.50	0.49	0.33	0.67	75	72	70	0.03	0.85	4.12
	2.27	0	0	0	0	0	109	0	0	0	0.85	3.78

Notes: The first row corresponds to values in our model, while the second row corresponds to values in the Krugman Benchmark in both autarky and X-integration.

Following Grossman and Helpman (2002), we assume that the matching function is given by $m(u^F, u^S) = u^F u^S/(u^F + u^S)$. With this specification, the rate at which unmatched firms meet unmatched suppliers is equal to $\mu^F = \theta/(1+\theta)$, and the rate at which unmatched suppliers meet unmatched firms is equal to $\mu^S = 1/(1+\theta)$. Using these rates, the expected share of matched firms and suppliers is expressed as $\phi^F = \theta/(1+\theta)(2\delta+1)$ and $\phi^S = 1/(1+\theta)(2\delta+1)$, respectively. Substituting these shares into the free entry condition in autarky (11) and that in X-integration (18), we can explicitly solve for the ratio of unmatched agents in both cases. For example, combining the two equalities in (11) and rearranging, the closed-form solution of θ in autarky is

$$\theta = \frac{(\varphi^{\sigma-1} - 1)f_e^F - (2\delta+1)f_e^S - f_d}{(\varphi^{\sigma-1} + 2\delta)f_e^S}.$$

The variable profit of unmatched firms is subsequently obtained by noting the equilibrium relationship between θ and r/σ in (14), which holds in both autarky and X-integration.

We choose standard values for the monopolistic competition model's parameters employed in the literature. We set the elasticity of substitution between varieties $\sigma = 4$. As the unit labor requirements are normalized to one ($a^F = a^S = 1$), we have the productivity level $\varphi = 1.25$. Given $\varphi = 1$ for unmatched firms, this shows that matched firms are 25 percent more productive than unmatched firms which seems to be reasonable in magnitude in view of the empirical literature.[14] Market size has no effect on key equilibrium variables of the model under constant returns to scale in matching (including the matching function above) and we simply choose $L = 1000$. In addition, following Bernard et al. (2007b), we set the probability of a bad shock $\delta = 0.025$, the fixed overhead cost $f_d = 0.1$, and the fixed entry cost for firms $f_e^F = 2$. We set the fixed entry cost for suppliers $f_e^S = 1$ so that the ratio of unmatched agents is positive ($\theta > 0$). Finally, in the equilibrium where only matched firms export, the level of trade costs must be intermediate so as to satisfy (19). For this reason, we set $f_x = 0.275, \tau_x = 1.8$, but the numerical results would not be sensitive to these values within the parameter range of (19).

Table 1 summarizes quantitative comparison between autarky and X-integration computed under the above specifications and parameter values. In autarky and X-integration, the values in the first and second rows are those with search and without search respectively where the latter is labelled as the Krugman Benchmark below. Though we examine the impact of X-integration by comparing equilibria between autarky and this integration, the impact is similar when comparing high and low levels of trade costs in X-integration satisfying (19).

Columns I–III show the variable profit of unmatched firms r/σ and that of matched firms $r_d(\varphi)/\sigma, r_x(\varphi)/\sigma$. In the Krugman Benchmark, the variable profit equals the fixed costs f_e^F in autarky and $f_e^F + f_x$ in X-integration. The profit level is higher in X-integration than in autarky since the introduction of fixed export costs raises

[14] As reported in Bernard et al. (2007a), exporters have 108 percent larger log shipments than non-exporters in US manufacturing (after including industry fixed effects). Plugging (1) and (16) into $\ln([r_d(\varphi) + r_x(\varphi)]/r) = 1.08$ yields $\varphi = 1.36$.

average firm output and hence average firm profit as well. In our model, in contrast, the variable profit equals the expected fixed cost given as (11) in autarky and (18) in X-integration. The profit level of unmatched firms is smaller in X-integration than in autarky due to shipment by (more efficient) exporters to the domestic market. Though matched firms earn smaller profit from the domestic market, their total profit is larger in X-integration than in autarky. This indicates that resources are reallocated from unmatched firms to matched firms.

Columns IV–VI show the ratio of unmatched agents θ and the associated probability of matches μ^F, μ^S. In the Krugman Benchmark, the ratio is zero and hence the probability of matches is also zero. In our model, in contrast, the ratio is higher in X-integration than in autarky as this integration raises the *ex ante* expected profit of suppliers relative to firms, which leads to the *ex post* large competitive pressure on suppliers relative to firms, increasing the ratio of unmatched suppliers to unmatched firms. Due to trade-induced industry restructuring, X-integration improves the probability of firms' matches by roughly 6 percent but it worsens the probability of suppliers' matches by the same amount.

Columns VII–IX then show the number of agents N^F, N^S as well as the number of matched firms n. In the Krugman Benchmark, the number of firms falls in X-integration as labor is used to fixed export costs f_x; but the total number of varieties (domestic plus foreign) rises to $(1+\tau_x^{1-\sigma})N_F$. For our parameter values, this number is 128, which is greater than 125 in autarky. In our model, however, the number of firms is smaller than that in the Krugman Benchmark in both autarky and X-integration because labor is used to develop non-core inputs that are not directly related to the number of varieties. At the same time, such development allows matched firms to improve their production efficiency relative to unmatched firms by exploiting a love-of-variety effect. Hence, the search opportunity creates a tradeoff between decreased product variety and improved production efficiency of matched firms. The total number of varieties that takes into account the tradeoff is $N^F - n + (1+\tau_x^{1-\sigma})\varphi^{\sigma-1}n$ which is inversely proportional to the price index. This number is 165 in our exercise, which is greater than 154 in autarky of our model and 128 in X-integration of the Krugman Benchmark.

Finally, Columns X–XII show the expenditure share on unmatched firms' goods s, the domestic expenditure share λ and welfare W. In the Krugman Benchmark, the gains from trade are due to increased product variety. For our parameter values, the welfare ratio between X-integration and autarky is 1.009 and thus the gains from trade are 0.9 percent, a comparable magnitude reported in Arkolakis et al. (2012). In our model, X-integration reduces unmatched firms' expenditure share by improving the matching probability of firms, which additionally increases the gains from trade. The welfare ratio between the two regimes given in (20) is 1.024 and hence the gains from trade are 2.4 percent.[15] Examining reductions in variable export costs from $\tau_x = 1.8$ in X-integration, we also find that the gains are roughly 1.5 percent higher in our model than those in the Krugman Benchmark. Note that not only is the trade elasticity but also the domestic expenditure share is (almost) the same between the two models. This confirms our theoretical result that endogenous firm matches have first-order significance for the gains from trade beyond the two sufficient statistics.

We conclude this exercise by briefly mentioning the case where both matched and unmatched firms export. If variable trade costs decrease from $\tau_x = 1.8$ to $\tau_x = 1.2$ (keeping all of the other parameter values the same), the condition in (19) is no longer satisfied and unmatched firms start exporting in X-integration. In that case, the gains from trade increase to 11.5 percent in the Krugman Benchmark and 13.1 percent in our model. While this setting is closer to Krugman (1980) in that all firms export, the baseline setting is preferable as selection into the export market is empirically ubiquitous. In addition, the gains from trade estimated there seem unrealistic, so long as we use parameter values based on the estimates obtained from US data.

[15] In the Krugman Benchmark, it can be easily shown that the welfare ratio is expressed as $W/W_a = \left[(f_e^F + f_x)\lambda/f_e^F\right]^{-1/(\sigma-1)}$. When there is no fixed export cost ($f_x = 0$), this ratio naturally reduces to (20) corresponding to Krugman (1980).

5 Integration of Matching Markets

5.1 Assumptions

We next study economic integration of matching markets referred to as *M-integration* in the paper, which allows firms to search for suppliers across borders (though they can search in only one country at any point in time), maintaining the assumption that final goods are traded within borders. In this integration, matched firms have an opportunity to source non-core inputs from matched suppliers both at home and abroad. As a consequence, non-core inputs are only tradable goods in M-integration.

In order to meaningfully contrast the implications of X- and M-integration, we impose the same assumptions as in Section 4, i.e., there is no difference in labor endowments and technology between two trading countries. The structure of trade costs is also similar to that in Section 4, in the sense that a firm wishing to import from a supplier matched abroad must incur an iceberg transport cost τ_m and a one-time fixed cost F_m (measured by units of labor) in this integration. The latter includes both an investment cost to start up the relationships like F_d and a communication cost to reach foreign suppliers. Due to additional resources included in F_m, agents are matched with foreign partners must incur a higher fixed cost, $F_m > F_d$ (Antràs and Helpman, 2004).[16] While the matching function is assumed to be the same between domestic and cross-border matches, the number of matches per unit of time in each country becomes half (as matches occur both within and across borders with identical search technology). Hence, the rate at which unmatched firms and suppliers meet unmatched partners in one country reduces to $\mu^F/2$ and $\mu^S/2 = \mu^F/2\theta$. In Figure 1, the number $\mu^F u^F/2$ of firms enters the matched pool with being matched with domestic suppliers and the same number of firms enters with being matched with foreign suppliers. Although we define $\theta \equiv (N^S - n)/(N^F - n)$ as before, we denote by $n \equiv N^F - u^F = N^S - u^S$ the *total* number of matched agents which includes both types of matches. Finally, when agents find partners in one country, they stop searching for partners in another country by implicitly assuming a large switching cost (Antràs and Costinot, 2011). Under these assumptions, aggregate revenue still equals market size in equilibrium, where the common wage rate is normalized to one (see Appendix A.2).

When M-integration takes place, there are two possible cases associated with different levels of trade costs: (i) no firm imports; and (ii) firms matched with foreign suppliers import. Below we focus on a more interesting case where cross-border matches are profitable. Clearly, such equilibrium occurs with the low level of trade costs, but M-integration entails not only trade costs but also search frictions that agents face to seek foreign partners. Autarky can be regarded as a special case where search frictions abroad are prohibitively large (i.e., $F_m = \infty$) so that no agents are able to search for foreign partners. M-integration then allows firms to have an additional opportunity to source inputs from another country. This differs from X-integration which allows firms to have an additional opportunity to sell final goods to another country, though the two forms of economic integration help to internationalize trading opportunities. With this difference in mind, we show that M-integration causes welfare losses for both countries by worsening the matching probability of firms, which stands in sharp contrast to the impact of X-integration. (As in X-integration, this contrasting welfare outcome occurs in M-integration only if firms incur some trade costs.) Assuming identical search technology between domestic and cross-border matches, we first analyze a simple case where the matching probability is the same within and across borders. We later analyze an extended case where the matching probability is smaller across borders than within borders. In both cases, equilibrium conditions are similar to those in autarky and detailed conditions are relegated to Appendix A.5.

[16] From this reason, if there is no fixed cost associated with M-integration, firms matched with domestic and foreign suppliers incur the same search cost in this integration.

5.2 Equilibrium Characterization

We mainly characterize and solve for M-integration equilibrium at home as in X-integration. In M-integration, however, there are six types of agents at home at any point in time: (i) unmatched firms; (ii) firms matched with home suppliers; (iii) firms matched with foreign suppliers; (iv) unmatched suppliers; (v) suppliers matched with home firms; and (vi) suppliers matched with foreign firms. It is clear that the behavior of agents classified as (i), (ii), (iv) and (v) is similar to that in autarky. Regarding the behavior of agents matched with foreign partners classified as (iii) and (vi), together with the fact that firms matched with foreign suppliers source x^S by incurring the transport cost τ_m, the CES production function implies the unit cost $c_m = 1/\varphi_m$ where

$$\varphi_m \equiv \left(1 + \left(\frac{1}{\tau_m}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}.$$

Recall that φ measures the unit cost differences between matched and unmatched firms. Noticing $\varphi > \varphi_m > 1$, cross-border matched firms are moderately efficient among three types of firms. The reason is straightforward: firms with cross-border matches are less efficient than those with domestic matches due to the transport cost τ_m; however, they are more efficient than unmatched firms due to the love-of-variety effect from input expansion. The first-order conditions for profit maximization yields the following equilibrium price for these firms:

$$p(\varphi_m) = \frac{\sigma}{\sigma-1}\frac{1}{\varphi_m},$$

Let $r(\varphi_m)$ denote the equilibrium revenue of cross-border matched firms. Since the productivity parameter only differs in equilibrium variables, the ratio of equilibrium revenue must satisfy

$$\frac{r(\varphi_m)}{r} = \varphi_m^{\sigma-1}, \tag{21}$$

which is very similar to (1).

We next specify the equilibrium conditions of M-integration. Since there are six types of agents at any point in time, we need to introduce six types of value functions. Below we focus on the Bellman equations for firms. Let $V^F(\varphi_m)$ denote the value function of cross-border matched firms. Then the Bellman equations for firms are

$$\begin{aligned}
\gamma V^F(\varphi) &= \frac{r^F(\varphi)}{\sigma} - \delta V^F(\varphi) + \dot{V}^F(\varphi),\\
\gamma V^F(\varphi_m) &= \frac{r^F(\varphi_m)}{\sigma} - \delta V^F(\varphi_m) + \dot{V}^F(\varphi_m),\\
\gamma V^F &= \frac{r}{\sigma} + \frac{\mu^F}{2}\Big(V^F(\varphi) - V^F - F_d\Big) + \frac{\mu^F}{2}\Big(V^F(\varphi_m) - V^F - F_m\Big) - \delta V^F + \dot{V}^F.
\end{aligned}$$

These Bellman equations have the following differences from those in autarky. The second equation shows that cross-border matched firms obtain a gain $r^F(\varphi_m)/\sigma$ which includes the transport cost τ_m. On the other hand, the last equation shows that unmatched firms become matched with suppliers at home or abroad at the same rate $\mu^F/2$ (due to identical search technology between two types of matches) at which point they obtain a gain $V^F(\varphi) - V^F - F_d$ for domestic matches or $V^F(\varphi_m) - V^F - F_m$ for cross-border matches. The Bellman equations also imply that, in the steady-state equilibrium with no discount factor, the value functions of either type of matched firms equal the present value of profit flows, $V^F(\varphi) = r^F(\varphi)/\delta\sigma$, $V^F(\varphi_m) = r^F(\varphi_m)/\delta\sigma$, just like (6).

In contrast, the value function for unmatched firms is now given by

$$V^F = \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{2(\delta+\mu^F)}\right)\left(\frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \left(\frac{\mu^F}{2(\delta+\mu^F)}\right)\left(\frac{r^F(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right),$$

where the probability of matches is divided by 2 in the second and third terms. This is because M-integration allows foreign firms to penetrate the domestic matching market, which lowers the probability of matches by half (second term). At the same time, this integration allows home firms to penetrate the foreign matching market, which lowers the probability of matches by half (third term).[17]

Agents matched with home or foreign partners determine their profit sharing by symmetric Nash bargaining. Since agents search for potential partners both within and across borders in M-integration, such expansion of search opportunities can alter effective bargaining power β. We find, however, that the bargaining power is the same as (8) in autarky. The reason is closely related to the bargaining power being endogenously determined by the number of agents in the model. On the one hand, M-integration increases the number of suppliers available for home firms by allowing foreign suppliers to penetrate the domestic matching market, which enhances the bargaining power. On the other hand, M-integration increases the number of firms seeking home suppliers by allowing foreign firms to penetrate the domestic matching market, which reduces the bargaining power. When penetration of each matching market occurs at the same rate between countries, these opposing forces exactly offset one another so that the effective bargaining power is the same as (8). Similarly, the steady-state number of agents is the same as (9) in M-integration, with the differences being that n is the total number of matches and the ratio of agents matched with either home or foreign partners reduces by half in each matching market, as shown in the value function of unmatched firms.

Finally, using the steady-state relationships, the free entry condition in (5) can be written as follows:

$$\begin{aligned} &\frac{r}{\sigma} + \frac{n}{2N^F}\beta\left(\frac{r(\varphi)}{\sigma} + \frac{r(\varphi_m)}{\sigma} - \frac{2r}{\sigma} - f_d - f_m\right) - f_e^F = 0, \\ &\frac{n}{2N^S}(1-\beta)\left(\frac{r(\varphi)}{\sigma} + \frac{r(\varphi_m)}{\sigma} - \frac{2r}{\sigma} - f_d - f_m\right) - f_e^S = 0, \end{aligned} \tag{22}$$

where $f_m \equiv \delta F_m$ denotes the amortized per-period portion of the one-time fixed import cost incurred at every point in time. Further, using (21) for (22), the variable profit of unmatched firms satisfies the following equalities:

$$\begin{aligned} \frac{r}{\sigma} &= \frac{f_e^F + (f_d + f_m)\frac{\phi^F}{2}}{1 + \left(\varphi^{\sigma-1} + \varphi_m^{\sigma-1} - 2\right)\frac{\phi^F}{2}}, \\ \frac{r}{\sigma} &= \frac{f_e^S + (f_d + f_m)\frac{\phi^S}{2}}{\left(\varphi^{\sigma-1} + \varphi_m^{\sigma-1} - 2\right)\frac{\phi^S}{2}}, \end{aligned} \tag{23}$$

where ϕ^F, ϕ^S are the same as before. (23) shows that the variable profit equals the expected fixed cost of agents, which includes the transport cost τ_m (via φ_m) as well as the fixed import cost f_m in M-integration. These two equalities simultaneously determine the ratio of unmatched agents θ and the profit of unmatched firms r/σ, where FF and SS curves from (23) can be illustrated in the $(\theta, r/\sigma)$ space as in Figure 2. We are now ready to address the welfare consequence of M-integration by exploring how this integration affects the two curves and hence two key equilibrium variables, θ and r/σ.

[17] We assume that cross-border matched firms have an enough incentive to sink the search cost abroad F_m in the dynamics, i.e., $r^F(\varphi_m)/\delta\sigma - r/\delta\sigma - F_m > 0$.

5.3 Losses from Trade in M-integration

To examine the impact of M-integration, we consider how the free entry condition in (22) is affected by the level of trade costs associated with this integration. In equilibrium where matched firms source inputs from abroad, agents matched with foreign partners earn higher profit in M-integration than in autarky ($r(\varphi_m)/\sigma - f_m > r/\sigma$). Using (21), this condition—a counterpart to (19) in X-integration—is expressed as

$$\frac{r}{\sigma} > \tau_m^{\sigma-1} f_m. \tag{24}$$

If trade costs are so large that (24) is violated, firms matched with foreign suppliers would immediately dissolve their partnerships. Comparing (10) and (22) under (24) then reveals that the economic rent of matched agents is greater in M-integration than in autarky. This raises the expected profit and induces further entry of agents, which leads to downward shifts in the FF and SS curves.

In M-integration, however, there is another force for the expected profit: the expected share becomes half, because of penetration by foreign agents of the domestic matching market. This reduces the expected profit and deters further entry of agents, which leads to upward shifts in the two curves. In equilibrium firms and suppliers strike a balance between these forces, one related to trade costs and another related to search frictions abroad. If the first dominates the second, the impact of M-integration is similar to that of X-integration; otherwise it is opposite. The question is which forces dominate.

To answer this, it suffices to consider the condition under which the expected profit in M-integration increases relative to autarky. If this is the case, a rise in the economic rent is greater than a fall in the expected share, shifting two curves downwards. From inspection of (10) and (22), the condition is equivalent with the following: the economic rent from cross-border matches is greater than the corresponding rent from domestic matches ($r(\varphi_m)/\sigma - r/\sigma - f_m > r(\varphi)/\sigma - r/\sigma - f_d$). Using (1) and (21), this is expressed as

$$\frac{\left(\varphi_m^{\sigma-1} - \varphi^{\sigma-1}\right) r}{\sigma} > f_m - f_d. \tag{25}$$

However, the inequality in (25) never holds in a symmetric-country setting, because firms matched with foreign suppliers are less efficient than those matched with home suppliers ($\varphi_m < \varphi$) and the search cost is greater for abroad ($f_m > f_d$). This implies that M-integration has an impact opposite to X-integration. In fact, comparing (11) and (23) under the opposite inequality in (25), the two curves in M-integration are located above those in autarky for a given ratio θ and thus the variable profit of unmatched firms rises in M-integration ($r/\sigma > r_a/\sigma$). Further, the equilibrium relationship between θ and r/σ in (14) continues to hold in M-integration from (22) and thus the ratio of unmatched agents falls in M-integration ($\theta < \theta_a$). From changes in two equilibrium variables, we find that M-integration increases the market share of unmatched firms (because $R = L$ in this integration) and worsens the matching probability of firms.

Intuition behind the result stems from resource reallocations in M-integration. With our search technology, M-integration generates exactly the same number of domestic and cross-border matches. As matched firms are split into two groups with productivity levels being $\varphi > \varphi_m$, resources are reallocated from the most efficient firms to the moderately efficient firms. As this split increases the price index, resources are further reallocated to the least efficient unmatched firms. In this way, M-integration hinders the resource-reallocation process of firms, which worsens the matching probability of firms by inducing firms to enter more than suppliers in response to the increased price index. The welfare channel is understood by considering welfare changes in the presence of search frictions. Since (15) holds for welfare changes occurring in M-integration, the expenditure share on goods produced by unmatched firms remains one of the sufficient statistics for welfare. That share is $s = (N^F - n)r/R$ as before, but aggregate expenditure is $R = (N^F - n)r + nr(\varphi)/2 + nr(\varphi_m)/2$ here. Using (1), (9) and (21),

$$s = \frac{\delta}{\delta + \left(\varphi^{\sigma-1} + \varphi_m^{\sigma-1}\right) \frac{\mu^F}{2}}.$$

From $\varphi > \varphi_m$, it follows that the expenditure share is higher in M-integration than in autarky. This is clearly seen in terms of the relative expenditure share $s/(1-s) = \delta/[(\varphi^{\sigma-1} + \varphi_m^{\sigma-1})\mu^F/2]$. As matched firms' revenues are $r(\varphi) + r_m(\varphi)$, simple average of revenues between two types of matched firms is given by $[r(\varphi) + r(\varphi_m)]/2$. Then, the relative share is decomposed into the intensive margin ratio $r/[r(\varphi) + r(\varphi_m)/2] = 2/\left(\varphi^{\sigma-1} + \varphi_m^{\sigma-1}\right)$ and the extensive margin ratio $(N^F - n)/n = \delta/\mu^F$, where both margins increase relative to those in autarky. The increase reflects that the least efficient unmatched firms expand relative to efficient matched firms through these margins in M-integration, which leaves consumers worse off.

Finally, it is important to stress that we have derived the impact of M-integration on welfare by comparing the equilibrium outcomes between autarky and this integration. This means that the welfare losses occur when each country experiences the transition from autarky to M-integration. However, the same does not occur when each country experiences trade liberalization in M-integration. Simple inspection of (23) shows that a decrease in trade costs (either variable τ_m or fixed f_m) shifts two curves downwards and hence trade liberalization is always welfare-enhancing. The gains from trade liberalization arise in M-integration through which firms matched with foreign suppliers source non-core inputs at low cost, reallocating resources in a desirable direction.

Proposition 3: *M-integration decreases welfare in both countries by worsening the matching probability of firms associated with resource reallocations from matched firms to unmatched firms.*

5.4 Extensions

We have shown that M-integration causes welfare losses. The result is obtained, however, by assuming identical search technology so that the contact rates are the same between domestic and cross-border matches. To check robustness of our result in M-integration, this section considers a more natural case where cross-border matches occur at a lower rate than domestic matches.

Let $m_d(u^F, u^S)$ and $m_m(u^F, u^S)$ denote the matching function of domestic matches and cross-border matches, respectively. To make the analysis tractable, we assume that these functions satisfy $m_m(u^F, u^S) = \kappa m_d(u^F, u^S)$ where $\kappa(< 1)$ is the difficulty of cross-border matches relative to domestic matches. Let n_d and n_m denote the number of matched agents within and across borders, respectively. Then the search technology means $n_m = \kappa n_d$. Keeping all other assumptions of our baseline model, we find that the free entry condition corresponding to (22) is given as follows (see Appendix A.5):

$$\frac{r}{\sigma} + \frac{n}{(1+\kappa)N^F}\beta\left(\frac{r(\varphi)}{\sigma} + \kappa\frac{r(\varphi_m)}{\sigma} - \frac{(1+\kappa)r}{\sigma} - f_d - \kappa f_m\right) - f_e^F = 0,$$
$$\frac{n}{(1+\kappa)N^S}(1-\beta)\left(\frac{r(\varphi)}{\sigma} + \kappa\frac{r(\varphi_m)}{\sigma} - \frac{(1+\kappa)r}{\sigma} - f_d - \kappa f_m\right) - f_e^S = 0,$$

where $n = n_d + n_m$ is the total number of matched agents. Several observations stand out from this expression.

First, not surprisingly, when the probability of cross-border matches is zero ($\kappa = 0$), the free entry condition reduces to (10) in autarky. On the other hand, when the probability of cross-border matches is identical with that of domestic matches ($\kappa = 1$), the free entry condition reduces to (22) in M-integration of the baseline case.

Table 2: Quantitative impact of M-integration

	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
	r/σ	$r(\varphi)/\sigma$	$r(\varphi_m)/\sigma$	θ	μ^F	μ^S	N^F	N^S	n	s	λ	W
Autarky	1.62	3.16	0	0.37	0.27	0.73	82	77	75	0.04	1	4.02
M-integration	1.76	3.44	2.75	0.23	0.19	0.81	81	74	71	0.06	1	3.90

Second, the impact of M-integration is the same as that in the baseline case, with additional comparative statics results with respect to κ: the easier are cross-border matches (the larger κ), the greater the *upward* shifts in two curves, and hence the greater the welfare losses from trade. The impact is opposite to that of trade liberalization (the smaller τ_m, f_m) as above. In terms of search frictions, this difference is captured by the search cost κf_m in the free entry condition. Finally, perhaps most interestingly, M-integration causes the welfare losses from trade even when search is more difficult across borders than within borders. This comes from noting that (25)—the condition under which the expected profit increases in M-integration relative to autarky—continues to hold in this extended case.

5.5 Numerical Solutions

To appreciate the impact of M-integration, we solve the model numerically using the same specifications and parameter values as those in X-integration. In M-integration where firms matched with foreign suppliers import, however, trade costs must be low enough to satisfy (24). In this exercise, hence, we set $f_m = 0.125,\ \tau_m = 1.2$.[18] The low level of import costs (relative to export costs) can be justified as follows. Antràs et al. (2017) find that the estimates of fixed import costs range from 10,000 to 56,000 USD, which are much smaller than those of fixed export costs, about 400,000 USD (Das et al., 2007). Regarding variable import costs, on the other hand, Grossman et al. (2024) show that tariffs imposed on final goods are more than four times as high as those on intermediate inputs in 2010–2017 in the United States. Note that when $\tau_m = 1.2$, we get $\varphi_m = 1.16 (< \varphi = 1.25)$ and cross-border matched firms are 16 percent more productive than unmatched firms. In addition, we set κ to adjust the difficulty of cross-border matches. Towards that end, we make use of data on the average fraction of firms that import among firms that also export in US manufacturing (0.4 as reported in Bernard et al. (2007a)). This fraction is $n_m/n_d = \kappa$ from Section 5.4 and hence $\kappa = 0.4$.

Table 2 summarizes quantitative comparison between autarky and M-integration computed in this setting. Clearly, the values in autarky are the same as those in Table 1 but the values in M-integration are different from those in X-integration. Columns I–III show that the profit level of any types of firms is higher in M-integration than in autarky. The increase in firm profit directly reflects that M-integration increases the price index relative to autarky and reallocates resources from efficient firms to less efficient firms in the industry. Columns IV–VI next show that the ratio of unmatched agents falls in M-integration because the increased price index raises the *ex post* profitability and induces a new entry of firms relative to suppliers. As a result, firms find it hard to search for suppliers and the probability of firms' matches falls but the probability of suppliers' matches rises. Columns VII–X show that the number of firms and suppliers falls, but the number of firms relatively rises to that of suppliers in M-integration reflecting the difference in entry patterns between different types of agents. While the number of matches also falls, it is split into domestic matches ($n_d = 51$) and cross-border matches ($n_m = 20$) with above search technology.

[18] Recall that we set $f_x = 0.275,\ \tau_x = 1.8$ in numerically solving for X-integration equilibrium.

Finally, Columns X–XII show that: (*i*) the expenditure share spent on unmatched firms' goods rises as the matching probability gets worse for firms; (*ii*) the domestic expenditure share is unity as the final-good markets are segregated in M-integration; and (*iii*) welfare falls in M-integration as this integration raises the price index. The welfare ratio between M-integration and autarky is 0.973 and hence welfare losses are 2.7 percent, a sizable magnitude.[19] This does not mean, however, that the welfare impact of M-integration is directly comparable to that of X-integration because the level of trade costs is different between these two forms of integration. In fact, if variable trade costs decrease from $\tau_x = 1.8$ to $\tau_x = 1.2$ in X-integration, the welfare gains increase from 2.4 percent to 13.1 percent as shown in the end of Section 4.4. Thus, while M-integration causes the welfare losses, they are quantitatively smaller than the welfare gains in X-integration conditional on the level of trade costs.

6 Country Asymmetry

We have focused on a symmetric-country setting to show new welfare implications from economic integration in the presence of search frictions most sharply. However our model is flexible enough to embed country asymmetry into the baseline model. Suppose that two countries have different unit labor requirements. To avoid a taxonomy of cases, assume $a^S/a^F < a^{S^*}/a^{F^*}$ so that the home country has a comparative advantage in producing a non-core input. Further, to make the analysis as simple as possible, keep all other structural parameters symmetric. In this setting, we show that M-integration can generate welfare gains for the home country.

Consider the autarky equilibrium. From $a^S/a^F < a^{S^*}/a^{F^*}$, productivity of matched firms is greater in the home country ($\varphi > \varphi^*$) because of the love-of-variety effect exploited more efficiently at home. This difference has several impacts on the autarky equilibrium. First, the wage rate is no longer equal to one between countries. As the home country is more productive, the wage rate is higher there ($w > w^*$).[20] Second, the two equilibrium variables of the model are no longer the same between countries. As fixed costs are measured in units of labor, (11) implies that the variable profit relative to the wage rate is smaller in the home country ($r/w\sigma < r^*/w^*\sigma$); in turn, (14) implies that the ratio of unmatched agents is higher there ($\theta > \theta^*$). Finally, welfare is higher in the home country ($W > W^*$) in light of (13). Hence, we have the following observation: *when* $a^S/a^F < a^{S^*}/a^{F^*}$, *the home country enjoys higher welfare than the foreign country in autarky because resources are relatively more allocated to matched firms and varieties are relatively more produced by these firms.*

Consider next the M-integration equilibrium. In this case, we must take into account productivity of firms matched abroad, which crucially depends not only on firms' matching status but also on suppliers' location. For example, productivity of home firms matched with foreign suppliers is expressed as

$$\varphi_m = \left(1 + \left(\frac{wa^F}{\tau_m w^* a^{S^*}}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}.$$

In contrast to the baseline model, cross-border matched firms are not always moderately efficient. In particular, $\varphi_m > \varphi$ if and only if $wa^S > \tau_m w^* a^{S^*}$ which is *more* likely, the larger is the relative wage w/w^*. This is because cross-border matched firms now have an advantage to use cheaper labor, which can dominate a disadvantage to pay transport costs. By the same reason, however, $\varphi_m^* > \varphi^*$ if and only if $w^* a^{S^*} > \tau_m wa^S$ which is *less* likely, the larger is w/w^*. Hence, it is possible to have the following ranking of firm productivity: $\varphi_m > \varphi > \varphi^* > \varphi_m^*$.

[19] The welfare ratio in (20) is modified in M-integration where the denominator is replaced by $N^F + (\varphi^{\sigma-1} - 1)n_d + (\varphi_m^{\sigma-1} - 1)n_m$. As $\lambda = 1$ in this integration, the welfare losses stem solely from industry restructuring.

[20] This follows from observing that the labor market clearing condition requires $R = wL$, $R^* = w^* L^*$ where $R > R^*$ from $\varphi > \varphi^*$ and $L = L^*$ by assumption.

Recall that, to know whether welfare rises as a result of M-integration, we just need to see the condition in (25), which holds only if $\varphi_m > \varphi$ and $\varphi_m^* > \varphi^*$. Noting that the condition can hold for the home country but cannot for the foreign country, we have the following observation: *when $a^S/a^F < a^{S^*}/a^{F^*}$, M-integration can generate welfare gains for the home country, whereas it may cause welfare losses for the foreign country.* The reason for welfare gains at home is very simple. In our baseline model, M-integration causes welfare losses by reallocating resources to moderately efficient matched firms. In the asymmetric-country setting, such misallocations can be fixed because cross-border matched firms are most efficient in the home country. Even in this setting, however, the possibility of welfare losses remains in the foreign country.

One of the broad welfare implications from this extension is that a less advanced country (with lower wage and productivity) is more likely to suffer from integration of matching markets. Thus policymakers need to pay closer attention to offshoring between developed and less developed countries.

7 Conclusion

This paper has described and analyzed the effect of search frictions on the characterization of aggregate welfare. The importance of search for firm performance and productivity has been documented in the era of globalization where drastic reductions in search frictions coupled with gradual reductions in trade frictions enable firms to profitably search for suppliers across the globe. Indeed, empirical work on search, networks and intermediation in international trade has extensively corroborated this importance using micro-level data on buyer–seller linkages from several countries.

We show that the introduction of search frictions into standard workhorse models of trade offers non-standard welfare results. In particular, depending upon whether globalization reduces trade or search frictions that firms must face, it generates the contrasting welfare implications by affecting industry structure in which firms operate. On the one hand, when globalization makes it easier for firms to ship varieties to another market, the gains from trade are amplified relative to those without search opportunities through a new mechanism arising from search: costly trade affects industry structure in such a way that the equilibrium ratio of suppliers to firms increases. This industry restructuring improves the matching probability of firms (and enhances the bargaining position of firms over suppliers in relationships), while simultaneously generating now-familiar resource reallocations from inefficient unmatched firms to efficient matched firms. On the other hand, when globalization makes it easier for firms to seek suppliers from another market, costly trade leads to the opposite impacts on industry structure, hindering the resource-reallocation process of firms and worsening the matching probability of firms. As a result, countries may suffer welfare losses from trade. It is also demonstrated that welfare changes triggered by above trade-induced industry restructuring are quantitatively substantial.

Although our analysis reveals that welfare losses due to integration of matching markets are relatively smaller than welfare gains due to integration of goods markets, it indicates that there is potential room to circumvent these welfare losses through trade policies. What kind of policy implications can we derive from our model? The implications are, of course, not that local governments should ban or restrict integration of matching markets. One of the reasons is that such integration is more likely to generate welfare gains for a more advanced country (though it comes at the expense of a less advanced country), in which case restrictions on this integration could result in aggregate losses in global welfare. The policy implications from our model are, instead, that integration of matching markets may as well be designed with integration of goods markets in order to soften welfare losses. In that sense, our model highlights a more critical role played by traditional trade liberalization in globalization where firms search for and match with suppliers.

References

Acemoglu D, Antràs P, Helpman E. 2007. Contracts and Technology Adoption. *American Economic Review*, 97, 916-943.

Allen T. 2014. Information Frictions in International Trade. *Econometrica*, 82, 2041-2083.

Antràs P. 2003. Firms, Contracts, and Trade Structure. *Quarterly Journal of Economics*, 118, 1375-1418.

Antràs P, Costinot A. 2011. Intermediated Trade. *Quarterly Journal of Economics*, 126, 1319-1374.

Antràs P, Fort TC, Tintelnot F. 2017. The Margins of Global Sourcing: Theory and Evidence from US Firms. *American Economic Review*, 107, 2514-2564.

Antràs P, Helpman E. 2004. Global Sourcing. *Journal of Political Economy*, 112, 552-580.

Ara T, Ghosh A. 2016. Tariffs, Vertical Specialization and Oligopoly. *European Economic Review*, 82, 1-23.

Arkolakis C, Costinot A, Rodriguez-Clare A. 2012. New Trade Models, Same Old Gains? *American Economic Review*, 102, 94-130.

Bernard AB, Dhyne E, Magerman G, Manova K, Moxnes A. The Origins of Firm Heterogeneity: A Production Network Approach. *Journal of Political Economy*, 130, 1765-1804.

Bernard AB, Jensen JB, Redding SJ, Schott SK. 2007a. Firms in International Trade. *Journal of Economic Perspectives*, 21, 105-130.

Bernard AB, Redding SJ, Schott PK. 2007b. Comparative Advantage and Heterogeneous Firms. *Review of Economic Studies*, 74, 31-66.

Chaney T. 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review*, 98, 1707-1721.

Chaney T. 2014. The Network Structure of International Trade. *American Economic Review*, 104, 3600-3634.

Costinot A. 2009. On the Origins of Comparative Advantage. *Journal of International Economics*, 77, 255-264.

Das S, Roberts MJ, Tybout JR. 2007. Market Entry Costs, Producer Heterogeneity, and Export Dynamics. *Econometrica*, 75, 837-873.

Eaton J, Kortum S, Kramarz F. 2022. Firm-to-Firm Trade: Imports, Exports, and the Labor Market. Mimeo.

Ethier WJ. 1982. National and International Returns to Scale in the Modern Theory of International Trade. *American Economic Review*, 72, 389-405.

Felbermayr G, Prat J, Schmerer HJ. 2011. Globalization and Labor Market Outcomes: Wage Bargaining, Search Frictions and Firm Heterogeneity. *Journal of Economic Theory*, 146, 39-73.

Grossman GM, Helpman, E. 2002. Integration Versus Outsourcing in Industry Equilibrium. *Quarterly Journal of Economics*, 117, 85-120.

Grossman GM, Helpman E. 2005. Outsourcing in a Global Economy. *Review of Economic Studies*, 72, 135-159.

Grossman GM, Helpman E, Redding SJ. 2024. When Tariffs Disrupt Global Supply Chains. *American Economic Review*, 114, 988-1029.

Halpern L, Koren M, Szeidl A. 2015. Imported Inputs and Productivity. *American Economic Review*, 105, 3660-3703.

Johnson RC, Noguera G. 2017. Accounting for Intermediates: Production Sharing and Trade in Value Added. *Review of Economics and Statistics*, 99, 896-911.

Krolikowski PM, McCallum AH. 2021. Goods-market Frictions and International Trade. *Journal of International Economics*, 129, 103411.

Krugman P. 1980. Scale Economies, Product Differentiation, and the Pattern of Trade. *American Economic Review*, 70, 950-959.

Levchenko AA. 2007. Institutional Quality and International Trade. *Review of Economic Studies*, 74, 791-819.

Melitz MJ. 2003. The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity. *Econometrica*, 71, 1695-1725.

Melitz MJ, Redding SJ. 2015. New Trade Models, New Welfare Implications. *American Economic Review*, 105, 1105-1146.

Nunn N. 2007. Relationship-Specificity, Incomplete Contracts, and the Pattern of Trade. *Quarterly Journal of Economics*, 122, 569-600.

Ornelas E, Turner JL. 2008. Trade Liberalization, Outsourcing, and the Hold-Up Problem. *Journal of International Economics*, 74, 225-241.

Ornelas E, Turner JL. 2012. Protection and International Sourcing. *Economic Journal*, 112, 26-63.

Pissarides CA. 2000. *Equilibrium Unemployment Theory*. MIT Press.

Sugita Y, Teshima K, Seira E. 2023. Assortative Matching of Exporters and Importers. *Review of Economics and Statistics*, 105, 1544-1561.

Wolinsky A. 1987. Matching, Search and Bargaining. *Journal of Economic Theory*, 42, 311-333.

A Online Appendix (Not for Publication)

A.1 Nash Bargaining Solution

We show that the solution to the Nash bargaining problem satisfies the constraint in (3) at any point in time. Clearly, the problem matched agents solve is equivalent for them to choose their revenues $r^F(\varphi), r^S(\varphi)$ subject to $r^F(\varphi) + r^S(\varphi) = r(\varphi)$. Let λ^N denote the Lagrange multiplier associated with the Nash bargaining problem. The first-order conditions associated with Nash bargaining are given by

$$\left(V^S(\varphi) - V^S\right) \frac{\partial V^F(\varphi)}{\partial r^F(\varphi)} = \lambda^N,$$
$$\left(V^F(\varphi) - V^F - F_d\right) \frac{\partial V^S(\varphi)}{\partial r^S(\varphi)} = \lambda^N.$$

Moreover, (2) implies $\partial V^F(\varphi)/\partial r^F(\varphi) = \partial V^S(\varphi)/\partial r^S(\varphi)$. Using this for the first-order conditions, we get

$$V^F(\varphi) - V^F - F_d = V^S(\varphi) - V^S.$$

Rearranging the above equality yields the condition given in (3).

A.2 Labor Market Clearing Condition

We show that aggregate revenue equals aggregate labor income (i.e., $R = L$) in the steady state equilibrium of autarky, X-integration, and M-integration.

A.2.1 Autarky

Labor is used for entry and search by entrants and matched firms, respectively, at every point in time. Labor is also used for production by incumbent agents at every point in time. Below, we first derive aggregate investment labor, next aggregate production labor, and finally aggregate labor in the industry.

Consider aggregate investment labor used by firms at every point in time. There is the number $N_e^F (= \delta N^F)$ of new entrants that pay the fixed entry cost F_e^F at every point in time. Equivalently, there is the number N^F of incumbent firms that pay the fixed entry cost $f_e^F (= \delta F_e^F)$ at every point in time. In either case, aggregate labor used for entry at every point in time is $\delta N^F F_e^F$ where F_e^F equals V^F from the free entry condition in (5) and V^F is given as the second equation in (6). Multiplying V^F by δN^F and rearranging,

$$\delta N^F V^F = \underbrace{\left(\frac{\delta N^F}{\delta + \mu^F}\right)}_{N^F - n} \frac{r}{\sigma} + \underbrace{\left(\frac{\mu^F N^F}{\delta + \mu^F}\right)}_{n} \left(\frac{r^F(\varphi)}{\sigma} - \delta F_d\right),$$

where the number of unmatched and matched firms follows from the steady-state relationship in (9). Similarly, there is the number n of matched firms that pay the fixed search cost $\delta F_d (= f_d)$ at every point in time, and thus aggregate labor used for search at every point in time is $\delta n F_d$. Taken together, aggregate labor used for entry and search by firms at every point in time is $L_e^F = \delta N^F F_e^F + \delta n F_d$ where the first term satisfies above $\delta N^F V^F$. Hence, aggregate investment labor used by firms equals aggregate profit earned by firms at every point in time:

$$L_e^F = \frac{(N^F - n) r}{\sigma} + \frac{n r^F(\varphi)}{\sigma}. \tag{A.1}$$

As for aggregate investment labor used by suppliers, there is the number N^S of incumbent suppliers that pay the fixed entry cost $f_e^S(= \delta F_e^S)$ at every point in time. As suppliers do not incur any fixed search cost, aggregate investment labor used by suppliers at every point in time is $L_e^S = \delta N^S F_e^S$. Using $F_e^S = V^S$ where the value of unmatched suppliers is given as the last equation in (6) and the steady-state relationship in (9),

$$L_e^S = \frac{nr^S(\varphi)}{\sigma}. \tag{A.2}$$

Note that revenue of matched agents is $r(\varphi) = r^F(\varphi) + r^S(\varphi)$ while aggregate revenue is $R = (N^F - n)r + nr(\varphi)$. Then, (A.1) and (A.2) suggest that aggregate investment labor equals aggregate profit:

$$L_e^F + L_e^S = \frac{R}{\sigma}. \tag{A.3}$$

In (A.3), $L_e^F + L_e^S$ represents not only aggregate investment labor but also aggregate investment cost (as the wage rate is normalized to one by choosing labor as the numéraire). Hence, (A.3) implies that there is no net investment income. It is worth emphasizing that a positive discount factor does not induce the result, in that the equality in (A.3) does not hold when $\gamma > 0$ for the Bellman equations in (2). This property of the model's stationary equilibrium is identical to that described by Melitz (2003).

Next, consider aggregate production labor. Let L_p^F and L_p^S denote aggregate labor used for production by firms and suppliers, respectively. There is the number $N^F - n$ of unmatched firms that use only core inputs and incur production cost $y(1) \equiv y$, whereas there is the number n of matched firms that use both core and non-core inputs and incur production cost $y(\varphi)/\varphi$ at every point in time. Adding them up, aggregate production labor is $L_p^F + L_p^S = (N^F - n)y + ny(\varphi)/\varphi$ where $y(\varphi) = \varphi^\sigma y$. Using $y(\varphi) = r(\varphi)/p(\varphi)$ and $R = (N^F - n)r + nr(\varphi)$ where $p(\varphi)$ satisfies the optimal pricing rule, we get

$$L_p^F + L_p^S = \left(\frac{\sigma - 1}{\sigma}\right) R. \tag{A.4}$$

Finally, consider aggregate labor used in the industry. Summing up (A.3) and (A.4),

$$L = (L_e^F + L_e^S) + (L_p^F + L_p^S) = R,$$

where L represents aggregate labor income as the wage rate is one. This establishes the desired result.

A.2.2 X-Integration

Since matched firms additionally incur a fixed export cost F_x in X-integration, aggregate labor used for entry by firms at any point in time is $L_e^F = \delta N^F F_e^F + \delta n F_d + \delta n F_x$. Using the value functions of unmatched firms in X-integration, we can express L_e^F as (A.1). Similarly, (A.2) holds since the Bellman equations for suppliers remain the same as those in autarky. Taken together, (A.3) also holds so that aggregate investment labor equals aggregate profit at every point in time in X-integration.

On the other hand, as matched firms incur iceberg transport cost τ_x to the export market in X-integration, aggregate labor used for production is $L_p^F + L_p^S = (N^F - n)y + ny_d(\varphi)/\varphi + n\tau_x y_x(\varphi)/\varphi$ where $y_x(\varphi) = \tau_x^{-\sigma}\varphi^\sigma y$ from the first-order condition. From $R = (N^F - n)r + nr_d(\varphi) + nr_x(\varphi)$ in X-integration, aggregate labor used for production also satisfies (A.4).

Finally, from (A.3) and (A.4), the equilibrium relationship $R = L$ continues to hold in X-integration.

A.2.3 M-Integration

Using the value functions of firms in M-integration (see (A.11) in Appendix A.5), we can show that aggregate labor used for entry by firms is written as (A.1). Similarly, using the value functions of suppliers in M-integration, aggregate labor used for entry by suppliers is written as (A.2). Thus (A.3) holds in M-integration.

On the other hand, aggregate labor used for production is $L_p^F + L_p^S = (N^F - n)y + ny(\varphi)/2\varphi + ny(\varphi_m)/2\varphi_m$, as the transport cost τ_m is included in φ_m and there is the number $n/2$ of firms matched at home and abroad. Using the optimal pricing rules and $R = (N^F - n)r + nr(\varphi)/2 + nr(\varphi_m)/2$, aggregate labor used for production satisfies (A.4).

Finally, from (A.3) and (A.4), the equilibrium relationship $R = L$ continues to hold in M-integration.

A.3 Number of Agents and Welfare

We show detailed derivations of the number of agents and welfare in autarky, X-integration, and M-integration.

A.3.1 Autarky

From the optimal pricing rule, the price index in autarky is

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma - 1}\right)^{1-\sigma} \left(N^F - n + \varphi^{\sigma-1} n\right). \tag{A.5}$$

Moreover, substituting $R = L$ and (A.5) into the optimal consumer expenditure $r = Ap^{1-\sigma}$ where $A = RP^{\sigma-1}$, the variable profit of unmatched firms is expressed as

$$\frac{r}{\sigma} = \frac{L}{\sigma(N^F - n + \varphi^{\sigma-1} n)}. \tag{A.6}$$

(A.5) and (A.6) are used to derive the number of agents in (12) and welfare per worker in (13). Regarding (12), rewrite the steady-state relationship in (9) as $N^F - n = \left(\delta/\mu^F\right) n$. Using this and solving (A.6) for n, we get

$$n = \frac{L}{r}\left(\frac{\mu^F}{\delta + \varphi^{\sigma-1}\mu^F}\right).$$

The number of matched agents is obtained by noting that the variable profit of unmatched firms given in the above equality must satisfy $r/\sigma = f$ in equilibrium. On the other hand, the total number of firms and suppliers in the industry is obtained by rewriting (9) as

$$N^F = \left(\frac{\delta + \mu^F}{\mu^F}\right) n, \quad N^S = \left(\frac{\delta + \mu^S}{\mu^S}\right) n.$$

As for (13), on the other hand, substituting (A.6) into (A.5) and rearranging,

$$\frac{1}{P} = \frac{\sigma - 1}{\sigma}\left(\frac{L}{r}\right)^{\frac{1}{\sigma-1}}.$$

Welfare per worker is obtained by noting that welfare is defined as an inverse of the CES price index and the variable profit of unmatched firms given in (A.6) must satisfy $r/\sigma = f$ under free entry.

A.3.2 X-Integration

In X-integration, the price index is written as

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[N^F - n + (1+\tau_x^{1-\sigma})\varphi^{\sigma-1}n\right].$$

Similarly, the variable profit of unmatched firms is written as

$$\frac{r}{\sigma} = \frac{L}{\sigma\left[N^F - n + (1+\tau_x^{1-\sigma})\varphi^{\sigma-1}n\right]}.$$

Using these equalities and following similar procedures shown above, we obtain the number of agents and welfare in X-integration where $\Xi \equiv \sigma f\left[\delta + \left(1+\tau_x^{1-\sigma}\right)\varphi^{\sigma-1}\mu^F\right]$ in (12) and f is the expected fixed cost in (18).

A.3.3 M-Integration

In M-integration, the price index is written as

$$P^{1-\sigma} = \left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma} \left[N^F - n + \left(\varphi^{\sigma-1} + \varphi_m^{\sigma-1}\right)\frac{n}{2}\right].$$

Similarly, the variable profit of unmatched firms is written as

$$\frac{r}{\sigma} = \frac{L}{\sigma\left[N^F - n + \left(\varphi^{\sigma-1} + \varphi_m^{\sigma-1}\right)\frac{n}{2}\right]}.$$

Using these equalities and following similar procedures shown above, we obtain the number of agents and welfare in M-integration where $\Xi \equiv \sigma f\left[\delta + \left(\varphi^{\sigma-1} + \varphi_m^{\sigma-1}\right)\mu^F/2\right]$ in (12) and f is the expected fixed cost in (23).

A.4 Free Entry Condition in X-integration

A.4.1 When Only Matched Firms Export

Consider the Bellman equations of agents under the assumption that only matched firms export. The Bellman equations for suppliers are the same as (2), whereas those for firms are given by

$$\begin{aligned}
\gamma V^F(\varphi) &= \frac{r^F(\varphi)}{\sigma} - \delta V^F(\varphi) + \dot{V}^F(\varphi), \\
\gamma V^F &= \frac{r}{\sigma} + \mu^F\left(V^F(\varphi) - V^F - F_d - F_x\right) - \delta V^F + \dot{V}^F.
\end{aligned}$$

Unmatched firms become matched at the rate μ^F at which point they obtain a gain $V^F(\varphi) - V^F - F_d - F_x$ where matched firms make a one-time investment F_x for entry into the export market, and they earn the variable profit $r^F(\varphi)/\sigma$ from the domestic market $r_d^F(\varphi)/\sigma$ and the export market $r_x^F(\varphi)/\sigma$.

Setting $\gamma = 0$ as well as $\dot{V}^F = \dot{V}^F(\varphi) = 0$, we get the value functions of firms corresponding to (6):

$$\begin{aligned}
V^F(\varphi) &= \frac{r^F(\varphi)}{\delta\sigma}, \\
V^F &= \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{\delta+\mu^F}\right)\left(\frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d - F_x\right).
\end{aligned}$$

Obviously the interpretation is similar to that of (6) in autarky, but we assume $r^F(\varphi)/\delta\sigma - r/\delta\sigma - F_d - F_x > 0$ to ensure that matched firms have an enough incentive to incur the fixed export cost F_x. Under the condition, the above value functions imply $V^F(\varphi) - V^F - F_d - F_x > 0$ so that the net value of matches is strictly positive. Like F_d, this is the only reason that firms consider sinking the fixed export cost, which is shared by suppliers at the bargaining stage.

We can describe how Nash bargaining between firms and suppliers affects the division of surplus. Following Appendix A.1, symmetric Nash bargaining imposes the following condition at any point in time:

$$V^F(\varphi) - V^F - F_d - F_x = \frac{1}{2}\Big(V^F(\varphi) - V^F - F_d - F_x + V^S(\varphi) - V^S\Big).$$

Using the value functions derived above for this bargaining constraint, the solution to the bargaining problem subject to $r^F(\varphi)/\sigma + r^S(\varphi)/\sigma = r(\varphi)/\sigma$ gives us the profit sharing rule. The result is obtained by noting that the effective bargaining power and the steady-state number of agents in X-integration are the same as those in autarky, given in (8) and (9), respectively.

A.4.2 When Both Unmatched and Matched Firms Export

Consider the Bellman conditions of agents under the assumption that both unmatched and matched firms export. In this case, we require the following modifications in (2). First, unmatched firms earn not only the domestic revenue r_d but also the export revenue $r_x = \tau_x^{1-\sigma} r_d$. Thus, unmatched firms obtain a gain $r/\sigma = (1+\tau_x^{1-\sigma})r_d/\sigma$ in the second equation of (2). As matched firms obtain a gain $r(\varphi)/\sigma = (1+\tau_x^{1-\sigma})\varphi^{\sigma-1} r_d/\sigma$, the ratio of the equilibrium revenue of matched firms to that of unmatched firms satisfies (1) in this setting just as in autarky. Second, unmatched firms pay the fixed export cost $f_x(=\delta F_x)$ to enter the export market at every point in time. Thus, firms' outside option is given by $r/\sigma - f_x$ in the second equation of (2).

Clearly, other equilibrium conditions are unchanged from the baseline case where only matched firms export. From the profit and trade costs seen above, the free entry condition (5) in this equilibrium is given by

$$\begin{aligned} \frac{r}{\sigma} - f_x + \frac{n}{N^F}\beta\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d\right) - f_e^F &= 0, \\ \frac{n}{N^S}(1-\beta)\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d\right) - f_e^S &= 0. \end{aligned} \tag{A.7}$$

In (A.7), note that the economic rent of matched agents does not include the fixed trade cost f_x, as the rent is defined as the difference in profits between matched and unmatched firms where both types of firms incur f_x in equilibrium. The expression in (A.7) is very similar to the free entry condition in autarky given as (10), but there exist two differences in firms' outside option: (*i*) the total profit r/σ includes both the domestic profit r_d/σ and the export profit r_x/σ; and (*ii*) the fixed export cost f_x is subtracted from r/σ as unmatched firms have to incur this fixed cost at every point in time.

Before proceeding, it is useful to consider a special case with no trade costs in X-integration ($\tau_x = 1, f_x = 0$). Then, unmatched firms also export and thus the free entry condition must be defined as (A.7), instead of (17). However, (A.7) is almost identical with (10) in autarky except that the total profit includes the export profit. This implies that, when there are no trade costs and firms freely export, X-integration is essentially the same as an increase in market size L. Even if the transport cost τ_x is positive, (A.7) is identical with (10) so long as the fixed export cost f_x is zero. In that case, the transport cost affects only the distribution of the total profit earned from the domestic and export markets, but the total profit remains the same as that in autarky.

Next, consider the equilibrium characterization. Substituting $r(\varphi) = (1+\tau_x^{1-\sigma})\varphi^{\sigma-1}r_d$ and $r = (1+\tau_x^{1-\sigma})r_d$ into (A.7) and using the definition of expected shares $\phi^F \equiv \beta n/N^F, \phi^S \equiv (1-\beta)n/N^S$, we can solve the free entry condition (A.7) for the *domestic* variable profit of unmatched firms:

$$
\begin{aligned}
\frac{r_d}{\sigma} &= \frac{f_e^F + f_x + f_d\phi^F}{(1+\tau_x^{1-\sigma})\left[1+(\varphi^{\sigma-1}-1)\phi^F\right]}, \\
\frac{r_d}{\sigma} &= \frac{f_e^S + f_d\phi^S}{(1+\tau_x^{1-\sigma})(\varphi^{\sigma-1}-1)\phi^S}.
\end{aligned}
\tag{A.8}
$$

(A.8) can be shown in the $(\theta, r_d/\sigma)$ space, where the first and second equalities are respectively downward- and upward-sloping and the intersection uniquely determines two equilibrium variables, θ and r_d/σ.

To address the impact of X-integration, we only need to compare the free entry condition associated with different levels of trade costs. Note that, for both unmatched and matched firms to have an enough incentive to export final goods, trade costs must be low enough to satisfy $(1+\tau_x^{1-\sigma})\varphi^{\sigma-1}r_d/\sigma - f_d - f_x > \varphi^{\sigma-1}r_d/\sigma - f_d$ for matched firms and $(1+\tau_x^{1-\sigma})r_d/\sigma - f_x > r_d/\sigma$ for unmatched firms, which are simplified as $(\tau_x/\varphi)^{1-\sigma}r_d/\sigma > f_x$ and $\tau_x^{1-\sigma}r_d/\sigma > f_x$ respectively. However, when the latter holds, the former always hold. Thus, we only require

$$
\frac{r_d}{\sigma} > \tau_x^{\sigma-1}f_x. \tag{A.9}
$$

Comparing (10) and (A.7) under (A.9) shows that not only is the economic rent but also the outside option of firms is greater in X-integration than in autarky. Then X-integration increases the *ex ante* expected profit and induces further entry of agents under free entry, which in turn decreases the *ex post* profit r_d/σ in X-integration relative to autarky. In fact, comparing (11) and (A.8) under (A.9) shows that both the FF and SS curves in X-integration are located *below* relative to those in autarky. Moreover, from (A.7), the negative relationship between r_d/σ and θ holds in this case. The relationship is similar to (14), though f_x enters the left-hand side. Hence, we can conclude that, even in equilibrium where both matched and unmatched firms export, θ is higher while r_d/σ is lower in X-integration than those in autarky, just as in Section 4.

Although the domestic profit of unmatched firms always decreases by X-integration, the *total* profit of them $r/\sigma = (1+\tau_x^{1-\sigma})r_d/\sigma$ increases by this integration. To see this, multiplying both sides of (A.8) by $(1+\tau_x^{1-\sigma})$, the free entry condition can be written in terms of the total variable profit of unmatched firms:

$$
\begin{aligned}
\frac{r}{\sigma} &= \frac{f_e^F + f_x + f_d\phi^F}{1+(\varphi^{\sigma-1}-1)\phi^F}, \\
\frac{r}{\sigma} &= \frac{f_e^S + f_d\phi^S}{(\varphi^{\sigma-1}-1)\phi^S}.
\end{aligned}
\tag{A.10}
$$

Similarly to (A.8), we can show (A.10) in the $(\theta, r/\sigma)$ space, which uniquely determines θ and r/σ. However, comparing (11) and (A.10) under (A.9) shows that the FF curve in X-integration is located *above* relative to that in autarky, while the SS curve is the same between the two regimes. Intuitively, if both unmatched and matched firms export, firms always have to incur the fixed export cost f_x regardless of their matching status. As a result, the expected profit of firms must reflect the fixed export cost, which shifts the FF curve upwards. Further, this additional fixed cost deters further entry of firms and decreases the number of firms N^F, thereby increasing the ratio of unmatched agents $\theta = (N^S - n)/(N^F - n)$. In sum, while θ always rises in X-integration, the domestic profit of unmatched firms r_d/σ falls due to the increased competition in the domestic market but their total profit r/σ rises due to the additional profit from the export market in that integration.

Once two equilibrium variables of the model—either $(\theta, r_d/\sigma)$ from (A.8) or $(\theta, r/\sigma)$ from (A.10)—determined, other endogenous variables can be written as a function of them. Here we use the latter to describe equilibrium. Free entry implies $r/\sigma = f$ where f represents the expected fixed cost, given as the right-hand side of (A.10). From the optimal pricing rules, the CES price index in (A.5) is expressed as

$$P^{1-\sigma} = (1+\tau_x^{1-\sigma})\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}\left(N^F - n + \varphi^{\sigma-1}n\right),$$

while the *domestic* variable profit of unmatched firms is expressed as

$$\frac{r_d}{\sigma} = \frac{L}{\sigma(1+\tau_x^{1-\sigma})(N^F - n + \varphi^{\sigma-1}n)}.$$

Since $r_x = \tau_x^{1-\sigma} r_d$, the *total* variable profit of unmatched firms $r/\sigma = (1+\tau_x^{1-\sigma})r_d/\sigma$ is still written as (A.6). Similarly to Appendix A.3, these equations are used to derive the number of agents and welfare. The number of agents is expressed as (12) where $\Xi \equiv \sigma f\left(\delta + \varphi^{\sigma-1}\mu^F\right)$ is the same as that in autarky. On the other hand, solving the price index for $1/P$ and using $r/\sigma = f$, the welfare expression in (13) is expressed as

$$W = \frac{\sigma-1}{\sigma}\left(\frac{(1+\tau_x^{1-\sigma})L}{\sigma f}\right)^{\frac{1}{\sigma-1}}.$$

This completes the characterization of X-integration equilibrium where unmatched firms also export.

If search frictions are prohibitively large, equilibrium properties of X-integration are identical to that shown by Krugman (1980). As the number of matched firms is zero ($n = 0$) and there is no fixed export cost ($f_x = 0$) in that case, $f = f_e^F$ by setting $\phi^F = 0$ in the first equality of (A.10). Further, as the number of agents in (12) satisfies $\Xi = \delta\sigma f_e^F$ (from $\mu^F = 0$) in autarky and X-integration, the number of firms is given by $N^F = L/\sigma f_e^F$. Finally, as $f = f_e^F$ in autarky and X-integration, comparing (13) and the above welfare expression shows that welfare is higher in X-integration than in autarky due solely to increased product variety.

A.5 Free Entry Condition in M-Integration

A.5.1 Baseline Case

Consider the Bellman conditions of agents under the assumption that the contact rates are the same between domestic and cross-border matches. In this baseline case, we have

$$\begin{aligned}
\gamma V^F(\varphi) &= \frac{r^F(\varphi)}{\sigma} - \delta V^F(\varphi) + \dot{V}^F(\varphi),\\
\gamma V^F(\varphi_m) &= \frac{r^F(\varphi_m)}{\sigma} - \delta V^F(\varphi_m) + \dot{V}^F(\varphi_m),\\
\gamma V^F &= \frac{r}{\sigma} + \frac{\mu^F}{2}\Big(V^F(\varphi) - V^F - F_d\Big) + \frac{\mu^F}{2}\Big(V^F(\varphi_m) - V^F - F_m\Big) - \delta V^F + \dot{V}^F,\\
\gamma V^S(\varphi) &= \frac{r^S(\varphi)}{\sigma} - \delta V^S(\varphi) + \dot{V}^S(\varphi),\\
\gamma V^S(\varphi_m) &= \frac{r^S(\varphi_m)}{\sigma} - \delta V^S(\varphi_m) + \dot{V}^S(\varphi_m),\\
\gamma V^S &= \frac{\mu^S}{2}\Big(V^S(\varphi) - V^S\Big) + \frac{\mu^S}{2}\Big(V^S(\varphi_m) - V^S\Big) - \delta V^S + \dot{V}^S.
\end{aligned}$$

Setting $\gamma = 0$ as well as $\dot{V}^F(\varphi) = \dot{V}^F(\varphi_m) = \dot{V}^F = \dot{V}^S(\varphi) = \dot{V}^S(\varphi_m) = \dot{V}^S = 0$ in the Bellman equations, the value functions of agents corresponding to (6) are

$$\begin{aligned}
V^F(\varphi) &= \frac{r^F(\varphi)}{\delta\sigma}, \\
V^F(\varphi_m) &= \frac{r^F(\varphi_m)}{\delta\sigma}, \\
V^F &= \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{2(\delta+\mu^F)}\right)\left(\frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \left(\frac{\mu^F}{2(\delta+\mu^F)}\right)\left(\frac{r^F(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right), \\
V^S(\varphi) &= \frac{r^S(\varphi)}{\delta\sigma}, \\
V^S(\varphi_m) &= \frac{r^S(\varphi_m)}{\delta\sigma}, \\
V^S &= \left(\frac{\mu^S}{2(\delta+\mu^S)}\right)\frac{r^S(\varphi)}{\delta\sigma} + \left(\frac{\mu^S}{2(\delta+\mu^S)}\right)\frac{r^S(\varphi_m)}{\delta\sigma}.
\end{aligned} \tag{A.11}$$

We assume not only $r^F(\varphi)/\delta\sigma - r/\delta\sigma - F_d > 0$ but also $r^F(\varphi_m)/\delta\sigma - r/\delta\sigma - F_m > 0$ in the third equation of (A.11), which ensures $\left(V^F(\varphi) - V^F - F_d\right) + \left(V^F(\varphi_m) - V^F - F_m\right) > 0$ and the net value of matches is strictly positive.

Agents matched with home and foreign partners determine profit sharing by symmetric Nash bargaining. Similarly to (3), this sharing imposes the following conditions for each type of matched agents:

$$\begin{aligned}
V^F(\varphi) - V^F - F_d &= \frac{1}{2}\left(V^F(\varphi) - V^F - F_d + V^S(\varphi) - V^S\right), \\
V^F(\varphi_m) - V^F - F_m &= \frac{1}{2}\left(V^F(\varphi_m) - V^F - F_m + V^S(\varphi_m) - V^S\right).
\end{aligned}$$

Adding up these two equalities and rearranging,

$$\left(V^F(\varphi) - V^F - F_d\right) + \left(V^F(\varphi_m) - V^F - F_m\right) = \left(V^S(\varphi) - V^S\right) + \left(V^S(\varphi_m) - V^S\right).$$

Substituting the value functions in (A.11) into the equality above and rearranging, we get the following profit sharing rule:

$$\begin{aligned}
\left(\frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \left(\frac{r^F(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right) &= \beta\left[\left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \left(\frac{r(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right)\right], \\
\frac{r^S(\varphi)}{\delta\sigma} + \frac{r^S(\varphi_m)}{\delta\sigma} &= (1-\beta)\left[\left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \left(\frac{r(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right)\right],
\end{aligned} \tag{A.12}$$

where β is the same as (8). Thus, (A.12) shows that matched agents split the total economic rent weighted by the effective bargaining power. Noting that the steady-state number of agents (9) is the same in M-integration, and using (A.11) and (A.12) for the free entry condition in (5) and rearranging, we get

$$\begin{aligned}
\frac{r}{\sigma} + \frac{n}{2N^F}\beta\left[\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d\right) + \left(\frac{r(\varphi_m)}{\sigma} - \frac{r}{\sigma} - f_m\right)\right] - f_e^F &= 0, \\
\frac{n}{2N^S}(1-\beta)\left[\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d\right) + \left(\frac{r(\varphi_m)}{\sigma} - \frac{r}{\sigma} - f_m\right)\right] - f_e^S &= 0.
\end{aligned} \tag{A.13}$$

As $r(\varphi)/\sigma - r/\sigma - f_d$ and $r(\varphi_m)/\sigma - r/\sigma - f_m$ are the economic rent of matched agents at home and abroad, (A.13) shows that the expected profit of agents consists of the outside option plus the expected economic rent of these two kinds of matches, which must be offset by the fixed entry cost. (22) follows immediately from (A.13).

A.5.2 Extended Case

Consider the Bellman conditions of agents under the assumption that the contact rates differ between domestic and cross-border matches. Let μ_d^F, μ_d^S denote the rate at which unmatched agents meet unmatched partners at home, whereas let μ_m^F, μ_m^S denote the rate at which unmatched agents meet unmatched partners from abroad. In this extended case, we have

$$\begin{aligned}
\gamma V^F(\varphi) &= \frac{r^F(\varphi)}{\sigma} - \delta V^F(\varphi) + \dot{V}^F(\varphi), \\
\gamma V^F(\varphi_m) &= \frac{r^F(\varphi_m)}{\sigma} - \delta V^F(\varphi_m) + \dot{V}^F(\varphi_m), \\
\gamma V^F &= \frac{r}{\sigma} + \mu_d^F\Big(V^F(\varphi) - V^F - F_d\Big) + \mu_m^F\Big(V^F(\varphi_m) - V^F - F_m\Big) - \delta V^F + \dot{V}^F, \\
\gamma V^S(\varphi) &= \frac{r^S(\varphi)}{\sigma} - \delta V^S(\varphi) + \dot{V}^S(\varphi), \\
\gamma V^S(\varphi_m) &= \frac{r^S(\varphi_m)}{\sigma} - \delta V^S(\varphi_m) + \dot{V}^S(\varphi_m), \\
\gamma V^S &= \mu_d^S\Big(V^S(\varphi) - V^S\Big) + \mu_m^S\Big(V^S(\varphi_m) - V^S\Big) - \delta V^S + \dot{V}^S.
\end{aligned}$$

Suppose that the matching function of domestic matches is $m_d(u^F, u^S) = m(u^F, u^S)/(1+\kappa)$ while that for cross-border matches is $m_m(m^F, u^S) = \kappa m(u^F, u^S)/(1+\kappa)$ where $m(u^F, u^S)$ is given in Section 2.3 and $\kappa(\leq 1)$ is the difficulty of cross-border matches relative to domestic matches, in that the smaller κ, the harder the cross-border matches. Using μ^F, μ^S, the probability of matches for each type of agents is defined as $\mu_d^F = \mu^F/(1+\kappa)$, $\mu_d^S = \mu^S/(1+\kappa)$, $\mu_m^F = \kappa\mu^F/(1+\kappa)$, $\mu_m^S = \kappa\mu^S/(1+\kappa)$. Then, the value functions corresponding to (A.11) are

$$\begin{aligned}
V^F(\varphi) &= \frac{r^F(\varphi)}{\delta\sigma}, \\
V^F(\varphi_m) &= \frac{r^F(\varphi_m)}{\delta\sigma}, \\
V^F &= \frac{r}{\delta\sigma} + \left(\frac{\mu^F}{(1+\kappa)(\delta+\mu^F)}\right)\left(\frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \left(\frac{\kappa\mu^F}{(1+\kappa)(\delta+\mu^F)}\right)\left(\frac{r^F(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right), \\
V^S(\varphi) &= \frac{r^S(\varphi)}{\delta\sigma}, \\
V^S(\varphi_m) &= \frac{r^S(\varphi_m)}{\delta\sigma}, \\
V^S &= \left(\frac{\mu^S}{(1+\kappa)(\delta+\mu^S)}\right)\frac{r^S(\varphi)}{\delta\sigma} + \left(\frac{\kappa\mu^S}{(1+\kappa)(\delta+\mu^F)}\right)\frac{r^S(\varphi_m)}{\delta\sigma}.
\end{aligned}$$

The profit sharing rule corresponding to (A.12) is now given by

$$\begin{aligned}
\left(\frac{r^F(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) + \kappa\left(\frac{r^F(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma}\right) &= \beta\left[\left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \kappa\left(\frac{r(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right)\right], \\
\left(\frac{r^S(\varphi)}{\delta\sigma} - F_d\right) + \kappa\left(\frac{r^S(\varphi_m)}{\delta\sigma} - F_m\right) &= (1-\beta)\left[\left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_d\right) + \kappa\left(\frac{r(\varphi_m)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_m\right)\right],
\end{aligned}$$

where β is the same as (8) which critically depends on our matching functions with $m_m(u^F, u^S) = \kappa m_d(u^F, u^S)$. Relative to the baseline case, the economic rent of cross-border matched agents is discounted by $\kappa < 1$ in the extended case. This, of course, comes from the search technology inducing a lower contact rate for cross-border matches than for domestic matches. Reflecting that, the steady-state number of agents (9) is affected by κ:

$$
\begin{aligned}
n_d &= \frac{1}{1+\kappa}\left(\frac{\mu^F}{\delta+\mu^F}\right)N^F = \frac{1}{1+\kappa}\left(\frac{\mu^S}{\delta+\mu^S}\right)N^S, \\
n_m &= \frac{\kappa}{1+\kappa}\left(\frac{\mu^F}{\delta+\mu^F}\right)N^F = \frac{\kappa}{1+\kappa}\left(\frac{\mu^S}{\delta+\mu^S}\right)N^S,
\end{aligned}
$$

where the total number of matches is $n = n_d + n_m$.

Finally, using the above equilibrium relationships for the free entry condition in (5), we obtain the following expression of free entry corresponding to (A.13):

$$
\begin{aligned}
\frac{r}{\sigma} + \frac{n}{(1+\kappa)N^F}\beta\left[\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d\right) + \kappa\left(\frac{r(\varphi_m)}{\sigma} - \frac{r}{\sigma} - f_m\right)\right] - f_e^F &= 0, \\
\frac{n}{(1+\kappa)N^S}(1-\beta)\left[\left(\frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d\right) + \kappa\left(\frac{r(\varphi_m)}{\sigma} - \frac{r}{\sigma} - f_m\right)\right] - f_e^S &= 0.
\end{aligned}
\tag{A.14}
$$

The free entry condition in Section 5.4 is directly obtained from (A.14). Comparing (10) and (A.14) shows that the condition under which M-integration improves welfare given in (25) is exactly the same even in this case, and thus κ does not affect the condition under which M-integration causes the welfare losses from trade. Intuition comes from observing that κ has opposing effects on the expected profit. On the one hand, the smaller is κ, the more difficult is for foreign firms to penetrate the home matching market. As foreign firms' penetration is less intense, the expected share of domestic matches is higher at home, which makes the expected profit greater. On the other hand, the smaller is κ, the more difficult is for home firms to penetrate the foreign matching market. As home firms' penetration is less intense, the expected share of cross-border matches is lower, which makes the expected profit smaller. With the matching functions considered above, these effects leave (25) independent of the difficulty of cross-border matches.

We can show that the impact of M-integration in the extended case is the same as that in the baseline case. Solving (A.14) for the variable profit of unmatched firms, we get the following equalities:

$$
\begin{aligned}
\frac{r}{\sigma} &= \frac{f_e^F + (f_d + \kappa f_m)\frac{\phi^F}{1+\kappa}}{1 + \left[\varphi^{\sigma-1} + \kappa\varphi_m^{\sigma-1} - (1+\kappa)\right]\frac{\phi^F}{1+\kappa}}, \\
\frac{r}{\sigma} &= \frac{f_e^S + (f_d + \kappa f_m)\frac{\phi^S}{1+\kappa}}{\left[\varphi^{\sigma-1} + \kappa\varphi_m^{\sigma-1} - (1+\kappa)\right]\frac{\phi^S}{1+\kappa}}.
\end{aligned}
\tag{A.15}
$$

Differentiating (A.15) suggests that the right-hand side is increasing not only in τ_m, f_m but also in κ. Hence reductions in trade costs (the smaller τ_m, f_m) shift down two curves while improvements in search technology (the larger κ) shift up the two curves. As in X-integration, a decrease in trade costs leads to an increase in the economic rent earned by matched firms, which in turn induces further entry and shifts two curves downwards in M-integration. In contrast, an improvement in search technology abroad leads to an increase of the share of cross-border matches in the overall expected profit. As firms matched with foreign suppliers are less efficient than those with domestic suppliers and their market share rises with such improvement, this in turn leads to weak competition in the industry by increasing the price index and hence shifts the two curves upwards.