Relationship Specificity, Market Thickness and International Trade*

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Abstract

We develop a dynamic, search-theoretical, general-equilibrium model to investigate the effect of trade liberalization in vertically-related industries, emphasizing differential impacts depending on the degree of relationship specificity of intermediates that are traded within matched pairs. The paper in particular unveils the role of search-and-matching and the resulting market restructuring of vertically-related industries in evaluating the impact of trade liberalization. We find that the higher the relationship specificity, the thinner the market transactions relative to the non-market transactions; and that a reduction in trade costs, either in final goods or in intermediate inputs, makes the market transactions thinner and enhances social welfare. We also examine how trade liberalization changes the trade volume of final goods and intermediate inputs, and whether they exhibit complementarity.

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1 Introduction

The volume of international trade has been steadily rising over the past fifty years. It is often argued that "offshoring" or "vertical specialization" in production processes contributes to this trade growth. For instance, using input-output tables, Hummels et al. (2001) estimate that offshoring in intermediates accounts for around 30 percent of countries' export growth of final goods between 1970 and 1990, and Johnson and Noguera (2012) show that intermediate inputs account for approximately two thirds of international trade by combining input-output tables and bilateral trade data for many countries. Similarly, Yi (2003) demonstrates that vertical specialization in intermediates amplifies the effect of trade liberalization on the growth of trade in final goods. While the evidence suggests that trade in final goods and trade in intermediates mutually increase the trade volume, few theoretical work has investigated this complementary effect and its consequences on the welfare gains from trade.

This paper proposes a dynamic general-equilibrium model in which matching with a partner in vertically-related production and heterogeneous product quality that arises from matching status play an important role in explaining the interaction between trade in final goods and intermediates. In particular, we build a search-theoretical framework where firms from a downstream sector search for firms in an upstream sector, and matching randomly takes place between the two types of firms. Matched upstream firms carry out a relationship-specific investment to raise the quality of final goods, which allows matched downstream firms to procure high-quality ("customized") inputs used for their production.[1] In contrast, unmatched downstream firms inevitably use low-quality ("generic") inputs that are sold in the market to produce their final goods, whose quality is poorer than that of matched downstream firms. In this model setting, we examine the effects of international trade both in final goods and in intermediates on the matching environment for both upstream and downstream firms, trade volumes and social welfare. Since the importance of matching varies across industries due to the differential degree of relationship specificity of industries, those effects of international trade also naturally vary with the relationship specificity of industries.

Our theoretical model tries to capture some important features of vertically-related industries. As a real-world example, consider smartphone manufacturing. Apple is a leading producer of high-end smartphones. As is well-known, Apple does not make all of its devices on its own, but instead procures intermediates and assembly from its carefully chosen suppliers to enhance product quality. There are also many producers of low-end smartphones, especially in China, that purchase most of intermediates from domestic markets sold at reference-priced. In contrast to Apple, many producers of low-end smartphones utilize intermediates that are produced by small local firms. *The Economist* (2017) describes this difference in sourcing strategies as follows: "zealous pursuit of quality would be expected of factories that produce phones for Apple – the world-class facilities run by Taiwan's Foxconn," whereas "teeming firms means vicious price competition, especially for cheaper phones." It is important to stress that these sourcing strategies are *not* related to vertical integration. Clearly, Apple does not vertically integrate Foxconn within the same boundary of the firm, but Apple searches for the best suppliers in the world and results in matching with Foxconn.

[1] Recent empirical evidence shows that producing high-quality products generally requires high-quality inputs. See, for example, Kugler and Verhoogen (2011), Bastos et al. (2018) and Fan et al. (2018).

We begin our analysis with a closed-economy model to establish the interactive relation between the input market and the matching environment, which also affects bargaining over surplus between partners. Customized inputs are traded between matched pairs, while generic inputs which can be inferior substitutes for customized inputs are traded in the market. To examine how the input market interacts with the matching environment that varies with industries, we pay special attention to cross-sectoral differences in input customization (or "relationship specificity") devised by Nunn (2007). He constructs a variable that measures the importance of relationship-specific investments by calculating the proportion of intermediates whose markets are thin; the less the intermediates are traded in the organized market or reference priced, the more relationship-specific are intermediates. We build a theoretical foundation of Nunn's construction of the relationship-specificity measure. We show that the more important the relationship-specific investment in the determination of product quality, the thinner the intermediates market *in equilibrium*. We also establish the negative relationship between the preferable matching environment for downstream firms and the thickness of the intermediates market. Moreover, our model predicts that the higher the relationship specificity of the industry, the higher the extensive margin of firms that use customized inputs and the higher the intensive margin of their sales in equilibrium.

Then, we examine the impact of costly trade in both final goods and intermediate inputs on the matching environment, trade volumes and social welfare. In the open economy, downstream firms have to incur a fixed export cost as well as a variable trade cost in order to ship their final goods to the foreign country. In addition, we also allow downstream firms to search their potential upstream partners in the foreign country as well as the home country. If matched with foreign upstream firms, downstream firms are able to use customized inputs by importing such inputs from upstream partners subject to a variable import cost and search frictions between borders. Because of this feature, not only are final goods but also intermediate inputs are internationally traded between foreign-matched pairs in our search-and-matching setup.

We find that costly trade facilitates resource reallocation from unmatched firms, which produce low-quality products, to matched firms, which produce high-quality products, which in turn improves the average product quality in each country simultaneously. Besides this well-known Melitz (2003) effect, we also find that costly trade induces relatively more entry to the upstream sector than to the downstream sector, so that the matching environment improves in favor for downstream firms. That leads to an increase in the proportion of high-quality products relative to low-quality products, which entails an improvement of social welfare. That is, we identify another novel channel of favorable resource reallocation through a change in the matching environment. Costly trade benefits exporting firms (i.e., matched downstream firms in our model), while it leads to foreign firms' penetration to the domestic market and gives a negative impact on unmatched downstream firms that has no choice but to sell only in the domestic market. Though costly trade gives such a mixed impact on downstream firms, it gives an unambiguously positive impact on upstream firms, as their profits are associated with those of matched downstream firms through the profit sharing, and thus an increase in partners' profits by costly trade only benefits upstream firms. Consequently, costly trade benefits upstream firms relatively more than downstream firms, inviting relatively more entry to the upstream sector than the downstream sector.

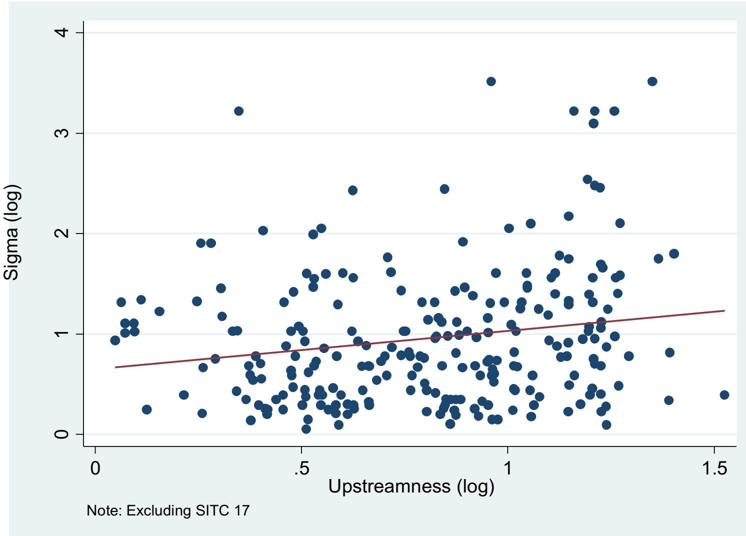


Figure 1 – Upstreamness and elasticity of substitution

Note: $\bar{R}^2 = 0.03$ and $n = 247$. We exclude the SITC 17 industry where the elasticity of substitution is 108 (the average elasticity is 4), and the slope coefficient is 0.38 (s.e. 0.143). Even if we include that industry, the slope coefficient is 0.339 (s.e. 0.148).

As for the trade volumes, we examine the impact of trade barriers on final goods and final goods. In our model of search and matching, trade liberalization refers not only to a reduction in trade costs but also to an improvement of search frictions. We find that any reductions in trade barriers induce a complementary effects on the trade volumes, which implies for example that a reduction in variable import costs increases not only intermediate-input trade but also final-good trade at the same time. Moreover, trade liberalization increases the trade volumes of intermediate inputs relatively more than those of final goods by facilitating search and matching between vertically-related pairs. This finding accords well with the empirical observations that intermediate-input trade has been growing faster than final-good trade under vertical disintegration. Our contribution is to shed new light on the role of trade in improving search-and-matched environments that are vital for offshoring.

We assume that the downstream sector is monopolistically competitive, but the upstream sector is in contrast perfectly competitive for generic inputs. Figure 1 provides suggestive evidence on this. In the figure, the horizontal axis measures the log of upstreamness, devised by Antràs et al. (2012), while the vertical axis measures the log of elasticity of substitution between varieties, estimated by Broda and Weinstein (2006). Since Antràs et al. (2012) and Broda and Weinstein (2006) respectively employ the U.S. I-O Table codes and the Standard International Trade Classification (SITC) codes, we use the publicly available concordance tables for the Harmonized System (HS) codes to make the product codes consistent between them. We find that there exists a statistically significant positive correlation between upstreamness and elasticity of substitution across 247 manufacturing industries: the degree of upstreamness is significantly higher, the higher the elasticity of substitution of industry. This suggests that, as our model presumes, the competition tends to be more fierce in the upstream sector than the downstream sector.

Our paper contributes to several strands of the recent literature of international trade. The first strand is firm heterogeneity and resource reallocation as a result of opening to trade. Our key novelty is that the firm distribution is endogenously determined through matching between upstream and downstream firms such that it varies across industries. This is worth stressing since recent empirical work unveils noticeable differences in the firm distribution across industries, which in turn affects the trade pattern (e.g., Helpman et al., 2004; Corcos et al., 2012). In our model, firm heterogeneity arises from the difference in the matching status of firms, and this varies systematically across industries depending on the relationship specificity of the industry. As a result of this, the model can explain heterogeneous impacts of trade liberalization in final goods and intermediates on industrial structures and social welfare in various industries. In addition, our model establishes a new sufficient statistic for welfare in trade models, namely the ratio of the mass of upstream firms to the mass of downstream firms, which plays a critical role in vertical specialization. We show that all the endogenous variables of the model are expressed as a function of the single sufficient statistic in our search-and-matching model. Although our sufficient statistic approach is parallel to that in Arkolakis et al. (2012), our new sufficient statistic is particularly informative when evaluating welfare impact of various policies for industries in which vertical linkages are prominent.

The second is firm exporting and importing. A growing body of firm-level evidence allows trade researchers to investigate various aspects of trade in intermediates as well as final goods. In particular, a series of work by Bernard et al. (2007, 2012, 2018a) reveal that importing firms exhibit many of the same features as exporting firms, e.g., only a small fraction of firms import and importers are more productive than non-importers.[2] Findings of such similarities between firm exporting and importing naturally lead to further investigation for whether trade liberalization in intermediates gives rise to a similar impact as trade liberalization in final goods. For instance, Kasahara and Lapham (2013) find that, because of import and export complementarities, policies that inhibit import of foreign intermediates can have large adverse effects on export of final goods (see also Bernard et al. (2018b) for the impact on vertical relationships). These papers however do not explicitly take account of the role of search and matching between downstream and upstream firms, which we believe is one of the most important aspects of offshoring or vertical specialization.

Finally, our model is closely related to the work that explores the interaction between downstream and upstream sectors in international trade models and examines the impact of trade liberalization on the market thickness; see McLaren (2000), Grossman and Helpman (2002, 2005), Ornelas and Turner (2008, 2012), and Alfaro et al. (2016). While these studies focus primarily on the impact of trade liberalization on the firm boundaries (i.e., firms' sourcing strategies between vertical integration and outsourcing) in incomplete-contract setups, we instead focus on how frictions in search and matching between firms from different production sectors endogenously characterize the market thickness of the industry, and its consequence to welfare. Moreover, our model is tractable enough to study the relationship between final-good trade and intermediate-input trade, which has not been analytically explored in the existing work.

[2] This evidence is also confirmed by Amiti and Konings (2007), Goldberg et al. (2010), Halpern et al. (2015), Kasahara and Lapham (2013), Kasahara and Rodrigue (2008), and Topalova and Khandelwal (2011) for different data from various countries.

2 Baseline Model

We develop a two-country, dynamic, general-equilibrium model in which firms in a downstream sector buy intermediates from those in an upstream sector to produce final goods in each industry. Upon paying relevant fixed entry costs, firms enter either the upstream or downstream sector, and search for their intermediate-input transaction partners at every instance. If firms are matched with their potential partner, they bargain over the profit sharing. Matched upstream firms make a relationship-specific investment to produce customized intermediates for matched downstream firms, if they have reached an agreement in the bargaining stage. In contrast, unmatched firms in the upstream and downstream sectors sell and buy generic intermediates in a competitive market, so that unmatched downstream firms can still produce their final products, albeit goods of lower quality due to the lack of customization of intermediate inputs. In this setting, we derive stationary equilibrium in autarky and then that in an open economy where both intermediates and final goods are traded between two countries. In the baseline model, we consider the case of symmetric countries, while we numerically analyze the case of asymmetric countries with respect to country size in Section 5.

2.1 Demand

There are multiple industries, indexed by $j \in \{1, 2, ..., J\}$, that produce differentiated final goods. The instantaneous preferences of a representative consumer in the economy are given by

$$U = \prod_{j=1}^{J} X_j^{\delta_j}, \qquad \sum_{j=1}^{J} \delta_j = 1, \ \delta_j > 0.$$

Within industry j, the consumer's preferences take the standard Dixit-Stiglitz form with constant elasticity of substitution (CES):

$$X_j = \left[\int_{\omega \in \Omega_j} \alpha_j(\omega)^{\frac{1}{\sigma_j}} x_j(\omega)^{\frac{\sigma_j - 1}{\sigma_j}} d\omega \right]^{\frac{\sigma_j}{\sigma_j - 1}}, \quad \sigma_j > 1,$$

where $\alpha_j(\omega)$ is the quality of variety ω in industry j, such that the greater is $\alpha_j(\omega)$, the higher is the quality and the larger is the demand for variety ω.

It follows from the upper-tier Cobb-Douglas preferences that the consumer allocates expenditure $E_j \equiv \delta_j \bar{E}$ to differentiated goods in industry j, where $\bar{E}$ represents aggregate expenditure. Moreover, it also follows from the lower-tier CES preferences that the consumer allocates sectoral expenditure E_j across varieties to maximize aggregate consumption X_j within industry j. As is well-known, this generates the following instantaneous demands for variety ω in industry j:

$$x_j(\omega) = E_j P_j^{\sigma_j - 1} \alpha_j(\omega) p_j(\omega)^{-\sigma_j}, \tag{1}$$

where

$$P_j = \left[\int_{\omega \in \Omega_j} \alpha_j(\omega) p_j(\omega)^{1-\sigma_j} d\omega \right]^{\frac{1}{1-\sigma_j}} \tag{2}$$

is the instantaneous price index associated with aggregate consumption X_j such that $P_j X_j = E_j$. In what follows, we focus on a representative firm in a particular industry at a time and hence drop a variety index ω and an industry subscript j from the variables, unless it is necessary to write them explicitly to avoid confusion.

2.2 Production

Every industry is composed of the upstream and downstream sectors, where the former specializes in producing intermediate inputs and the latter specializes in producing final goods. Intermediates are produced only from labor, which is a sole production factor in the economy. One unit of intermediates is produced from one unit of labor. Final goods are produced from intermediates and labor with the Cobb-Douglas production function, such that the unit cost of production is given by $w^{1-\kappa} q^{\kappa}$, where w and q denote a wage rate and an intermediate-input price, while $\kappa \in (0, 1)$ is a share parameter. The economy is endowed with $\bar{L}$ units of perfectly mobile labor across the upstream and downstream sectors, and we choose labor as a numeraire of the model so that $w = 1$.

There are two possible ways for downstream firms to procure intermediate inputs. One way is to buy generic intermediate inputs in a perfectly competitive market. Another way is to source inputs from partner upstream firms, which is possible only for firms that have been successfully matched. As will be described shortly, both upstream and downstream firms search for their respective partners. When they are matched, two firms negotiate how to split joint profits. If they agree, an upstream firm makes a one-time fixed investment K (measured in labor units) and provide customized intermediate inputs with which a partner downstream firm can produce final goods of high quality.

The quality of final goods is normalized to $\alpha = 1$ if a downstream firm uses generic intermediate inputs. The quality of final goods is α if that firm is matched with an upstream firm and instead uses customized intermediate inputs. The quality is a random variable which is drawn from a probability distribution $G(\alpha)$ with support in $[\alpha_{\min}, \infty)$ where an infimum of support of the distribution satisfies $\alpha_{\min} > 1$. The matching between the upstream and downstream firms can be quite successful if the needs on the side of the downstream firm are nicely matched with the strength of the upstream firm, or the corporate culture is similar between the two firms, allowing them to collaborate effectively. In such cases, the product quality α is expected to be a high value. However, these firms are not always that lucky to meet their best partners, in which cases α is likely to take a low value. The probability distribution $G(\alpha)$ also depends on some industry characteristics. For instance, industries might vary in the degree of customization in their core intermediate inputs. This difference is reflected by the difference in $\alpha_{\min}$ which varies across industries.

While n_e firms successfully find partner firms among M upstream firms and N downstream firms, whether the matching is successful depends on the level of quality. In particular, high-quality firms above $\alpha_d \in (\alpha_{\min}, \infty)$ can profitably make a fixed investment and use customized intermediate inputs. In contrast, low-quality firms below the cutoff have to dissolve their partnership and inevitably use generic intermediate inputs. Thus $n = [1 - G(\alpha_d)]n_e$ firms use customized inputs, while remaining $N - n$ firms use generic inputs in final-good production, due to either low quality or failure of match. All of these numbers are endogenously determined in the model.

Regardless of their matching status, downstream firms choose the price for individual final goods to maximize the respective instantaneous profits. Given the CES preferences, downstream firms set a common constant markup over the marginal cost, which is one for unmatched downstream firms, because the input price is $q = w = 1$. The marginal cost is also one for matched downstream firms, because matched downstream firms take the marginal cost for partner upstream firms as the input price when choosing the price for final goods to maximize the instantaneous joint profits. Thus both matched and unmatched firms charge a constant markup $\sigma/(\sigma-1)$ over the unit cost $c = w^{1-\kappa}q^{\kappa} = 1$. Noting that the quality of final goods is α and 1 for matched and unmatched firms respectively, the price index P in (2) is rewritten as

$$\begin{aligned} P^{1-\sigma} &= \left(\frac{\sigma c}{\sigma - 1}\right)^{1-\sigma} \left[n_e \int_{\alpha_d}^{\infty} \alpha dG(\gamma) + (N-n)\right] \\ &= \left(\frac{\sigma c}{\sigma - 1}\right)^{1-\sigma} \left[n\tilde{\alpha}_d + (N-n)\right], \end{aligned} \tag{3}$$

where

$$\tilde{\alpha}_d = \int_{\alpha_d}^{\infty} \alpha \frac{dG(\alpha)}{1 - G(\alpha_d)}$$

is the average product quality of matched firms.

Using (3), we obtain the instantaneous value of the profits. A matched firm with quality α and an unmatched downstream firm with quality 1 respectively earn the following profits at every instance:

$$\begin{aligned} \pi(\alpha) &= \frac{E\alpha}{\sigma}\left(\frac{\frac{\sigma c}{\sigma-1}}{P}\right)^{1-\sigma} = \frac{E\alpha}{\sigma(n\tilde{\alpha}_d + N - n)}, \\ \pi(1) &= \frac{E}{\sigma}\left(\frac{\frac{\sigma c}{\sigma-1}}{P}\right)^{1-\sigma} = \frac{E}{\sigma(n\tilde{\alpha}_d + N - n)}, \end{aligned} \tag{4}$$

which indicates that the ratio of the profits for firms is captured only by the quality parameter:

$$\frac{\pi(\alpha)}{\pi(1)} = \alpha. \tag{5}$$ [3]

Let k denote the instantaneous value of the fixed investment. We then define the cutoff value α_d at which matched firms are indifferent about whether or not to use customized inputs:

$$\pi(\alpha_d) - \pi(1) = k,$$

where the left-hand side is positive as $\alpha_d > 1$. Using (5), the cutoff value is explicitly solved for

$$\alpha_d = \frac{\pi(1) + k}{\pi(1)}.$$

[3] The results of our analysis remain qualitatively the same even if we consider a type of the fixed investment that lowers the unit cost (rather than raising the product quality). Suppose that the unit cost of final-good production is reduced to $\alpha^{\frac{1}{1-\sigma}} (< 1)$ for firms that use customized inputs, while keeping the product quality the same for all firms. With this specification, we can show that (4) and (5) remain the same as before.

From this cutoff, we define the average profits of matched firms:

$$\begin{aligned}\tilde{\pi} &= \int_{\alpha_d}^{\infty} \pi(\alpha) \frac{dG(\alpha)}{1-G(\alpha_d)} \\ &= \int_{\alpha_d(\pi(1))}^{\infty} \alpha\pi(1) \frac{dG(\alpha)}{1-G(\alpha_d)} \\ &= \tilde{\alpha}_d \pi(1).\end{aligned} \tag{6}$$

2.3 Search and matching

We assume free entry to both the upstream and downstream sectors in every industry. Entrants to the upstream and downstream sectors respectively incur one-time fixed entry costs, F^U, F^D (measured in labor units), and then enter the respective unmatched pool. At every instance, one-to-one matching occurs randomly between upstream and downstream firms. In the meantime, the fraction λ of firms are randomly chosen to go bankrupt either in the sectors, regardless of their matching status.

Search technology denoted by $\nu(M-n, N-n)$ assigns the number of newly matched pairs from the number of unmatched upstream firms $M-n$ and that of unmatched downstream firms $N-n$. Following the existing literature (Grossman and Helpman, 2002), we assume that search technology is characterized by an increasing and concave function of homogeneous of degree 1. These properties of search technology implies complementarity or supermodularity in matching, i.e., $\frac{\partial^2 \nu(M-n,N-n)}{\partial(M-n)\partial(N-n)} > 0$. As originally demonstrated by Shimer and Smith (2000), supermodular functions play a key role in search-and-matching environments.

Due to constant-returns-to-scale search technology, the hazard rates of matching for downstream and upstream firms are respectively given by

$$\begin{aligned}\breve{\mu}^D &\equiv \frac{\nu(M-n, N-n)}{N-n} = \nu\left(\frac{M-n}{N-n}, 1\right), \\ \breve{\mu}^U &\equiv \frac{\nu(M-n, N-n)}{M-n} = \frac{N-n}{M-n}\nu\left(\frac{M-n}{N-n}, 1\right).\end{aligned} \tag{7}$$

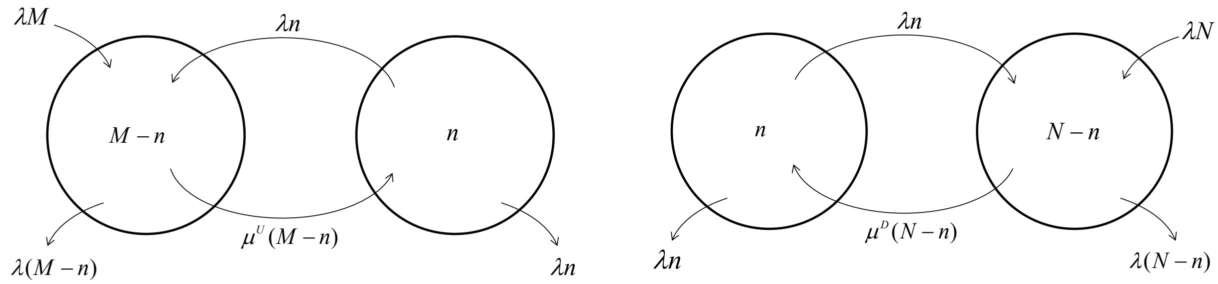
Let $z \equiv (M-n)/(N-n)$ denote the ratio of the number of unmatched upstream firms to that of unmatched downstream firms. Then the hazard rates in (7) are expressed in terms of z only:

$$\breve{\mu}^D = \breve{s}(z), \quad \breve{\mu}^U = \frac{\breve{s}(z)}{z}.$$

From the properties of search technology, it directly follows that the hazard rate for downstream firms $\breve{\mu}^D = \breve{s}(z)$ is increasing and concave in z, while that for upstream firms $\breve{\mu}^U = \breve{s}(z)/z$ is decreasing and convex in z. Further, recall that only high-quality firms above the cutoff α_d can use customized inputs. It is thus useful to define the hazard rates of using customized inputs among matched firms:

$$\mu^D \equiv [1-G(\alpha_d)]\breve{\mu}^D = s(z,\pi), \quad \mu^U \equiv [1-G(\alpha_d)]\breve{\mu}^U = \frac{s(z,\pi)}{z}, \tag{8}$$

where we notice the fact that α_d is a function of $\pi = \pi(1)$. These hazard rates are assumed to exhibit the same features as $\breve{\mu}^D$ and $\breve{\mu}^U$.



(a) Upstream sector (b) Downstream sector

Figure 2 – Search processes

We explore a simple dynamic model in which matching, dissolving, and exiting of firms occur stochastically over the continuous time. Figure 2 illustrates the search processes in the two sectors. Consider first the upstream sector in panel (a). At every instance, the fraction $\breve{\mu}^U$ of $M-n$ unmatched firms find their partner, but only high-quality firms above α_d can sustain the partnership, so that $\mu^U(M-n)$ firms enter the matched pool. Every firm is also faced with an exogenous shock at a hazard rate of λ, such that firms that are hit by the shock go bankrupt. Since a fraction λ of $M-n$ unmatched firms are hit by this shock, $\lambda(M-n)$ firms exit the unmatched pool and leave the market. Matched firms are also faced with the shock, so that, at every instance, λn matched upstream firms exit from the market as they themselves are hit by the shock; at the same time, λn matched upstream firms exit from the matched pool and enter the unmatched pool as their downstream partners are hit by the shock and hence the partnership has to be dissolved. Altogether, λM firms go bankrupt and the same number of firms enter the market in the stationary equilibrium. The same search process applies to the downstream sector as shown in panel (b).

We only consider the stationary equilibrium in which all the endogenous variables (including the number of matched and unmatched firms) remain constant over time. This implies in particular that the number of firms that enter the matched pool is equal to the number of firms that dissolve the partnership and leave the market at every instance. Noting from Figure 2 that $2\lambda n$ matched firms exit the matching pool at every instance, we have

$$\begin{cases} \mu^U(M-n) = 2\lambda n, \\ \mu^D(N-n) = 2\lambda n. \end{cases}$$

Solving the above steady-state relationships for n yields

$$n = \left(\frac{\mu^U}{\mu^U + 2\lambda}\right) M = \left(\frac{\mu^D}{\mu^D + 2\lambda}\right) N. \tag{9}$$

Equation (9) describes how the number of matched firms n in each sector is tied to the hazard rates of matching, μ^U, μ^D, and the total numbers of firms, M, N, in the dynamics where all the variables in (9) are endogenously determined.

3 Closed-economy equilibrium

This section considers a closed-economy version of our model to derive some key features that arise in equilibrium. Then the model will be extended to an open-economy version in the next section to examine the impact of intermediate-input trade as well as final-good trade.

3.1 Equilibrium conditions

We first derive the equilibrium value functions for downstream and upstream firms among different matching status, taking account of bargaining within matched pairs over the joint surplus to be split. Then we characterize the equilibrium with free entry.

Let $\tilde{V}^D$ and V^D denote the average values of matched and unmatched firms in the downstream sector, and let $\tilde{V}^U$ and V^U denote those of matched and unmatched firms in the upstream sector. These average values must satisfy the following no-arbitrage conditions:

$$\begin{aligned} r\tilde{V}^D &= \tilde{\pi}^D - \lambda\left(\tilde{V}^D - V^D\right) - \lambda\tilde{V}^D + \dot{\tilde{V}}^D, \\ rV^D &= \pi + \mu^D\left(\tilde{V}^D - V^D\right) - \lambda V^D + \dot{V}^D, \\ r\tilde{V}^U &= \tilde{\pi}^U - \lambda\left(\tilde{V}^U - V^U\right) - \lambda\tilde{V}^U + \dot{\tilde{V}}^U, \\ rV^U &= \mu^U\left(\tilde{V}^U - K - V^U\right) - \lambda V^U + \dot{V}^U, \end{aligned}$$

where r is a common discount rate, while $\pi = \pi(1)$ in (4) and $\tilde{\pi}^D + \tilde{\pi}^U = \tilde{\pi}$ in (6). The system of these four equations represents how matched and unmatched firms gain and lose from matching, dissolving and exiting in the dynamics. The first equation, for example, shows that matched downstream firms have (i) instantaneous profits of $\tilde{\pi}^D$; (ii) losses of $\tilde{V}^D - V^D$ from dissolving the relationship, which occurs at a hazard rate of λ; (iii) losses of $\tilde{V}^D$ from exiting the market, which occurs at a hazard rate of λ; and (iv) capital gains of $\dot{\tilde{V}}^D$ from remaining matched. While the other equations are similarly interpreted, the last equation also shows that unmatched upstream firms make the one-time fixed investment K when matched with their downstream partners.

There are no capital gains in the stationary equilibrium so that $\dot{\tilde{V}}^D = \dot{V}^D = \dot{\tilde{V}}^U = \dot{V}^U = 0$. For simplicity, we assume that the discount rate is zero ($r = 0$).[4] Solving the no-arbitrage conditions for $\tilde{V}^D$ and V^D simultaneously, we obtain the value functions of downstream firms:

$$\begin{aligned} \tilde{V}^D &= \frac{\pi}{\lambda} + \left(\frac{\mu^D + \lambda}{\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda}\right), \\ V^D &= \frac{\pi}{\lambda} + \left(\frac{\mu^D}{\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda}\right). \end{aligned} \tag{10}$$

Because the hazard rate λ works as a discount rate in our dynamic model, $\frac{\tilde{\pi}^D}{\lambda}$ and $\frac{\pi}{\lambda}$ in (10) represent the present values of profit flows of matched and unmatched downstream firms respectively.

[4] The assumption would not qualitatively affect our results. Under the assumption, aggregate profit equals aggregate sunk cost paid in individual industries at each instance, so that aggregate expenditure equals aggregate labor income, simplifying the general-equilibrium analysis. Melitz (2003) also makes the same assumption in his dynamic model.

Similarly, the value functions of upstream firms can be rewritten as

$$\begin{aligned} \tilde{V}^U - K &= \left(\frac{\mu^U + \lambda}{\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right), \\ V^U &= \left(\frac{\mu^U}{\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right), \end{aligned} \tag{11}$$

where K is subtracted from $\tilde{V}^U$ so as to represent the *net* expected present value of matched upstream firms. Note that $2K$ is subtracted from $\frac{\tilde{\pi}^U}{\lambda}$ because the value that is lost when moving out of the matched pool is greater than $\tilde{V}^U - K$ by K, and this happens in two occasions (i.e., when an upstream firm goes bankrupt and its downstream partner is forced to exit the market).

To derive the instantaneous profits for individual matched firms, we next consider bargaining over profit sharing within matched pairs. The outcome of this bargaining is characterized as a symmetric Nash bargaining solution in which downstream and upstream firms have the same bargaining power over ex-post gains from the relationship. Disagreement would force the pairs to be dissolved, so that the threat point of the Nash bargaining is (V^D, V^U). Let $\tilde{\pi}^{D'}$ and $\tilde{\pi}^{U'}$ denote the respective parties' profits to be determined by the bargaining, and let $\tilde{V}^{D'}$ and $\tilde{V}^{U'}$ denote the corresponding values in (10) and (11). Then the equilibrium profits for matched firms are given by

$$\begin{aligned} (\tilde{\pi}^D, \tilde{\pi}^U) \; &\in \; \arg \max_{\tilde{\pi}^{D'}, \tilde{\pi}^{U'}} \; \left(\tilde{V}^{D'} - V^D\right)\left(\tilde{V}^{U'} - K - V^U\right), \\ &\text{s.t.} \quad \tilde{\pi}^{D'} + \tilde{\pi}^{U'} = \tilde{\pi}. \end{aligned}$$

Using (10) and (11), the solution to this problem gives us the optimal profit sharing rule:

$$\begin{aligned} \frac{\tilde{\pi}^D}{\lambda} &= \frac{\pi}{\lambda} + \beta\left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K\right), \\ \frac{\tilde{\pi}^U}{\lambda} - K &= (1-\beta)\left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K\right), \end{aligned} \tag{12}$$

where

$$\beta \equiv \frac{\mu^D + 2\lambda}{\mu^D + \mu^U + 4\lambda} \tag{13}$$

is the effective bargaining power of downstream firms.[5] In this model, $\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K$ can be thought of as the present value of economic rents generated by the relationship. Thus (12) indicates that the equilibrium net profits for matched firms equal the outside option plus the distribution of the rents, weighted by the effective bargaining power. Substituting (12) into (10) and (11), we have $\tilde{V}^D > V^D$ and $\tilde{V}^U - K > V^U$ if and only if $\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K > 0$, which is our focus in the analysis below.

[5] Even though matched downstream and upstream firms have the same bargaining power, the effective bargaining power β is not one-half and is affected by the number of firms in each production sector. From the firm dynamics (9),

$$\beta \gtreqless \frac{1}{2} \iff \mu^D \gtreqless \mu^U \iff M \gtreqless N.$$

In addition, from the properties of search technology, β is increasing in $z = (M-n)/(N-n)$. Thus, β increases as the upstream sector becomes relatively thicker, while it decreases as the downstream sector becomes relatively thicker.

In the stationary equilibrium, free entry implies that the expected value of an entering firm into the unmatched pool must equal the fixed entry cost in each sector:

$$V^D = F^D, \quad V^U = F^U.$$

Using (9), (10), (11) and (12), we can rewrite them in terms of the instantaneous values:

$$\begin{aligned} \pi + \frac{n}{N}\beta\left(\tilde{\pi} - \pi - k\right) &= f^D, \\ \frac{n}{M}(1-\beta)\left(\tilde{\pi} - \pi - k\right) &= f^U, \end{aligned} \tag{14}$$

where $f^D \equiv \lambda F^D$, $f^U \equiv \lambda F^U$ and $k \equiv 2\lambda K$ represent the instantaneous values of the fixed entry costs and fixed investment, respectively. A fraction $\frac{n}{N}$ of matched downstream firms earn the instantaneous rents of $\beta(\tilde{\pi} - \pi - k)$ in addition to the instantaneous profit of π. Under free entry, the expected instantaneous profits must equal the instantaneous value of entry cost f^D. The similar interpretation applies to upstream firms, except that they earn nothing when being unmatched.

Together with search technology in (7), the free entry conditions in (14) simultaneously determine the total numbers of firms in the two sectors M, N as well as the number of matched pairs n. Since $z = (M-n)/(N-n)$ is endogenous, the hazard rates of matching μ^D, μ^U are endogenous. Moreover, the price index is also endogenously determined so as to achieve the zero expected profits, which in turn implies that the cutoff quality α_d, average quality $\tilde{\alpha}_d$, and average profit $\tilde{\pi}$ are also endogenous. Finally, aggregate profit equals aggregate sunk cost under free entry, and hence aggregate expenditure equals aggregate labor income $E = \delta\bar{L}$ in the stationary equilibrium (see Appendix).

3.2 Equilibrium characterizations

Having described the equilibrium conditions, we now characterize the closed-economy equilibrium. Though the equilibrium characterizations so far apply for a general distribution $G(\alpha)$, we will restrict our attention to a Pareto distribution in the following analysis:

$$G(\alpha) = 1 - \left(\frac{\alpha_{\min}}{\alpha}\right)^{\gamma},$$

where $\alpha \geq \alpha_{\min} > 0$ and $\gamma > \sigma - 1$. Given this distributional assumption, the average quality $\tilde{\alpha}_d$ and the average profits of matched firms $\tilde{\pi}$ are explicitly expressed as a function of the profits of unmatched downstream firms π:

$$\begin{aligned} \tilde{\alpha}_d &= \frac{\gamma}{\gamma - 1}\left(\frac{\pi + k}{\pi}\right), \\ \tilde{\pi} &= \frac{\gamma}{\gamma - 1}\left(\pi + k\right). \end{aligned} \tag{15}$$

As a result, the economic rents are always positive:

$$\tilde{\pi} - \pi - k = \frac{\pi + k}{\gamma - 1}.$$

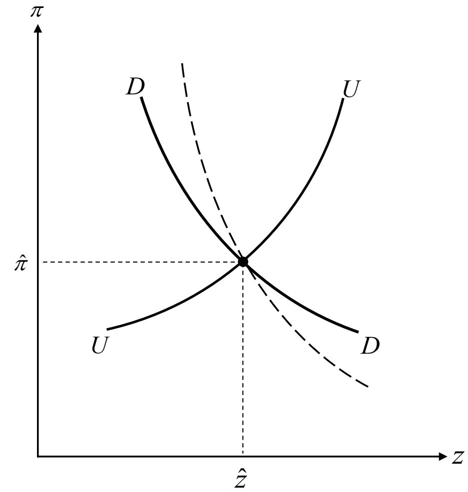


Figure 3 – Equilibrium in the closed economy

The equilibrium is characterized by the numbers of firms M, N, n which are pinned down by search technology (7) and the free entry conditions (14). We find it more convenient however to work with the ratio of unmatched firms $z = (M - n)/(N - n)$ and the profits of unmatched downstream firms π. Using (5), (8), (9), and (13), we can express (14) as a function of z and π:

$$(\gamma - 1)\pi + \phi^D(z, \pi)(\pi + k) = (\gamma - 1)f^D, \tag{16}$$

$$\phi^U(z, \pi)(\pi + k) = (\gamma - 1)f^U, \tag{17}$$

where

$$\phi^D(z, \pi) \equiv \frac{n}{N}\beta = \frac{zs(z, \pi)}{s(z, \pi) + zs(z, \pi) + 4\lambda z}, \quad \phi^U(z, \pi) \equiv \frac{n}{M}(1 - \beta) = \frac{s(z, \pi)}{s(z, \pi) + zs(z, \pi) + 4\lambda z},$$

which can be thought of as weighted effective bargaining power of the respective firms. Since $s(z, \pi)$ is an increasing function of z, we have $\frac{\partial \phi^D(z,\pi)}{\partial z} > 0$ and $\frac{\partial \phi^U(z,\pi)}{\partial z} < 0$ for any z.

Figure 3 illustrates the relationship between z and π depicted in equations (16) and (17), labeled as DD and UU, respectively. The DD curve is downward-sloping, whereas the UU curve is upward-sloping. For downstream firms, the greater z implies the higher probability of a match $\mu^D = s(z, \pi)$ and hence the higher expected profits. This induces entry into the downstream sector, which drives down the ex-post profits of downstream firms, offsetting the initial increase in the expected profits. Thus, the DD curve is downward-sloping. It is clear that the opposite is true for upstream firms. These features of the DD and UU curves ensure the existence and uniqueness of the closed-economy equilibrium where the intersection of the two curves determines the two endogenous variables $\{\hat{z}, \hat{\pi}\}$ as illustrated in Figure 3.

Once these two endogenous variables are determined, other endogenous variables can be written as a function of $\hat{z}$ only by noticing that $\hat{\pi} = \pi(\hat{z})$. The hazard rates of matching are $\mu^D = s(\hat{z}, \pi(\hat{z}))$ and $\mu^U = s(\hat{z}, \pi(\hat{z}))/\hat{z}$, which are expressed as $\mu^D = s(\hat{z})$ and $\mu^U = s(\hat{z})/\hat{z}$ below for simplicity. Similarly, the average quality $\tilde{\alpha}_d$ and average profits $\tilde{\pi}$ of matched pairs are expressed in terms of $\hat{\pi}$

and hence $\hat{z}$ (see (15)). We also calculate the number of matched pairs n from (4) using $E = \delta\bar{L}$ and $\frac{N-n}{n} = \frac{2\lambda}{s(\hat{z})}$ (which can be derived from (9)):

$$n = \left[\frac{s(\hat{z})}{\sigma\pi(\hat{z})(\tilde{\alpha}_d s(\hat{z}) + 2\lambda)}\right]\delta\bar{L}. \tag{18}$$

From (7), (9) and (18), it follows immediately that the total numbers of downstream and upstream firms are respectively given by

$$M = \left[\frac{s(\hat{z}) + 2\lambda\hat{z}}{\sigma\pi(\hat{z})(\tilde{\alpha}_d s(\hat{z}) + 2\lambda)}\right]\delta\bar{L}, \quad N = \left[\frac{s(\hat{z}) + 2\lambda}{\sigma\pi(\hat{z})(\tilde{\alpha}_d s(\hat{z}) + 2\lambda)}\right]\delta\bar{L}. \tag{19}$$

Moreover, using (4) and $E = \delta\bar{L}$, the price index P can be written as

$$P = \frac{\sigma c}{\sigma - 1}\left(\frac{\sigma\pi(\hat{z})}{\delta\bar{L}}\right)^{\frac{1}{\sigma-1}}. \tag{20}$$

Finally, social welfare in this economy is defined by

$$W = \prod_{j=1}^{J} P_j^{-\delta_j}, \tag{21}$$

which is proportional to the representative consumer's utility U. Since the price index in (20) depends solely on $\hat{z}$ through its impact on the profits of unmatched firms, social welfare in (21) implies that $\hat{z}$ is a sufficient statistic for welfare in our model.[6] Thus, any changes in welfare can be inferred from changes in $\hat{z}$. This completes the characterization of the unique closed-economy equilibrium.

It is important to emphasize that market size $\delta\bar{L}$ has no effect on the key endogenous variable $\hat{z}$. The free entry conditions (16) and (17) do not involve $\delta\bar{L}$, so the intersection of DD and UU curves is independent of $\delta\bar{L}$. Intuitively, when $\delta\bar{L}$ increases, the numbers of upstream and downstream firms both increase proportionately, and so do matched firms under the constant-returns-to-scale matching technology. Since M, N, and n grow proportionately with $\delta\bar{L}$, $z = (M - n)/(N - n)$ remains fixed. The equilibrium profits for unmatched firms, $\hat{\pi} = \pi(\hat{z})$, also remain fixed because an increase in the market expenditure is exactly offset by a fall in the price index caused by new entry (see (1) and (3)). Despite that an increase in $\delta\bar{L}$ does not affect the equilibrium values of these endogenous variables, it contributes to the welfare gains since the number of varieties increases and hence the price index falls with $\delta\bar{L}$ (see (20) and (21)).

3.3 Comparative statics

Building on the above equilibrium characterizations, we examine how aggregate outputs produced by matched and unmatched firms vary with industry characteristics. In particular, we are interested in the cross-industry difference in $\alpha_{\min}$ and γ.

[6] This "sufficient statistic" approach is reminiscent of that in Arkolakis et al. (2012). Our sufficient statistic differs from theirs, however, since we spotlight search and matching in vertical specialization which are absent in their models. The feature of our model plays a key role in evaluating the welfare gains from trade in the next section.

Let us first consider the impact of $\alpha_{\min}$. As $\alpha_{\min}$ rises and the degree of customization is higher, the fraction that matched firms use customized inputs $1 - G(\alpha_d) = \alpha_{\min}^{\gamma}\alpha_d^{-\gamma}$ and the hazard rates of matching $\mu^D = [1 - G(\alpha_d)]\check{\mu}^D$ and $\mu^U = [1 - G(\alpha_d)]\check{\mu}^U$ are higher. Since this induces more firms to be matched with potential partners, both $\phi^D(z, \pi)$ and $\phi^U(z, \pi)$ go up in an industry featured with a large $\alpha_{\min}$. Simple inspection of (16) and (17) reveals that the DD and UU curves shift down as a result, which leads to a fall in $\hat{\pi}$. This in turn raises $\hat{z}$, because we have the following equilibrium relationship derived from canceling out $\tilde{\pi} - \pi - k$ from (14) using (8), (9), and (13):

$$\hat{\pi} = f^D - f^U \hat{z}. \tag{22}$$

The relationship between $\hat{z}$ and $\hat{\pi}$ in (22) is illustrated by the dotted curve in Figure 3. This suggests that an industry with a larger $\alpha_{\min}$ is associated with a higher $\hat{z}$ and a lower $\hat{\pi}$. The key to this result lies in a differential impact of an increase in $\alpha_{\min}$ on the upstream and downstream sectors. For both upstream and downstream firms, the resulting increase in economic rents, $\tilde{\pi} - \pi - k = (\pi - k)/(\gamma - 1)$ contributes to greater expected profits, inducing further entry to both production sectors. Increases in the numbers of upstream and downstream firms give rise to downward shifts of the UU and DD curves. However, this effect is not of the same magnitude and is weaker in the downstream sector, since the profits of unmatched *downstream* firms π decrease, which dampens their entry incentive, while the profits of unmatched *upstream* firms remain the same. Consequently, the UU curve shifts down more than the DD curve does, thereby decreasing $\hat{\pi}$ and increasing $\hat{z}$. Though we have focused on an increase in $\alpha_{\min}$, a similar argument applies to a decrease in γ: an industry with a lower γ – which implies a greater dispersion of product quality – is associated with a higher $\hat{z}$ and a lower $\hat{\pi}$. These imply that the upstream sector is relatively thicker than in the downstream sector (i.e., $\hat{z}$ is greater), the higher the degree of input customization of the industry.

Our model also allows us to examine how market thickness varies with industry characteristics. Following Nunn (2007), we consider input markets as being thick if more inputs are traded through market transactions relative to non-market transactions. In our framework, generic inputs are traded among unmatched firms through markets, while customized inputs are traded among matched firms through search and matching. It thus suffices to calculate the aggregate output values produced by matched and unmatched firms. From the above equilibrium characterizations, the aggregate values of these transactions are respectively given by

$$n_e \int_{\alpha_d}^{\infty} \tilde{p} x(\alpha) dG(\alpha) = n\tilde{p}\tilde{x} = \left(\frac{\tilde{\alpha}_d}{\tilde{\alpha}_d + \frac{2\lambda}{s(\hat{z})}} \right) \delta \bar{L},$$

$$(N - n)px = \left(\frac{\frac{2\lambda}{s(\hat{z})}}{\tilde{\alpha}_d + \frac{2\lambda}{s(\hat{z})}} \right) \delta \bar{L}.$$

where $n\tilde{p}\tilde{x} + (N - n)px = \delta \bar{L} (= E)$ and

$$\tilde{x} = \int_{\alpha_d}^{\infty} x(\alpha) \frac{dG(\alpha)}{1 - G(\alpha_d)} = \tilde{\alpha}_d x$$

is the average output level by matched firms. In our model, input markets are thicker if the aggregate value of generic inputs $(N-n)px$ is higher relative to the aggregate value of customized inputs $n\tilde{p}\tilde{x}$. In this paper, as in Nunn (2007), we regard the ratio between these two aggregate values as a possible proxy to measure the market thickness:

$$\frac{(N-n)px}{n\tilde{p}\tilde{x}} = \frac{2\lambda}{\tilde{\alpha}_d s(\hat{z})}. \tag{23}$$

This ratio decreases with $\alpha_{\min}$ and γ since both $\hat{z}$ (hence $s(\hat{z})$) and $\tilde{\alpha}_d$ increases with these variables. Thus, we can show, as an equilibrium outcome, that the higher the degree of input customization, the thinner the input market. We can also examine the impact of $\alpha_{\min}$ or γ on the market thickness through the effects on the extensive margin (i.e., the number of firms) and intensive margin (i.e., the average output value per firm). The effect on the extensive margin can be measured by a change in $(N-n)/n$. From (18) and (19), the relative extensive margin of the market transaction is

$$\frac{N-n}{n} = \frac{2\lambda}{s(\hat{z})}. \tag{24}$$

Since $\hat{z}$ increases with $\alpha_{\min}$ or γ and $s(z)$ is an increasing function of z, the input market is thinner at the extensive margin, the higher is the degree of input customization. As for the intensive margin, it follows from $px = \sigma\pi$ and $\tilde{p}\tilde{x} = \tilde{\alpha}_d\sigma\pi$ that

$$\frac{px}{\tilde{p}\tilde{x}} = \frac{1}{\tilde{\alpha}_d}, \tag{25}$$

which implies that the market is thinner also at the intensive margin, the higher is the degree of input customization. Therefore, the input market is thinner in an industry featured with the higher degree of input customization at both margins. This finding is summarized in the following proposition.

Proposition 1 *The higher the degree of input customization (either in a higher $\alpha_{\min}$ or a lower γ) of the industry, the smaller the market transaction is relative to the bilateral transaction within the vertically related pairs.*

Our model provides a theoretical foundation of Nunn's (2007), associating the market thickness to the degree of input customization ("relationship specificity" in his terminology). Nunn measures the proportion of intermediate inputs sold in markets to those traded in other non-market mechanisms across a variety of industries, and use this measure of the market thickness to define the relationship specificity. However, he does not explain why this measure varies with industries. Our model shows that the market thickness is endogenously decreasing in the relationship specificity, identifying the underlying mechanism through which this measure varies with relationship specificity of the industry, namely, search and matching between vertically related pairs.

The result can be stated alternatively in terms of cross-industry distributional differences. While we have focused on comparative statics with respect to $\alpha_{\min}$ and γ in Proposition 1, the following lemma is related to differences in $G(\alpha)$.

Lemma 1 *Suppose that industry 1 is the first-stochastic dominance over industry 2,* $G_1(\alpha) < G_2(\alpha)$ *for* $\forall\alpha \in (\alpha_{\min}, \infty)$*, so that industry 1 is more relationship-specific than industry 2. Then, we have:*

(i) $\hat{z}_1 > \hat{z}_2$;

(ii) $\tilde{\alpha}_d(\hat{z}_1) > \tilde{\alpha}_d(\hat{z}_2)$;

(iii) $n_1\tilde{p}_1\tilde{x}_1 > n_2\tilde{p}_2\tilde{x}_2$;

(iv) $(N_1 - n_1)p_1x_1 < (N_2 - n_2)p_2x_2$.

Lemma 1 says that an industry with the higher relationship specificity endogenously entails the higher average quality and thereby more (less) final goods produced by customized (generic) inputs. As a result of these composition effects, the price index is lower, the higher the relationship specificity of the industry (see (20)).

We have so far characterized the closed-economy equilibrium and uncovered some of its important properties. Our main interest of this paper, however, is to understand whether an increase in final-good trade stimulates intermediate-input trade and vice versa, a key question that will be addressed in the next section. One may think that intermediate-input trade would naturally stimulates final-good trade. This is not always the case and it is possible that intermediate-input trade substitutes final-good trade by enabling countries to produce final goods for their own sake with imported inputs.

4 Open-economy equilibrium

This section explores a global economy in which two symmetric countries previously described engage in international trade. We allow upstream firms to export intermediate inputs whereas downstream firms to import these inputs and export final goods in order to discuss the complementarity between trade in final goods and trade in intermediate inputs.

Recall that country size $\delta\bar{L}$ has no impact on the key endogenous variables $\{\hat{z}, \hat{\pi}\}$. If firms incur no trade costs, opening to trade has exactly the same impact as an increase in country size, without affecting the two endogenous variables. As a result, countries gain from trade solely from a decline in the price index caused by an increase in varieties in consumption (see Section 3.2). If firms incur trade costs, in contrast, both final-good trade and intermediate-input trade have a critical impact on $\hat{z}$ and $\hat{\pi}$ in such a way as to reinforce the welfare gains from trade.

4.1 Production, exporting, and importing

Exporting firms incur an iceberg transport cost τ_x and a one-time fixed export cost F_x (both measured in labor units). Similarly, importing firms incur an iceberg transport cost of τ_m (measured in labor units) but they do not incur a fixed import cost. This treatment follows some of the existing literature that focuses on search and matching (e.g., Bernard et al., 2018b), which assumes that exporting firms incur a match-specific fixed cost for matched partners, while importing firms are sorted by a matching function without incurring such a fixed cost. This assumption, however, does not critically affect our result, so long as exporting firms incur a fixed export cost.

In the open economy where intermediate inputs are traded across borders, there are two types of matched firms: *domestic-matched* firms who use domestic inputs and *foreign-matched* firms who use foreign inputs. While n_e firms successfully find partner firms, high-quality firms above $\alpha_x \in (\alpha_d, \infty)$ can profitably incur a fixed export cost and export their final goods. In contrast, low-quality firms below the cutoff have to inevitably sell only in the domestic market. Thus the number of domestic-matched firms operating in the the foreign market is $n_x = [1 - G(\alpha_x)]n_e$, and that operating in the domestic market is $n = [1 - G(\alpha_d)]n_e$. Similarly, these numbers of foreign-matched firms are given by $n_x^* = \theta[1 - G(\alpha_x^*)]n_e$ and $n^* = \theta[1 - G(\alpha_d^*)]n_e$ respectively, where $\theta \in (0, 1)$ measures the difficulty achieving a foreign match relative to a domestic match.

As before, all types of firms charge a constant markup $\sigma/(\sigma - 1)$ over the unit cost. Since foreign-matched firms incur τ_m to procure foreign inputs, the unit cost of final-good production and hence the pricing rule of foreign-matched firms are τ_m^κ times higher than that of domestic-matched firms under our Cobb-Douglas production assumption. As usual, the pricing rule of exporting firms is τ_x times higher than that of non-exporting firms. Noting that the product quality is α for domestic-matched and foreign-matched firms and 1 for unmatched firms, the price index is expressed as

$$\begin{aligned} P^{1-\sigma} &= \left(\frac{\sigma c}{\sigma - 1}\right)^{1-\sigma} \left[n_e\left(\int_{\alpha_d}^{\infty} \alpha dG(\alpha) + \int_{\alpha_x}^{\infty} \tau_x^{1-\sigma} \alpha dG(\alpha)\right) + n_e \tau_m^{\kappa(1-\sigma)} \left(\int_{\alpha_d^*}^{\infty} \alpha dG(\alpha) + \int_{\alpha_x^*}^{\infty} \tau_x^{1-\sigma} \alpha dG(\alpha)\right) + (N - n - n^*)\right] \\ &= \left(\frac{\sigma c}{\sigma - 1}\right)^{1-\sigma} \left[n(\tilde{\alpha}_d + \chi \tau_x^{1-\sigma} \tilde{\alpha}_x) + n^* \tau_m^{\kappa(1-\sigma)} (\tilde{\alpha}_d^* + \chi^* \tau_x^{1-\sigma} \tilde{\alpha}_x^*) + (N - n - n^*)\right], \end{aligned} \tag{26}$$

where

$$\chi = \frac{1 - G(\alpha_x)}{1 - G(\alpha_d)}, \quad \chi^* = \frac{1 - G(\alpha_x^*)}{1 - G(\alpha_d^*)},$$

are the proportion of exporting firms among domestic-matched firms and foreign-matched firms, and

$$\begin{aligned} \tilde{\alpha}_d &= \int_{\alpha_d}^{\infty} \alpha \frac{dG(\alpha)}{1 - G(\alpha_d)}, \quad \tilde{\alpha}_x = \int_{\alpha_x}^{\infty} \alpha \frac{dG(\alpha)}{1 - G(\alpha_x)} \\ \tilde{\alpha}_d^* &= \int_{\alpha_d^*}^{\infty} \alpha \frac{dG(\alpha)}{1 - G(\alpha_d^*)}, \quad \tilde{\alpha}_x^* = \int_{\alpha_x^*}^{\infty} \alpha \frac{dG(\alpha)}{1 - G(\alpha_x^*)} \end{aligned}$$

are the average product quality among each type of matched firms.

From the pricing rules, we can immediately obtain the instantaneous profits. Denoting the profits of unmatched firms as $\pi(1) = \pi$, the profits of domestic-matched firms and foreign-matched firms with quality α are respectively expressed as

$$\pi_d(\alpha) = \alpha\pi, \quad \pi_x(\alpha) = \tau_x^{1-\sigma}\alpha\pi, \quad \pi_d^*(\alpha) = \tau_m^{\kappa(1-\sigma)}\alpha\pi, \quad \pi_x^*(\alpha) = \tau_x^{1-\sigma}\tau_m^{\kappa(1-\sigma)}\alpha\pi.$$

Since (5) similarly holds in the open economy, we define the cutoff values $\alpha_d, \alpha_x, \alpha_d^*, \alpha_x^*$ as $\pi_d(\alpha_d) - \pi(1) = k$, $\pi_x(\alpha_x) = f_x$, $\pi_d^*(\alpha_d^*) - \pi(1) = k$, $\pi_x^*(\alpha_x^*) = f_x$ respectively. From the instantaneous profits, the cutoff values can be explicitly solved as follows:

$$\alpha_d = \frac{\pi + k}{\pi}, \quad \alpha_x = \frac{\tau_x^{\sigma-1} f_x}{\pi}, \quad \alpha_d^* = \frac{\tau_m^{\kappa(\sigma-1)}(\pi + k)}{\pi}, \quad \alpha_x^* = \frac{\tau_m^{\kappa(\sigma-1)}\tau_x^{\sigma-1} f_x}{\pi}.$$

It is important to notice from the cutoff values that

$$\frac{\alpha_d^*}{\alpha_d} = \frac{\alpha_x^*}{\alpha_x} = \tau_m^{\kappa(\sigma-1)} > 1.$$

This suggests that the average product quality of foreign-matched firms is necessarily higher than that of domestic-matched firms. On the other hand,

$$\frac{\alpha_x}{\alpha_d} = \frac{\alpha_x^*}{\alpha_d^*} = \frac{\tau_x^{\sigma-1} f_x}{\pi + k} > 1 \quad \Longleftrightarrow \quad \pi < \tau_x^{\sigma-1} f_x - k.$$

Under this condition, a fraction of firms export among domestic-matched firms and foreign-matched firms (i.e., $\chi < 1$, $\chi^* < 1$). Since selection into the export market ubiquitously observed in empirical data, we will hereafter assume the condition, which also ensures that the average product quality is higher for exporting firms than that of non-exporting firms for both types of firms.

From these cutoffs, we define the average profits of domestic-matched firms:

$$\begin{aligned}
\tilde{\pi} &= \int_{\alpha_d}^{\infty} \alpha\pi \frac{dG(\alpha)}{1 - G(\alpha_d)} + \chi \int_{\alpha_x}^{\infty} \tau_x^{1-\sigma} \alpha\pi \frac{dG(\alpha)}{1 - G(\alpha_x)} \\
&= \tilde{\alpha}_d \pi + \chi \tau_x^{1-\sigma} \tilde{\alpha}_x \pi \\
&= \left(\tilde{\alpha}_d + \chi \tau_x^{1-\sigma} \tilde{\alpha}_x\right) \pi,
\end{aligned}$$

and those of foreign-matched firms:

$$\begin{aligned}
\tilde{\pi}^* &= \int_{\alpha_d^*}^{\infty} \alpha \tau_m^{\kappa(1-\sigma)} \pi \frac{dG(\alpha)}{1 - G(\alpha_d^*)} + \chi^* \int_{\alpha_x^*}^{\infty} \alpha \tau_m^{\kappa(1-\sigma)} \tau_x^{1-\sigma} \pi \frac{dG(\alpha)}{1 - G(\alpha_x^*)} \\
&= \tau_m^{\kappa(1-\sigma)} \tilde{\alpha}_d^* \pi + \chi^* \tau_m^{\kappa(1-\sigma)} \tau_x^{1-\sigma} \tilde{\alpha}_x^* \pi \\
&= \tau_m^{\kappa(1-\sigma)} \left(\tilde{\alpha}_d^* + \chi^* \tau_x^{1-\sigma} \tilde{\alpha}_x^*\right) \pi.
\end{aligned}$$

4.2 Search, matching, and offshoring

We continue to assume free entry to both the upstream and downstream sectors. At every instance, one-to-one matching occurs randomly between upstream and downstream firms, and dissolving their relationship also occurs randomly with an exogenous probability of λ. Since there are $M - n - n^*$ unmatched upstream firms and $N - n - n^*$ unmatched downstream firm in the open economy, search technology denoted by $\nu(M - n - n^*, N - n - n^*)$ assigns the number of newly domestic-matched firms. As a result, the hazard rates of matching in (7) is redefined as

$$\begin{aligned}
\check{\mu}^D &\equiv \frac{\nu(M - n - n^*, N - n - n^*)}{N - n - n^*} = \nu\left(\frac{M - n - n^*}{N - n - n^*}, 1\right), \\
\check{\mu}^U &\equiv \frac{\nu(M - n - n^*, N - n - n^*)}{M - n - n^*} = \frac{N - n - n^*}{M - n - n^*} \nu\left(\frac{M - n - n^*}{N - n - n^*}, 1\right).
\end{aligned}$$

The hazard rates in (8), $\mu^D = [1 - G(\alpha_d)]\check{\mu}^D$ and $\mu^U = [1 - G(\alpha_d)]\check{\mu}^U$, are also redefined by setting $z \equiv (M - n - n^*)/(N - n - n^*)$.

Search technology denoted by $\nu^*(M-n-n^*, N-n-n^*)$ simultaneously assigns the number of newly foreign-matched firms. This search technology satisfies $\nu^*(\cdot,\cdot) = \theta\nu(\cdot,\cdot)$ and generates the associated hazard rates of matching, $\mu^{D^*} \equiv [1-G(\alpha_d^*)]\breve{\mu}^{D^*}$ and $\mu^{U^*} \equiv [1-G(\alpha_d^*)]\breve{\mu}^{U^*}$. Using (8), $\breve{\mu}^{D^*} = \theta\breve{\mu}^D$ and $\breve{\mu}^{U^*} = \theta\breve{\mu}^U$, these hazard rates are expressed in terms of $s(z)$:

$$\mu^{D^*} = \theta\left(\frac{1-G(\alpha_d^*)}{1-G(\alpha_d)}\right)\mu^D = \zeta s(z), \quad \mu^{U^*} = \theta\left(\frac{1-G(\alpha_d^*)}{1-G(\alpha_d)}\right)\mu^U = \frac{\zeta s(z)}{z},$$

where $\zeta \equiv \theta\left(\frac{1-G(\alpha_d^*)}{1-G(\alpha_d)}\right) \in (0,1)$ measures the quality-adjusted difficulty achieving a foreign match relative to a domestic match. From the property of search technology, μ^{D^*} is increasing and concave in z whereas μ^{U^*} is decreasing and convex in z, just as in μ^D and μ^U.

In the stationary equilibrium, the number of newly matched firms equals the number of exiting firms at every instance, so we have

$$\begin{cases} \mu^U(M-n-n^*) = 2\lambda n, \\ \mu^D(N-n-n^*) = 2\lambda n, \end{cases} \qquad \begin{cases} \mu^{U^*}(M-n-n^*) = 2\lambda n^*, \\ \mu^{D^*}(N-n-n^*) = 2\lambda n^*, \end{cases}$$

where the first and second systems of equations describe the firm dynamics of domestic-matched and foreign-matched firms, respectively. Solving these relationships for n and n^*, and recalling that a χ (χ^*) fraction of domestic-matched (foreign-matched) firms export, we obtain

$$\begin{aligned} n &= \left(\frac{\mu^U}{(1+\zeta)\mu^U + 2\lambda}\right) M = \left(\frac{\mu^D}{(1+\zeta)\mu^D + 2\lambda}\right) N, \\ n^* &= \zeta n, \quad n_x = \chi n, \quad n_x^* = \chi^*\zeta n. \end{aligned} \tag{27}$$

As with the closed economy, equation (27) similarly describes how the number of domestic-matched firms n in each sector is tied to the hazard rates of matching, μ^U, μ^D. Once this number is determined, the numbers of the other types of matched pairs n^*, n_x, n_x^*, the total numbers of firms, M, N, and the cutoff values $\alpha_d, \alpha_d^*, \alpha_x, \alpha_x^*$ are pinned down simultaneously in the dynamics.

4.3 Open-economy equilibrium characterizations

The free entry conditions equate the expected instantaneous profits and the instantaneous values of entry costs in the downstream and upstream sectors ($V^D = F^D$, $V^U = F^U$). As shown in Appendix, the conditions corresponding to (14) are given by

$$\begin{aligned} \pi + \frac{n}{N}\beta(\tilde{\pi} - \pi - k - \chi f_x) + \frac{n^*}{N}\beta(\tilde{\pi}^* - \pi - k - \chi^* f_x) &= f^D, \\ \frac{n}{M}(1-\beta)(\tilde{\pi} - \pi - k - \chi f_x) + \frac{n^*}{M}(1-\beta)(\tilde{\pi}^* - \pi - k - \chi^* f_x) &= f^U, \end{aligned} \tag{28}$$

where $f_x \equiv 2\lambda F_x$ represents the instantaneous value of the fixed export cost, while $\tilde{\pi}^* - \pi - k - \chi^* f_x$ represents the economic rents generating from foreign matching, which depend upon export costs τ_x, f_x as well as an import cost τ_m. On the other hand, the effective bargaining power (the counterpart of (13) in the closed economy) is given by

$$\beta \equiv \frac{(1+\zeta)\mu^D + 2\lambda}{(1+\zeta)(\mu^D + \mu^U) + 4\lambda}.$$

Fractions $\frac{n}{N}$ and $\frac{n^*}{N}$ of domestic-matched and foreign-matched downstream firms respectively earn the instantaneous rents of $\beta(\tilde{\pi} - \pi - k - \chi f_x)$ and $\beta(\tilde{\pi}^* - \pi - k - \chi^* f_x)$, in addition to the instantaneous profit of π. Under free entry, the expected instantaneous profits must equal the instantaneous value of entry cost f^D. The similar interpretation applies to upstream firms, except that they earn nothing when being unmatched. As in the closed economy, these two conditions simultaneously determines all the endogenous variables of the model, thereby realizing that aggregate expenditure equals aggregate labor income ($E = \delta \bar{L}$) in the stationary equilibrium.

While the above equilibrium characterizations hold for a general distribution $G(\alpha)$, we will restrict our attention to a Pareto distribution in the following analysis. Given this distributional assumption, all measures of the average quality for domestic-matched and foreign-matched firms $\tilde{\alpha}_d$, $\tilde{\alpha}_x$, $\tilde{\alpha}_d^*$, $\tilde{\alpha}_x^*$ are explicitly expressed as a function of the profits of unmatched downstream firms π:

$$\begin{aligned} \tilde{\alpha}_d &= \frac{\gamma}{\gamma-1}\left(\frac{\pi+k}{\pi}\right), \quad \tilde{\alpha}_x = \frac{\gamma}{\gamma-1}\left(\frac{\tau_x^{\sigma-1} f_x}{\pi}\right) \\ \tilde{\alpha}_d^* &= \frac{\gamma}{\gamma-1}\left(\frac{\tau_m^{\kappa(\sigma-1)}(\pi+k)}{\pi}\right), \quad \tilde{\alpha}_x^* = \frac{\gamma}{\gamma-1}\left(\frac{\tau_m^{\kappa(\sigma-1)}\tau_x^{\sigma-1} f_x}{\pi}\right). \end{aligned} \tag{29}$$

Similarly, the average profits for these matched firms are expressed as

$$\tilde{\pi} = \frac{\gamma}{\gamma-1}\left(\pi + k + \chi f_x\right), \quad \tilde{\pi}^* = \frac{\gamma}{\gamma-1}\left(\pi + k + \chi^* f_x\right), \tag{30}$$

where

$$\chi = \chi^* = \left(\frac{\pi+k}{\tau_x^{\sigma-1} f_x}\right)^{\gamma}.$$

It follows immediately from (30) that the economic rents are the same between domestic-matched and foreign-matched firms under the Pareto distribution:

$$\tilde{\pi} - \pi - k - \chi f_x = \tilde{\pi}^* - \pi - k - \chi^* f_x = \frac{\pi + k + \chi f_x}{\gamma - 1}.$$

Furtrhermore, substituting (30) into (28) and rearranging, the free entry conditions corresponding to (16) and (17) are rewritten as

$$(\gamma-1)\pi + (1+\zeta)\phi^D(z)(\pi + k + \chi f_x) = (\gamma-1) f^D, \tag{31}$$

$$(1+\zeta)\phi^U(z)(\pi + k + \chi f_x) = (\gamma-1) f^U, \tag{32}$$

where $\zeta = \theta \tau_m^{\gamma\kappa(1-\sigma)}$ and

$$\phi^D(z) = \frac{z s(z)}{(1+\zeta)(1+z)s(z) + 4\lambda z}, \quad \phi^U(z) = \frac{s(z)}{(1+\zeta)(1+z)s(z) + 4\lambda z}.$$

While $\phi^D(z)$ and $\phi^U(z)$ are the counterparts of the corresponding ones in the case of autarky, the functional forms are different due to the possibility of foreign matching, although we continue to use the same notations for simplicity. Despite of this difference, $\phi^{D'}(z) > 0$ and $\phi^{U'}(z) < 0$ for any z, and the graph of $\pi(z)$ represented by (31) is downward-sloping, whereas that by (32) is upward-sloping, so the intersection of the two curves uniquely determines $\{\hat{z}, \hat{\pi}\}$.

To examine the impact of trade, we need to compare the two free entry conditions, (14) and (28). Note that the open-economy equilibrium entails the higher economic rents generated from matching relative to the closed-economy equilibrium, so long as some firms profitably export ($\chi = \chi^* > 0$) and some firms profitably import ($\zeta > 0$). The resulting increase in the economic rents contributes to greater expected profits, inducing further entry to both production sectors. As with the case of an increase in $\alpha_{\min}$ or a decrease in γ in the closed economy, increases in the numbers of upstream and downstream firms induces intense competition and gives rise to downward shifts on the DD and UU curves relative to autarky, which leads to a fall in $\hat{\pi}$. In addition, simple inspection of (31) and (32) immediately reveals that the equilibrium relationship in (22) still holds in the open economy, and hence a decline in $\hat{\pi}$ always increases $\hat{z}$. The intuition why $\hat{z}$ rises in the open economy is explained as follows. Opening to costly trade not only benefits downstream firms due to extra export profits but also causes some harms to them due to foreign firms' penetration to the final-good market, which dampens their entry incentive. In contrast, upstream firms unambiguously benefit from opening to trade because their expected profits hinge on the profits when they are matched, which necessarily increase by opening to trade through which international trade benefits the most-competitive firms. As a result, opening to trade raises the total number of upstream firms M more sharply relative to the total number of upstream firms N, and, consequently, $z = (M - n)/(N - n)$ increases when opening the countries to trade.

Once these two endogenous variables are determined, other endogenous variables can be written as a function of $\hat{z}$ because $\hat{\pi} = \pi(\hat{z})$ from (22) in equilibrium. In the open economy, the number of matched pairs n is calculated as

$$n = \left[\frac{s(\hat{z})}{\sigma\pi(\hat{z})[(1+\zeta)(\tilde{\alpha}_d + \chi\tau_x^{1-\sigma}\tilde{\alpha}_x)s(\hat{z}) + 2\lambda]} \right] \delta\bar{L}, \tag{33}$$

where $\tilde{\alpha}_d, \tilde{\alpha}_x$ and χ in (33) are endogenously determined, as they are a function of $\hat{\pi}$ (and hence $\hat{z}$). The numbers of the other types of matched firms are

$$n_x = \chi n, \quad n^* = \zeta n = \theta\tau_m^{\gamma\kappa(1-\sigma)} n, \quad n_x^* = \chi^*\zeta n = \chi\theta\tau_m^{\gamma\kappa(1-\sigma)} n. \tag{34}$$

From (27) and (33), it also follows immediately that the total numbers of downstream and upstream firms are respectively given by

$$\begin{aligned} M &= \left[\frac{(1+\zeta)s(\hat{z}) + 2\lambda\hat{z}}{\sigma\pi(\hat{z})[(1+\zeta)(\tilde{\alpha}_d + \chi\tau_x^{1-\sigma}\tilde{\alpha}_x)s(\hat{z}) + 2\lambda]} \right] \delta\bar{L}, \\ N &= \left[\frac{(1+\zeta)s(\hat{z}) + 2\lambda}{\sigma\pi(\hat{z})[(1+\zeta)(\tilde{\alpha}_d + \chi\tau_x^{1-\sigma}\tilde{\alpha}_x)s(\hat{z}) + 2\lambda]} \right] \delta\bar{L}. \end{aligned} \tag{35}$$

Moreover, using (27) and (34), the price index P in (26) can be written as

$$P^{1-\sigma} = \left(\frac{\sigma c}{\sigma - 1}\right)^{1-\sigma} n \left[(1+\zeta)(\tilde{\alpha}_d + \chi \tau_x^{1-\sigma}\tilde{\alpha}_x) + \frac{2\lambda}{s(\hat{z})}\right].$$

Substituting (33) into this, we find that the expression of the price index in (20) is exactly the same between the closed-economy and open-economy equilibria. Since $\hat{z}$ is a sufficient statistic for welfare, a rise in $\hat{z}$ as a result of opening to costly trade implies that social welfare in (21) is higher in the open economy than the closed economy.

While we have thus far compared the closed-economy and open-economy equilibria, we can also examine the impact of gradual trade liberalization in the open-economy equilibrium. In our model, trade liberalization refers not only to a reduction in trade costs (a decrease in τ_x, τ_m) but also to an improvement of a foreign match (an increase in θ). Simple inspection of (31) and (32) reveals that these changes shift the DD and UU curves downwards, generating a similar impact on $\{\hat{z}, \hat{\pi}\}$ as that in the transition from autarky to costly trade:

$$\begin{array}{lll} \dfrac{d\hat{z}}{d\tau_x} < 0, & \dfrac{d\hat{z}}{d\tau_m} < 0, & \dfrac{d\hat{z}}{d\theta} > 0, \\ \dfrac{d\hat{\pi}}{d\tau_x} > 0, & \dfrac{d\hat{\pi}}{d\tau_m} > 0, & \dfrac{d\hat{\pi}}{d\theta} < 0. \end{array} \tag{36}$$

In any cases, such trade liberalization raises $\hat{z}$ and hence leads to higher social welfare.

4.4 Sources of welfare gains

We have noticed that each country enjoys welfare gains from trade. What are sources of these welfare gains in the presence of matching and search that are prevalent in offshoring? The next proposition summarizes the welfare gains in the current model.

Proposition 2 *Social welfare is higher in the open economy than in the closed economy due to:*

(i) lower price indices;

(ii) more profit reallocations from unmatched firms to matched firms;

(iii) higher average product quality;

(iv) more consumptions of high-quality products (relative to low-quality products).

Recall first that country size has no impact on the key endogenous variables $\{\hat{z}, \hat{\pi}\}$. As seen in the last section, if firms do not incur any trade costs, opening to trade is essentially the same as an increase in country size and each country gains from trade solely from a decline in the price index. However, if firms do incur some trade costs, on top of this impact operating through the price index, opening to trade decreases $\hat{\pi}$ and increases $\hat{z}$, which works to reinforce the welfare gains from trade. In the following, we spell out a specific mechanism through which the changes in $\{\hat{z}, \hat{\pi}\}$ give rise to additional welfare gains from trade.

A decline in $\hat{\pi}$ implies that the profits are relatively more reallocated from unmatched firms to matched firms by opening to costly trade. From (15) and (30), we can easily show that

$$\frac{\tilde{\pi}}{\pi} = \frac{\tilde{\pi}^*}{\pi} > \frac{\tilde{\pi}_a}{\pi_a} \quad \Longleftrightarrow \quad \frac{\hat{\pi} + k + \chi f_x}{\hat{\pi}} > \frac{\hat{\pi}_a + k}{\hat{\pi}_a}, \tag{37}$$

where the subscript a denotes autarky. Since $\hat{\pi}_a > \hat{\pi}$ and $\chi \in (0, 1)$, matched firms earn relatively more profits than unmatched firms in the open-economy equilibrium. Thus, opening to costly trade benefits each country by allowing matched firms to increase their profits (and hence hight-quality outputs) relatively more than unmatched firms. A decline of $\hat{\pi}$ also implies that the average product quality is improved by opening to costly trade. From (15) and (29), it follows immediately that the average product quality of domestic-matched firms is necessarily higher in the open economy:

$$\tilde{\alpha}_d > \tilde{\alpha}_{da} \quad \Longleftrightarrow \quad \frac{\hat{\pi} + k}{\hat{\pi}} > \frac{\hat{\pi}_a + k}{\hat{\pi}_a}.$$

Noting that $\tilde{\alpha}_x > \tilde{\alpha}_d$ and $\tilde{\alpha}^*_x > \tilde{\alpha}^*_d > \tilde{\alpha}_d$, the average product quality of domestic-matched or foreign-matched firms is even higher through selection into the export and import markets. These welfare gains are similar to those stressed in Melitz (2003).

New welfare gains stem from a rise in $\hat{z}$ in our model, which has an impact on search and matching between upstream and downstream firms. Comparing (18) and (19) in the closed economy with (33) and (35) in the open economy, we have that a fraction of high-quality products relative to low-quality products necessarily increases in the open-economy equilibrium:

$$\frac{n + n^*}{N - n - n^*} > \frac{n_a}{N_a - n_a} \quad \Longleftrightarrow \quad \frac{(1 + \zeta)s(\hat{z})}{2\lambda} > \frac{s(\hat{z}_a)}{2\lambda}.$$

Since $\hat{z} > \hat{z}_a$ (hence $s(\hat{z}) > s(\hat{z}_a)$) and $\zeta \in (0, 1)$, the open-economy equilibrium entails a relatively larger share of high-quality varieties produced by customized inputs. This suggests that opening to costly trade leads to restructuring in the market structure in the upstream and downstream sectors. Intuitively, the higher $\hat{z}$, the thicker the upstream sector relative to the downstream sector (i.e., the higher $(M - n)/(N - n)$). Thus the matching environment is improved in favor of downstream firms, which in turn allows each country to consume relatively more high-quality products. Moreover, since $px = \sigma\pi$, $\tilde{p}\tilde{x} = \sigma\tilde{\pi} = \left(\tilde{\alpha}_d + \chi\tau_x^{1-\sigma}\tilde{\alpha}_x\right)\sigma\pi$, $\tilde{p}^*\tilde{x}^* = \sigma\tilde{\pi}^* = \tau_m^{\kappa(1-\sigma)}\left(\tilde{\alpha}^*_d + \chi^*\tau_x^{1-\sigma}\tilde{\alpha}^*_x\right)\sigma\pi$ where $\tilde{\pi} = \tilde{\pi}^*$, (37) shows that these gains arise not only at the extensive margin but also at the intensive margin:

$$\frac{\tilde{p}\tilde{x}}{px} = \frac{\tilde{p}^*\tilde{x}^*}{px} > \frac{\tilde{p}_a\tilde{x}_a}{p_a x_a}.$$

The finding also implies that market transactions relative to non-market transactions are thinner in the open economy by facilitating search and matching between vertically related pairs.

While the welfare gains occur in every industry, the magnitude of these gains is not uniform but varies across industries. Applying the comparative statics in Proposition 1, it is straightforward to show that the welfare gains from trade are greater, the higher the degree of input customization of the industry, because the changes in $\{\hat{z}, \hat{\pi}\}$ are more significant in that industry.

4.5 Aggregate trade flows

Building on the equilibrium characterizations in the open economy, we finally examine how aggregate exports of final goods and intermediate inputs are affected by trade barriers. More specifically, we are interested in investigating whether trade liberalization in τ_x, τ_m or θ has a complementary effect on the trade volumes.

To see this, let us first derive the aggregate export values of final goods. Noting that only high-quality firms above the cutoff α_x are able to profitably export among n_e matched firms, the aggregate export values of final goods by domestic-matched firms are give by

$$n_e \int_{\alpha_x}^{\infty} \tilde{p}_x x_x(\alpha) dG(\alpha) = n_x \tilde{p}_x \tilde{x}_x = \left(\frac{\chi \tau_x^{1-\sigma} \tilde{\alpha}_x}{(1+\zeta)(\tilde{\alpha}_d + \chi \tau_x^{1-\sigma} \tilde{\alpha}_x) + \frac{2\lambda}{s(\hat{z})}} \right) \delta \bar{L},$$

where

$$\tilde{x}_x = \int_{\alpha_x}^{\infty} x_x(\alpha) \frac{dG(\alpha)}{1 - G(\alpha_x)} = \tau_x^{-\sigma} \tilde{\alpha}_x x$$

is the average export volume by domestic-matched firms. Similarly, we can also derive the aggregate export values of final goods by foreign-matched firms:

$$n_e \int_{\alpha_x^*}^{\infty} \tilde{p}_x^* x_x^*(\alpha) dG(\alpha) = n_x^* \tilde{p}_x^* \tilde{x}_x^* = \left(\frac{\zeta \chi \tau_x^{1-\sigma} \tilde{\alpha}_x}{(1+\zeta)(\tilde{\alpha}_d + \chi \tau_x^{1-\sigma} \tilde{\alpha}_x) + \frac{2\lambda}{s(\hat{z})}} \right) \delta \bar{L},$$

where

$$\tilde{x}_x^* = \int_{\alpha_x^*}^{\infty} x_x^*(\alpha) \frac{dG(\alpha)}{1 - G(\alpha_x^*)} = \tau_x^{-\sigma} \tau_m^{-\kappa\sigma} \tilde{\alpha}_x^* x$$

is the average export volume by foreign-matched firms. Combining these two values and rearranging, the aggregate export values of final goods are expressed as

$$n_x \tilde{p}_x \tilde{x}_x + n_x^* \tilde{p}_x^* \tilde{x}_x^* = \left(\frac{\chi \tau_x^{1-\sigma} \tilde{\alpha}_x}{\tilde{\alpha}_d + \chi \tau_x^{1-\sigma} \tilde{\alpha}_x + \frac{2\lambda}{(1+\zeta)s(\hat{z})}} \right) \delta \bar{L}. \tag{38}$$

As for the aggregate export values of intermediate inputs, on the other hand, we use the marginal costs $\tau_x \tau_m^{\kappa} c$ to evaluate the trade volume since intermediate inputs are internationally traded between foreign-matched firms and hence there is no explicit input price that can be used for the evaluation. Together with our Cobb-Douglas production assumption, the aggregate export values of intermediate inputs are computed as

$$\kappa n_x^* \tau_x \tau_m^{\kappa} c \tilde{x}_x^* = \kappa \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{\zeta}{1+\zeta} \right) \left(\frac{\chi \tau_x^{1-\sigma} \tilde{\alpha}_x}{\tilde{\alpha}_d + \chi \tau_x^{1-\sigma} \tilde{\alpha}_x + \frac{2\lambda}{(1+\zeta)s(\hat{z})}} \right) \delta \bar{L}. \tag{39}$$

From (38) and (39), it follows that the ratio of the aggregate export values of intermediate inputs to those of final goods is given by

$$\frac{\kappa n_x^* \tau_x \tau_m^{\kappa} c \tilde{x}_x^*}{n_x \tilde{p}_x \tilde{x}_x + n_x^* \tilde{p}_x^* \tilde{x}_x^*} = \kappa \left(\frac{\sigma - 1}{\sigma} \right) \left(\frac{\zeta}{1+\zeta} \right). \tag{40}$$

Armed with (38) and (39), we are ready to explore the impact of trade barriers on the two types of aggregate exports. Applying the comparative static results in (36) to the aggregate export values in (38) and (39), we have that (see the Appendix for proof):

$$\frac{d(n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^*)}{d\tau_x} < 0, \quad \frac{d(n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^*)}{d\tau_m} < 0, \quad \frac{d(n\tilde{p}_x\tilde{x}_x + n^*\tilde{p}_x^*\tilde{x}_x^*)}{d\theta} > 0,$$
$$\frac{d(\kappa n^*\tau_x\tau_m^\kappa c\tilde{x}_x^*)}{d\tau_x} < 0, \quad \frac{d(\kappa n^*\tau_x\tau_m^\kappa c\tilde{x}_x^*)}{d\tau_m} < 0, \quad \frac{d(\kappa n^*\tau_x\tau_m^\kappa c\tilde{x}_x^*)}{d\theta} > 0.$$

Thus a reduction in τ_x raises not only the aggregate export value of final goods, expressed in (38), but also the aggregate export value of intermediate inputs, represented by (39). Similarly, a reduction in τ_m or a rise in θ simultaneously increases these two aggregate export values. From this observation, we can say that final-good trade and intermediate-input trade are complementary in the sense that a reduction in trade costs of final-good trade stimulates not only final-good trade but also intermediate-input trade, and vice versa.

Moreover, applying (36) to the ratio of the two aggregate export values in (40) and noticing that $\zeta = \theta\tau_m^{\gamma\kappa(1-\sigma)}$, we also have that

$$\frac{d\left(\frac{\kappa n_x^*\tau_x\tau_m^\kappa c\tilde{x}_x^*}{n_x\tilde{p}_x\tilde{x}_x + n_x^*\tilde{p}_x^*\tilde{x}_x^*}\right)}{d\tau_x} = 0, \quad \frac{d\left(\frac{\kappa n_x^*\tau_x\tau_m^\kappa c\tilde{x}_x^*}{n_x\tilde{p}_x\tilde{x}_x + n_x^*\tilde{p}_x^*\tilde{x}_x^*}\right)}{d\tau_m} < 0, \quad \frac{d\left(\frac{\kappa n_x^*\tau_x\tau_m^\kappa c\tilde{x}_x^*}{n_x\tilde{p}_x\tilde{x}_x + n_x^*\tilde{p}_x^*\tilde{x}_x^*}\right)}{d\theta} > 0.$$

This suggests that trade liberalization increases the aggregate export values of intermediate inputs relatively more than those of final goods. In other words, the trade elasticity of intermediate inputs is greater than the trade elasticity of final goods. Intuitively, a reduction in τ_x equally affects the two aggregate export volumes, a reduction in τ_m or a rise in θ induces firms to realize a foreign match easily, which in turn prompts intermediate-input trade relatively more than final-good trade. This finding accords well with the empirical observations by for example Johnson and Noguera (2012) that intermediate-input trade has been growing faster than final-good trade under vertical disintegration. Our contribution is in shedding new light on the role of search and matching that would particularly play an important role in offshoring.

We summarize these results in the following proposition.

Proposition 3 *Trade liberalization (in terms of a reduction in τ_x, τ_m or a rise in θ) gives rise to the following impacts on the aggregate exports of final goods and intermediate inputs:*

(i) A reduction in τ_x increases the aggregate export values of final goods, and in turn increases those of intermediate inputs as a result.

(ii) A reduction in τ_m or a rise in θ increases the aggregate export values of intermediate inputs, and in turn increases those of final goods as a result.

(iii) Trade liberalization increases the aggregate export values of intermediate inputs relatively more than those of final goods.

5 Country asymmetry

To be added soon.

6 Conclusion

This paper investigated the effect of trade liberalization in vertically-related industries, emphasizing differential impacts depending on the degree of relationship specificity of intermediate inputs that are traded within matched pairs. We showed that the higher is the relationship specificity, the thinner is the market; and that a reduction in trade costs, either in final goods or in intermediate inputs, makes the market transactions thinner relative to the non-market transactions and enhances social welfare. We also found that a reduction in trade costs in either final-good trade or intermediate-input trade entails an increase in both types of trade simultaneously, i.e., trade in final goods and trade in intermediate inputs are complementary. Our main emphasis is that these effects of international trade are not only through a resource reallocation from low-quality firms to high-quality firms as stressed by Melitz (2003) but also through a change in matching environments and market restructuring in upstream and downstream sectors.

To analyze vertical relationships between downstream and upstream sectors, we resort to a rather strong assumption that while the downstream sector is monopolistically competitive, the upstream sector is perfectly competitive in trading generic inputs in anonymous markets, which we believe is a reasonable approximation of the reality. Relaxing the assumption of perfectly competitive markets in the upstream sector might change the impact of trade liberalization on the market thickness and on trade volumes of final goods and intermediate inputs. Even in such circumstances, however, we believe it is possible to show that our key results continue to hold as long as the market competition is tougher in the upstream sector than in the downstream sector.

It would be interesting to analyze the effect of country asymmetry on the location of production of final goods and intermediate inputs. Will a larger country host disproportionally more downstream firms or upstream firms than a smaller country? Which country, large or small, benefits relatively more from trade liberalization that takes place in either production sector in the vertically-related world? We shall address such questions in future research.

Appendix

A Proofs of the closed-economy equilibrium

A.1 Proof of the Nash bargaining solution

Setting $r = 0$ and $\dot{\tilde{V}}^D = \dot{V}^D = \dot{\tilde{V}}^U = \dot{V}^U = 0$ in the no-arbitrage conditions and solving for $\tilde{V}^D, V^D, \tilde{V}^U$ and V^U gives

$$\tilde{V}^D = \frac{\tilde{\pi}^D}{2\lambda} + \frac{1}{2}V^D, \tag{41}$$

$$V^D = \frac{\pi}{\mu^D + \lambda} + \frac{\mu^D}{\mu^D + \lambda}\tilde{V}^D, \tag{42}$$

$$\tilde{V}^U = \frac{\tilde{\pi}^U}{2\lambda} + \frac{1}{2}V^U, \tag{43}$$

$$V^U = \frac{\mu^U}{\mu^U + \lambda}(\tilde{V}^U - K). \tag{44}$$

Solving (41) and (42) simultaneously gives (10), whereas solving (43) and (44) simultaneously gives (11). Substituting (41), (43) and the constraint $\tilde{\pi}^{U'} = \tilde{\pi} - \tilde{\pi}^{D'}$ into the bargaining problem, the equilibrium profit of downstream firms is

$$\tilde{\pi}^D \in \arg\max_{\tilde{\pi}^{D'}} \left(\frac{\tilde{\pi}^{D'}}{2\lambda} - \frac{V^D}{2}\right)\left(\frac{\tilde{\pi} - \tilde{\pi}^{D'}}{2\lambda} - \frac{V^U}{2} - K\right).$$

The solution to this problem yields

$$\tilde{\pi}^{D'} = \frac{1}{2}\left[\tilde{\pi} + \lambda\left(V^D - V^U\right) - 2\lambda K\right],$$
$$\tilde{\pi}^{U'} = \frac{1}{2}\left[\tilde{\pi} + \lambda\left(V^U - V^D\right) + 2\lambda K\right].$$

The result follows from noting $\tilde{\pi}^{D'} = \tilde{\pi}^D$ and $\tilde{\pi}^{U'} = \tilde{\pi}^U = \tilde{\pi} - \tilde{\pi}^D$ and substituting (10) and (11) into the above.

Although we derive the profit sharing by solving the bargaining problem, suppose that the outcome of bargaining over the division of the total surplus from the match satisfies the "surplus-splitting" rule (Felbermayr et al., 2011):

$$\frac{1}{2}\left(\tilde{V}^D - V^D\right) = \frac{1}{2}\left(\tilde{V}^U - K - V^U\right),$$

where 1/2 implies the same bargaining power between downstream and upstream firms. Substituting (10) and (11) into the above equality yields the same profit sharing as that obtained by solving the bargaining problem.

A.2 Proof of the labor market clearing condition

We first show that aggregate labor used for entry equals aggregate profit in each industry at every instance. Let L_e^D denote aggregate labor used for entry by downstream firms. In the downstream sector, since λN new entrants pay the fixed entry cost F^D at every instance, aggregate amount of labor used for entry at every instance is $L_e^D = \lambda N F^D$. Using the free entry condition ($V^D = F^D$), we have

$$\begin{aligned} L_e^D &= \lambda N V^D \\ &= \lambda N \left[\frac{\pi}{\lambda} + \left(\frac{\mu^D}{\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda}\right)\right] \qquad \text{(using (10))} \\ &= \left(\frac{\mu^D N}{\mu^D + 2\lambda}\right)\tilde{\pi}^D + \left(\frac{2\lambda N}{\mu^D + 2\lambda}\right)\pi. \end{aligned}$$

From the firm dynamics (9), we have $\frac{\mu^D N}{\mu^D+2\lambda} = n$ and $\frac{2\lambda N}{\mu^D+2\lambda} = N - n$. Substituting these equalities into the above equation, we find that aggregate labor equals aggregate profit in the downstream sector:

$$L_e^D = n\tilde{\pi}^D + (N - n)\pi. \tag{45}$$

Similarly, let L_e^U denote aggregate amount of labor used for entry and investment by upstream firms. In the upstream sector, since λM new entrants pay the fixed entry cost F^U and n new matched pairs pay the fixed investment cost $2K$ at every instance, aggregate amount of labor used for entry and investment is $\lambda M F^U$ and $2\lambda n K$ respectively, and thus $L_e^U = \lambda M F^U + 2\lambda n K$. Using (9), (11) and (13) and rearranging, we have

$$L_e^U = n\tilde{\pi}^U. \tag{46}$$

Then, it immediately follows from (45) and (46) that

$$\begin{aligned} L_e^D + L_e^U &= n\tilde{\pi} + (N-n)\pi \\ &= \frac{E}{\sigma}. \qquad \text{(using (4))} \end{aligned}$$

Next, we show that aggregate expenditure equals aggregate labor income in every industry. Let L_p^D and L_p^U denote aggregate amount of labor used for production by downstream and upstream firms respectively. Recalling that the unit cost of producing final goods is $c(=1)$ for both matched and unmatched firms, the amount of labor used for production by matched firms with quality α is $cx(\alpha)$ and that by unmatched firms is $cx(= cx(1))$. Aggregating these, we have

$$\begin{aligned} L_p^D + L_p^U &= n_e c \int_{\alpha_d}^{\infty} x(\alpha) dG(\alpha) + (N-n)cx \\ &= \left[n\tilde{\alpha}_d + (N-n)\right] cx \\ &= \left(\frac{\sigma-1}{\sigma}\right) E. \qquad \text{(using (1) and (3))} \end{aligned}$$

Summing up aggregate amount of labor for production, entry, and investment establishes the desired result that

$$\begin{aligned} L &\equiv (L_p^D + L_p^U) + (L_e^D + L_e^U) \\ &= E, \end{aligned}$$

where L represents aggregate labor income since we choose labor as a numeraire of the model. Noting that this equality holds for each industry, summing up both sides of the equation over all industries, we have that aggregate labor income equals aggregate expenditure of the country $\bar{L} = \bar{E}$. This also means that $E = \delta\bar{E} = \delta\bar{L}$ for each industry.

A.3 Proof of Lemma 1

To prove Lemma 1, it suffices to show that $\hat{z}_1 > \hat{z}_2$ under the following condition:

$$G_1(\alpha) < G_2(\alpha) \quad \Longleftrightarrow \quad 1 - G_1(\alpha) > 1 - G_2(\alpha), \tag{47}$$

where $1 - G(\alpha) = \left(\frac{\alpha_{\min}}{\alpha}\right)^{\gamma}$ under the Pareto distribution. Suppose first that only $\alpha_{\min}$'s are different across industries (while keeping γ the same). Then (47) implies that industry 1 is the first-stochastic dominance over industry 2 if

$$\left(\frac{\alpha_{\min_1}}{\alpha}\right)^{\gamma} > \left(\frac{\alpha_{\min_2}}{\alpha}\right)^{\gamma}.$$

Since $\gamma > \sigma - 1$, (47) holds if and only if $\alpha_{\min_1} > \alpha_{\min_2}$. Applying Proposition 1 directly establishes the desired result. Similarly, suppose that only γ's are different across industries (while keeping $\alpha_{\min}$ the same). Then industry 1 is the first-stochastic dominance over industry 2 if

$$\left(\frac{\alpha_{\min}}{\alpha}\right)^{\gamma_1} > \left(\frac{\alpha_{\min}}{\alpha}\right)^{\gamma_2}.$$

Since $\alpha_{\min} < \alpha$ and hence $\frac{\alpha_{\min}}{\alpha} < 1$, (47) holds if and only if $\gamma_1 < \gamma_2$. Applying Proposition 1 again directly establishes the desired result. These suggest that $\hat{z}_1 > \hat{z}_2$ for $\{\alpha_{\min_1}, \alpha_{\min_2}, \gamma_1, \gamma_2\}$ satisfying the following condition:

$$1 - G_1(\alpha) = \left(\frac{\alpha_{\min_1}}{\alpha}\right)^{\gamma_1} > 1 - G_2(\alpha) = \left(\frac{\alpha_{\min_2}}{\alpha}\right)^{\gamma_2}.$$

Once this result is obtained, the other equilibrium outcomes follow immediately from the equilibrium characterizations.

B Proofs of the open-economy equilibrium

B.1 Proof of the free entry condition

We first derive equation (28). The no-arbitrage conditions in the open economy are given by

$$
\begin{aligned}
r\tilde{V}^D &= \tilde{\pi}^D - \lambda\left(\tilde{V}^D - V^D\right) - \lambda\tilde{V}^D + \dot{\tilde{V}}^D,\\
r\tilde{V}^{D^*} &= \tilde{\pi}^{D^*} - \lambda\left(\tilde{V}^{D^*} - V^D\right) - \lambda\tilde{V}^{D^*} + \dot{\tilde{V}}^{D^*},\\
rV^D &= \pi + \mu^D\left(\tilde{V}^D - \chi F_x - V^D\right) + \mu^{D^*}\left(\tilde{V}^{D^*} - \chi^* F_x - V^D\right) - \lambda V^D + \dot{V}^D,\\
r\tilde{V}^U &= \tilde{\pi}^U - \lambda\left(\tilde{V}^U - V^U\right) - \lambda\tilde{V}^U + \dot{\tilde{V}}^U,\\
r\tilde{V}^{U^*} &= \tilde{\pi}^{U^*} - \lambda\left(\tilde{V}^{U^*} - V^U\right) - \lambda\tilde{V}^{U^*} + \dot{\tilde{V}}^{U^*},\\
rV^U &= \mu^U\left(\tilde{V}^U - K - V^U\right) + \mu^{U^*}\left(\tilde{V}^{U^*} - K - V^U\right) - \lambda V^U + \dot{V}^U,
\end{aligned}
$$

where $\tilde{\pi}^{D^*} + \tilde{\pi}^{U^*} = \tilde{\pi}^*$ and we use the fact that a proportion χ (χ^*) of domestic-matched (foreign-matched) downstream firms profitably incur the fixed export cost F_x. Setting $r = 0$ and $\dot{\tilde{V}}^D = \dot{\tilde{V}}^{D^*} = \dot{V}^D = 0$ in the no-arbitrage conditions and using $\mu^{D^*} = \zeta\mu^D$, solving the first three equations gives us the value functions for each type of downstream firms:

$$
\begin{aligned}
\tilde{V}^D - \chi F_x &= \frac{\pi}{\lambda} - + \left(\frac{(1+\frac{1}{2}\zeta)\mu^D + \lambda}{(1+\zeta)\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2\chi F_x\right) + \left(\frac{\frac{1}{2}\zeta\mu^D}{(1+\zeta)\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2\chi^* F_x\right),\\
\tilde{V}^{D^*} - \chi^* F_x &= \frac{\pi}{\lambda} + \left(\frac{\frac{1}{2}\mu^D}{(1+\zeta)\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2\chi F_x\right) + \left(\frac{(\frac{1}{2}+\zeta)\mu^D + \lambda}{(1+\zeta)\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2\chi^* F_x\right),\\
V^D &= \frac{\pi}{\lambda} + \left(\frac{\mu^D}{(1+\zeta)\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2\chi F_x\right) + \left(\frac{\zeta\mu^D}{(1+\zeta)\mu^D + 2\lambda}\right)\left(\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2\chi^* F_x\right).
\end{aligned}
\tag{48}
$$

Similarly, the last three equations can be solved for $\tilde{V}^U, \tilde{V}^{U^*}$ and V^U:

$$
\begin{aligned}
\tilde{V}^U - K &= \left(\frac{(1+\frac{1}{2}\zeta)\mu^U + \lambda}{(1+\zeta)\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right) + \left(\frac{\frac{1}{2}\zeta\mu^U}{(1+\zeta)\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K\right),\\
\tilde{V}^{U^*} - K &= \left(\frac{\frac{1}{2}\mu^U}{(1+\zeta)\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right) + \left(\frac{(\frac{1}{2}+\zeta)\mu^U + \lambda}{(1+\zeta)\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K\right),\\
V^U &= \left(\frac{\mu^U}{(1+\zeta)\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right) + \left(\frac{\zeta\mu^U}{(1+\zeta)\mu^U + 2\lambda}\right)\left(\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K\right).
\end{aligned}
\tag{49}
$$

As in the closed economy, both domestic-matched and foreign-matched pairs determine the profit sharing by the symmetric Nash bargaining so as to maximize ex-post gains from the relationship. These ex-post gains are given by $\tilde{V}^D - \chi F_x - V^D$ and $\tilde{V}^U - K - V^U$ for domestic-matched pairs, while $\tilde{V}^{D^*} - \chi^* F_x - V^D$ and $\tilde{V}^{U^*} - K - V^U$ for foreign-matched pairs. Solving the bargaining problem gives us the following profit sharing rules:

$$
\begin{aligned}
\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2\chi F_x &= \beta\left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K - 2\chi F_x\right) - \frac{\zeta\left(\beta - \frac{1}{2}\right)}{1+\zeta}\left(\frac{\tilde{\pi}}{\lambda} - \frac{\tilde{\pi}^*}{\lambda}\right),\\
\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2\chi^* F_x &= \beta\left(\frac{\tilde{\pi}^*}{\lambda} - \frac{\pi}{\lambda} - 2K - 2\chi^* F_x\right) + \frac{\beta - \frac{1}{2}}{1+\zeta}\left(\frac{\tilde{\pi}}{\lambda} - \frac{\tilde{\pi}^*}{\lambda}\right),\\
\frac{\tilde{\pi}^U}{\lambda} - 2K &= (1-\beta)\left(\frac{\tilde{\pi}}{\lambda} - \frac{\pi}{\lambda} - 2K - 2\chi F_x\right) + \frac{\zeta\left(\beta - \frac{1}{2}\right)}{1+\zeta}\left(\frac{\tilde{\pi}}{\lambda} - \frac{\tilde{\pi}^*}{\lambda}\right),\\
\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K &= (1-\beta)\left(\frac{\tilde{\pi}^*}{\lambda} - \frac{\pi}{\lambda} - 2K - 2\chi^* F_x\right) - \frac{\beta - \frac{1}{2}}{1+\zeta}\left(\frac{\tilde{\pi}}{\lambda} - \frac{\tilde{\pi}^*}{\lambda}\right).
\end{aligned}
\tag{50}
$$

where

$$
\beta \equiv \frac{(1+\zeta)\mu^D + 2\lambda}{(1+\zeta)(\mu^D + \mu^U) + 4\lambda}.
$$

Finally, substituting the firm dynamics (27) into (48) and (49), the free entry conditions ($V^D = F^D, V^U = F^U$) can be written as

$$\frac{\pi}{\lambda} + \frac{n}{N}\left(\frac{\tilde{\pi}^D}{\lambda} - \frac{\pi}{\lambda} - 2\chi F_x\right) + \frac{n^*}{N}\left(\frac{\tilde{\pi}^{D^*}}{\lambda} - \frac{\pi}{\lambda} - 2\chi^* F_x\right) = F^D,$$
$$\frac{n}{M}\left(\frac{\tilde{\pi}^U}{\lambda} - 2K\right) + \frac{n^*}{M}\left(\frac{\tilde{\pi}^{U^*}}{\lambda} - 2K\right) = F^U.$$

Substituting the profit sharing rule (50) into above equations and noting that $n^* = \zeta n$ (from (27)), we obtain the free entry condition in the open-economy equilibrium, given by (28).

B.2 Proof of the labor market clearing condition

We show that in each industry, aggregate expenditure equals aggregate labor income $E = \delta\bar{L}$, in the open-economy equilibrium. Aggregate labor used for investment in the downstream sector is $L_e^D = \lambda N F^D + 2\lambda n\chi F_x + 2\lambda n^*\chi^* F_x$. Using (27) and (48), we find that (45) is in the open economy expressed as

$$L_e^D = n\tilde{\pi}^D + n^*\tilde{\pi}^{D^*} + (N - n - n^*)\pi.$$

Similarly, noting that $L_e^U = \lambda M F^U + 2\lambda nK + 2\lambda n^* K$ in the upstream sector, (46) is expressed as

$$L_e^U = n\tilde{\pi}^U + n^*\tilde{\pi}^{U^*}.$$

These implies that

$$L_e^D + L_e^U = \frac{E}{\sigma}.$$

As for aggregate labor used for production, the total amount of labor by domestic-matched firms is

$$n_e c\int_{\alpha}^{\infty} x_d(\alpha)dG(\alpha) + n_e\tau_x c\int_{\alpha_x} x_x(\alpha)dG(\alpha) = nc\tilde{\alpha}_d x + n_x\tau_x^{1-\sigma}c\tilde{\alpha}_x x,$$

whereas the total amount of labor by foreign-matched firms is

$$n^*\tau_m^\kappa c\int_{\alpha_d^*}^{\infty} x_d^*(\alpha)dG(\alpha) + n^*\tau_x\tau_m^\kappa c\int_{\alpha_x^*}^{\infty} x_x^*(\alpha)dG(\alpha) = n^*\tau_m^{\kappa(1-\sigma)}c\alpha_d^* x + n^*\tau_x^{1-\sigma}\tau_m^{\kappa(1-\sigma)}c\alpha_x^* x.$$

Noting that the total amount of labor by unmatched firms is $(N - n - n^*)cx$, adding up all of these amounts of labor and rearranging, we obtain

$$\begin{aligned} L_p^D + L_p^U &= \left[n(\tilde{\alpha}_d + \chi\tau_x^{1-\sigma}\tilde{\alpha}_x) + n^*\tau_m^{\kappa(1-\sigma)}(\tilde{\alpha}_d^* + \chi^*\tau_x^{1-\sigma}\tilde{\alpha}_x^*) + (N - n - n^*)\right]cx \\ &= \left(\frac{\sigma - 1}{\sigma}\right)E. \qquad \text{(using (1) and (26))} \end{aligned}$$

Finally, the desired result follows from observing that $L \equiv (L_p^D + L_p^U) + (L_e^D + L_e^U) = E$. Given that the wage rate is normalized to one and the equality holds for each industry, we have that aggregate labor income equals aggregate expenditure of the country $\bar{L} = \bar{E}$ and hence $E = \delta\bar{E} = \delta\bar{L}$ in the open-economy equilibrium.

B.3 Proof of Proposition 3

Let us first consider the impact of the variable export trade cost τ_x on the aggregate export values of final goods (38) and those of intermediate inputs (39). It is useful to rewrite (38) as

$$n_x\tilde{p}_x\tilde{x}_x + n_x^*\tilde{p}_x^*\tilde{x}_x^* = \left(\frac{\eta\tilde{\alpha}_d}{(1+\eta)\tilde{\alpha}_d + \frac{2\lambda}{(1+\zeta)s(\hat{z})}}\right)\delta\bar{L}, \qquad (51)$$

where

$$\eta \equiv \frac{\chi\tau_x^{1-\sigma}\tilde{\alpha}_x}{\tilde{\alpha}_d} = \tau_x^{1-\sigma}\chi^{\frac{\gamma-1}{\gamma}}.$$

Differentiating (51) with respect to τ_x, we obtain

$$\frac{d(n_x\tilde{p}_x\tilde{x}_x + n_x^*\tilde{p}_x^*\tilde{x}_x^*)}{d\tau_x} = \left(\frac{\left((\tilde{\alpha}_d)^2 + \frac{2\lambda}{(1+\zeta)s(\hat{z})}\right)\frac{d\eta}{d\tau_x} + \eta\left(\frac{2\lambda}{(1+\zeta)s(\hat{z})}\right)\frac{d\tilde{\alpha}_d}{d\tau_x} - \eta\tilde{\alpha}_d\frac{d\left(\frac{2\lambda}{(1+\zeta)s(\hat{z})}\right)}{d\tau_x}}{\left((1+\eta)\tilde{\alpha}_d + \frac{2\lambda}{(1+\zeta)s(\hat{z})}\right)^2} \right)\delta\bar{L}. \quad (52)$$

In (52), it follows immediately from (36) that $\frac{d\tilde{\alpha}_d}{d\tau_x} < 0$ and $\frac{d\left(\frac{2\lambda}{(1+\zeta)s(\hat{z})}\right)}{d\tau_x} > 0$. To show that $\frac{d\eta}{d\tau_x} < 0$, it suffices to show that $\frac{d\chi}{d\tau_x} < 0$. Note that $\chi = \left(\frac{\hat{\pi}+k}{\tau_x^{\sigma-1}f_x}\right)^{\gamma}$ is decreasing in τ_x if

$$\frac{\tau_x}{\hat{\pi}+k}\frac{d\hat{\pi}}{d\tau_x} < \sigma - 1. \quad (53)$$

Since $\hat{\pi}$ is endogenously determined by the two free entry conditions, (31) and (32), we need to differentiate these two conditions with respect to τ_x, and then solve for $\frac{d\hat{z}}{d\tau_x}$ and $\frac{d\hat{\pi}}{d\tau_x}$ in order to see whether (53) holds in equilibrium. The easier way is, however, to recall that the key endogenous variables $\{\hat{z}, \hat{\pi}\}$ satisfies (22) in the open-economy equilibrium:

$$\hat{\pi} = f^D - f^U\hat{z}.$$

Thus we use (32) and (22) (rather than (31) and (32)) to prove that $\frac{d\chi}{d\tau_x} < 0$. Differentiating (32) and using (22),

$$\frac{d\hat{\pi}}{d\tau_x} = \frac{(\sigma-1)\phi^U\frac{\gamma\chi}{\tau_x}f_x}{\left(-\frac{\partial\phi^U}{\partial z}\frac{1}{f^U} + \frac{\partial\phi^U}{\partial\pi}\right)(\hat{\pi}+k+\chi f_x) + \phi^U\left(1 + \frac{\gamma\chi}{\hat{\pi}+k}f_x\right)}, \quad (54)$$

where the denominator is positive (because $\frac{\partial\phi^U}{\partial z} < 0$ and $\frac{\partial\phi^U}{\partial\pi} > 0$). Using (54), we find that (53) holds and hence $\frac{d\chi}{d\tau_x} < 0$ and $\frac{d\eta}{d\tau_x} < 0$. Since all the terms in the numerator of (52) are negative, we have that

$$\frac{d(n_x\tilde{p}_x\tilde{x}_x + n_x^*\tilde{p}_x^*\tilde{x}_x^*)}{d\tau_x} < 0.$$

As for the aggregate export values of intermediate inputs (39), on the other hand, let us rewrite (39) as

$$\kappa n_x^*\tau_x\tau_m^{\kappa}c\tilde{x}_x^* = \kappa\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\zeta}{1+\zeta}\right)\left(\frac{\eta\tilde{\alpha}_d}{(1+\eta)\tilde{\alpha}_d + \frac{2\lambda}{(1+\zeta)s(\hat{z})}}\right)\delta\bar{L}. \quad (55)$$

Since (51) is decreasing in τ_x, it is straightforward from the above proof to show that (55) is also decreasing in τ_x:

$$\frac{d(\kappa n_x^*\tau_x\tau_m^{\kappa}c\tilde{x}_x^*)}{d\tau_x} < 0.$$

Next, we consider the impact of the variable import trade cost τ_m and the relative difficulty of a foreign match θ on (51) and (55). Note that τ_m and θ have the same indirect effect through η, $\tilde{\alpha}_d$ and $s(\hat{z})$ as those with (52):

$$\frac{d\eta}{d\tau_m} < 0, \quad \frac{d\tilde{\alpha}_d}{d\tau_m} < 0, \quad \frac{d\left(\frac{2\lambda}{(1+\zeta)s(\hat{z})}\right)}{d\tau_m} > 0,$$
$$\frac{d\eta}{d\theta} > 0, \quad \frac{d\tilde{\alpha}_d}{d\theta} > 0, \quad \frac{d\left(\frac{2\lambda}{(1+\zeta)s(\hat{z})}\right)}{d\theta} < 0.$$

Note that τ_m and θ also have an indirect effect through $\zeta = \theta\tau_m^{\kappa(1-\gamma)}$. Since ζ is decreasing in τ_m and is increasing in θ, simple inspection of (51) shows that a reduction in τ_m or a rise in θ reinforces the impact on the aggregate export values of final goods. From the functional form in (55), the same claim also applies to the aggregate export values of intermediate inputs.

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