# The Product Cycle with Firm Heterogeneity

## — JEA Meetings —

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# Motivation

- Recent empirical evidence shows that the following factors have large effects on international trade flow:
  1. *Firm-level heterogeneity*
     - Melitz (2003); Helpman, Melitz, and Yeaple (2004)
  2. *International outsourcing*
     - Antràs (2003, 2005); Antràs and Helpman (2004)
  3. *Degree of contract incompleteness*
     - Levchenko (2007); Nunn (2007)
- This paper investigates a new product cycle in which each firm faces different productivity and endogenous organization

## Model (Based on Antràs and Helpman, 2004)

- Final-good producer & Component supplier
- Production: $x = \theta \left(\frac{h}{\eta}\right)^{\eta} \left(\frac{m}{1-\eta}\right)^{1-\eta}, \quad 0 < \eta < 1$
  - Two organizations: $k \in \{V, O\}$
  - Two countries: $\ell \in \{N, S\}$
- Demand: $x = Ap^{-1/(1-\alpha)}, \quad 0 < \alpha < 1$
- Revenue: $R(h, m) = A^{1-\alpha}\theta^{\alpha} \left(\frac{h}{\eta}\right)^{\alpha\eta} \left(\frac{m}{1-\eta}\right)^{\alpha(1-\eta)}$

- Wage: $w^N > w^S$
- Fixed cost: $f_V^S > f_O^S > f_V^N > f_O^N$
- Legal protection: $\delta^N > \delta^S \quad (\delta^\ell \in (0,1))$
- Nash bargaining solution:

$$\begin{cases} \beta_V^\ell = \underbrace{(\delta^\ell)^\alpha}_{\text{outside option}} + \frac{1}{2} \underbrace{[1-(\delta^\ell)^\alpha]}_{\text{gains from relationship}} \\ \qquad = \frac{1}{2}[1+(\delta^\ell)^\alpha] \\ \\ \beta_O^\ell = \frac{1}{2} \end{cases}$$

- Profit-maximizing problems:
  - Final-good producer

$$\max_{h} \quad \beta_k^{\ell} R(h, m) - w^N h$$

  - Component supplier

$$\max_{m} \quad (1 - \beta_k^{\ell}) R(h, m) - w^{\ell} m$$

- From the first-order conditions, we have

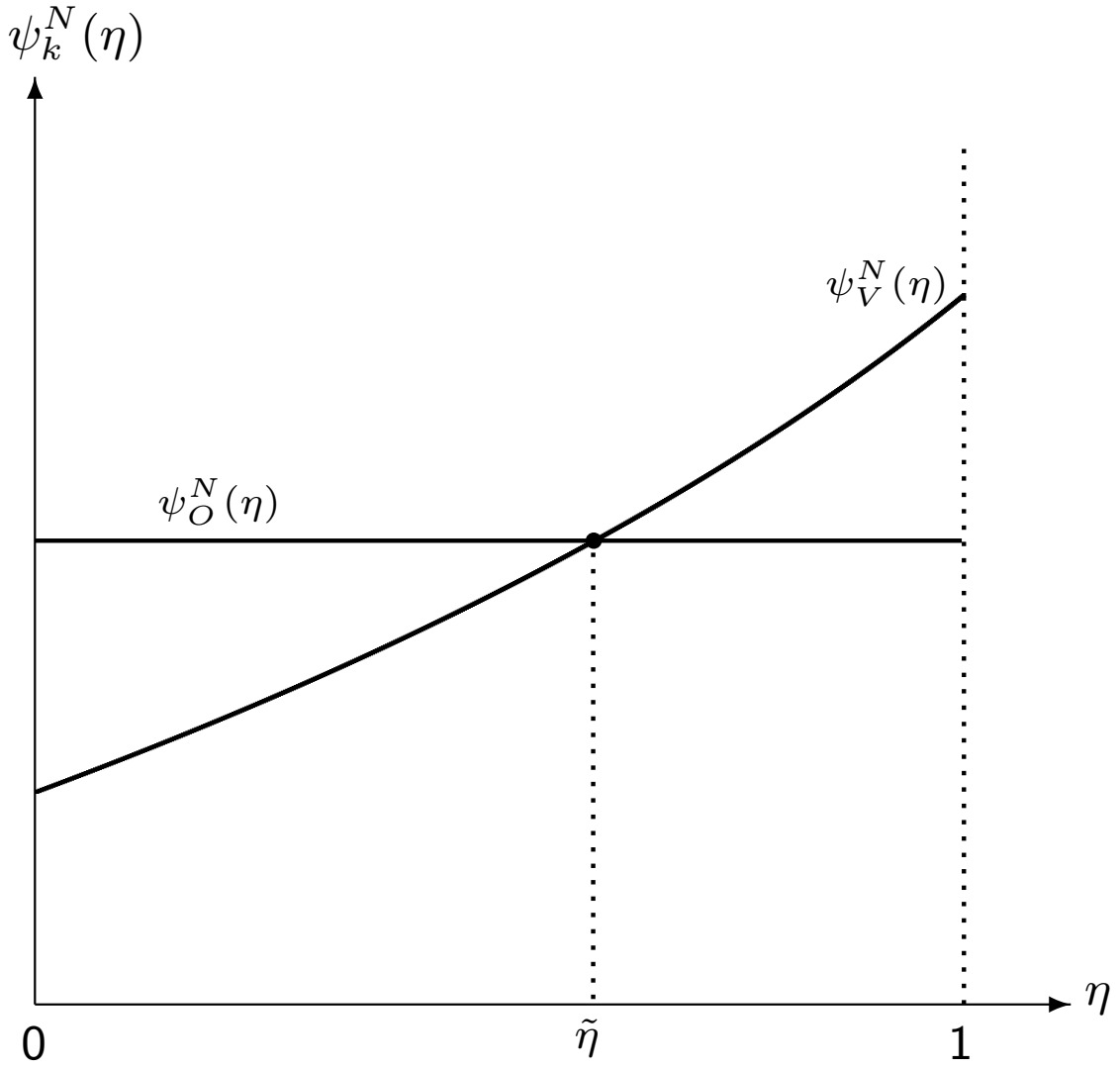
$$\pi_k^{\ell} = A\theta^{\alpha/(1-\alpha)} \psi_k^{\ell}(\eta) - w^N f_k^{\ell}$$

where

$$\psi_k^{\ell}(\eta) = \frac{1 - \alpha[\beta_k^{\ell}\eta + (1 - \beta_k^{\ell})(1 - \eta)]}{(p_k^{\ell}\theta)^{\alpha/(1-\alpha)}},$$

$$p_k^{\ell} = \left(\frac{1}{\theta\alpha}\right) \left(\frac{w^N}{\beta_k^{\ell}}\right)^{\eta} \left(\frac{w^{\ell}}{1 - \beta_k^{\ell}}\right)^{1-\eta}$$

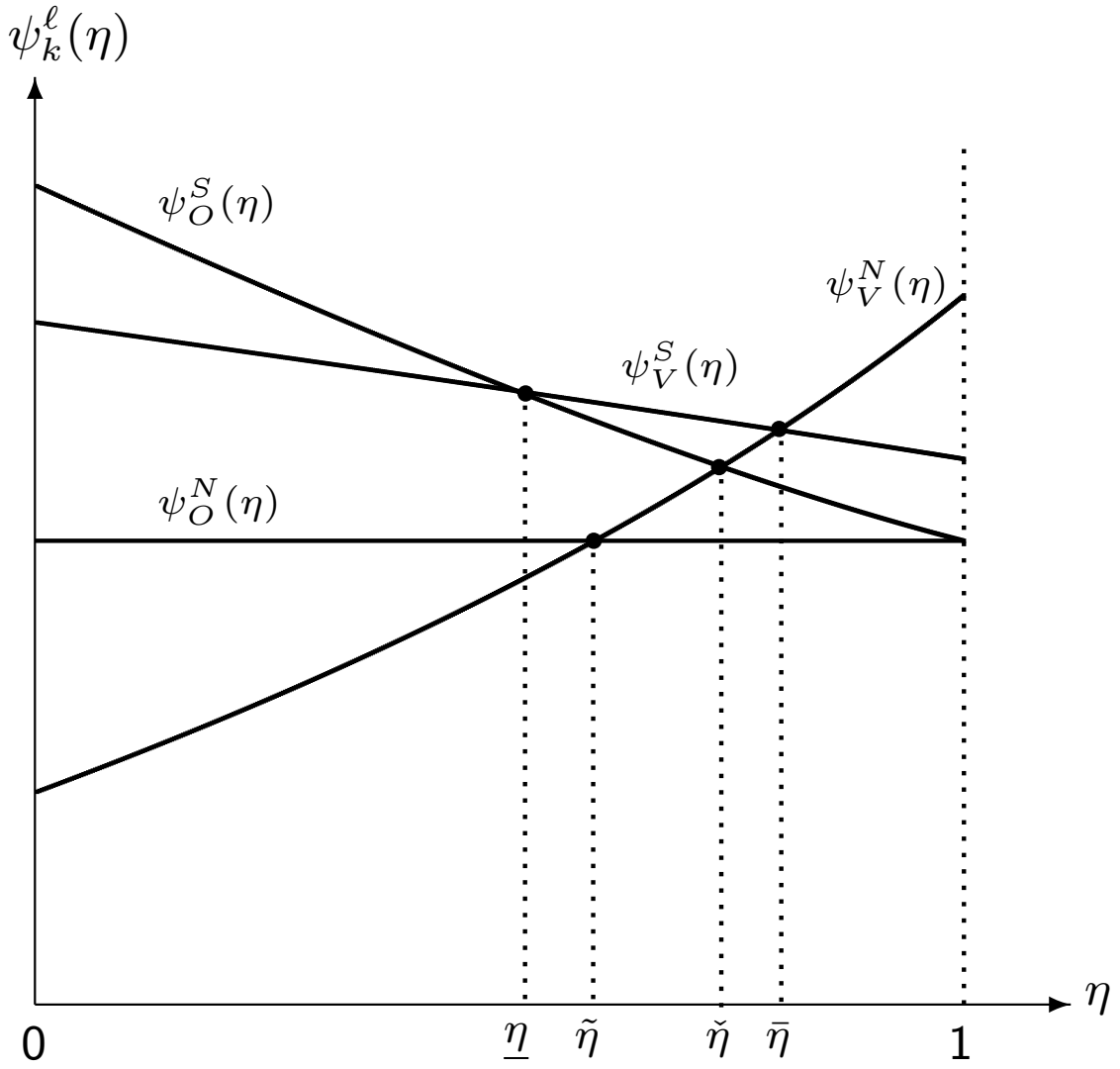
## Relationship between $\psi_k^N(\eta)$




- $\begin{cases} \frac{\partial}{\partial \eta}\psi_V^N(\eta) > 0 \\ \frac{\partial}{\partial \eta}\psi_O^N(\eta) = 0 \\ \psi_V^N(1) > \psi_O^N(1) \\ \psi_V^N(0) < \psi_O^N(0) \end{cases}$

- *Holdup problem* (Hart, 1995)

# Relationship among $\boldsymbol{\psi_k^\ell(\eta)}$




- $\begin{cases} \frac{\partial}{\partial \eta}\psi_V^S(\eta) < 0 \\ \frac{\partial}{\partial \eta}\psi_O^S(\eta) < 0 \\ \psi_V^S(1) > \psi_O^S(1) \\ \psi_V^S(0) < \psi_O^S(0) \end{cases}$
- $\psi_k^S(\eta)$ shifts up due to the lower marginal cost in South

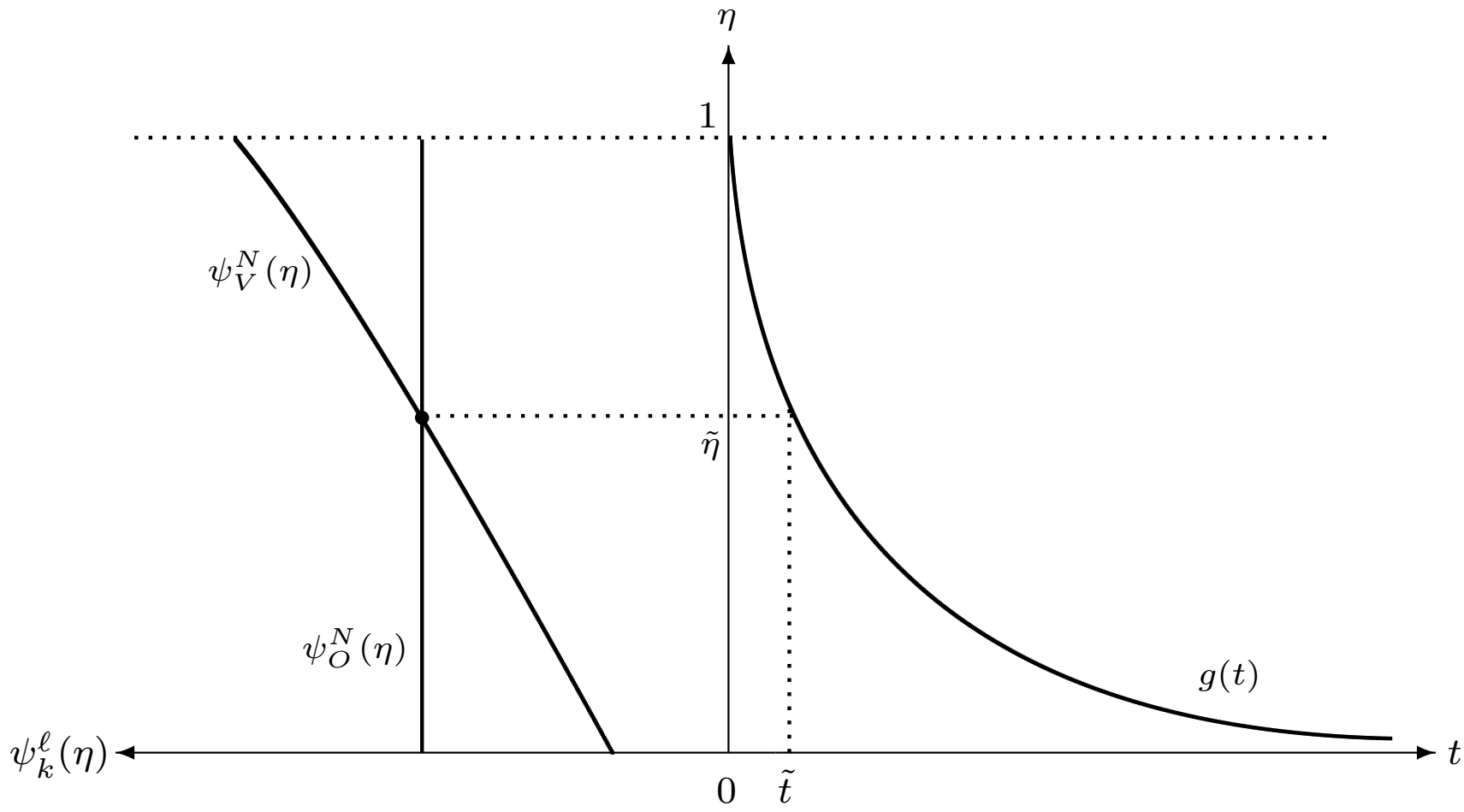
## Dynamics

- As a good matures, the firms' organization of production gradually changes according to their productivity levels
- Following Antràs (2005), the dynamics is captured by $\eta = g(t)$ with

$$g'(t) < 0,\ g(0) = 1,\ \text{and}\ \lim_{t \to \infty} g(t) = 0$$

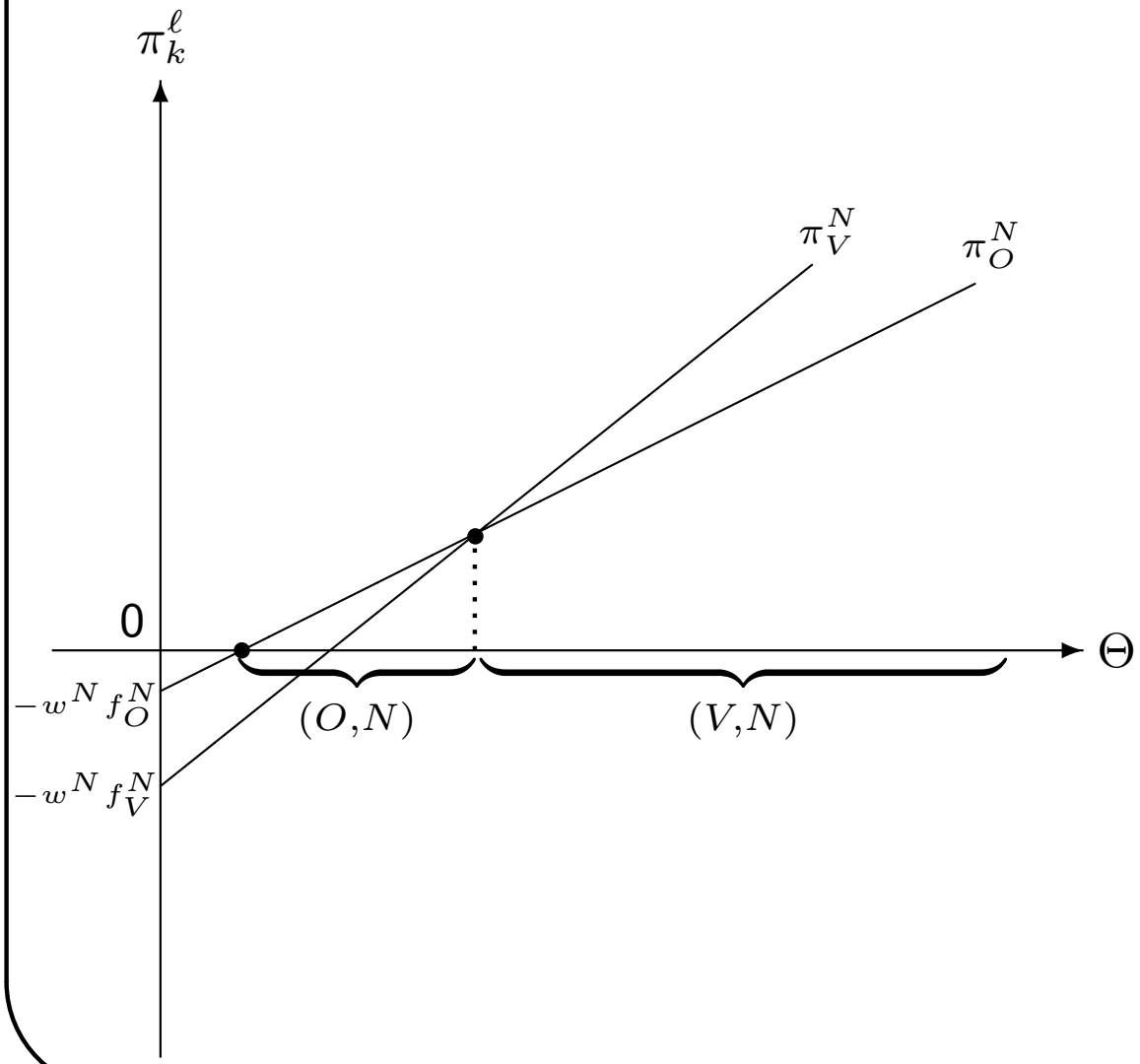
- Time thresholds: $\bar{t},\ \underline{t},\ \tilde{t},\ \check{t}$, where e.g. $\bar{t} = g^{-1}(\bar{\eta})$

# Time threshold



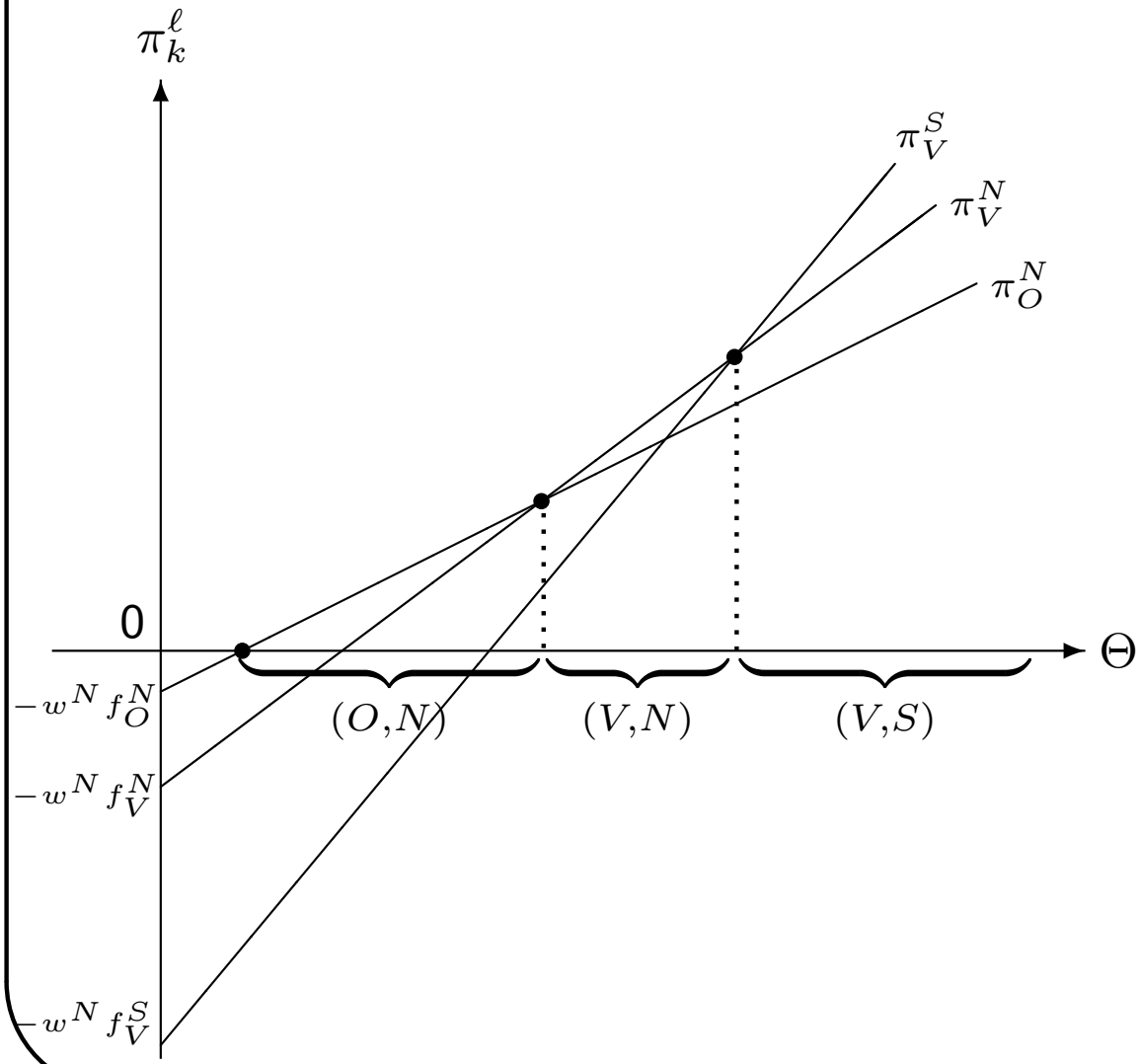

■ $0 \leq \bar{t} < \check{t} < \tilde{t} < \underline{t} < +\infty$

$$0 \leq t < \bar{t}$$




- $\pi_k^\ell = A\Theta\psi_k^\ell(\eta) - w^N f_k^\ell$
- $\Theta = \theta^{\alpha/(1-\alpha)}$
- In this stage, $\psi_k^\ell(\eta)$ is
  $\psi_V^N(\eta) > \psi_V^S(\eta) > \psi_O^S(\eta) > \psi_O^N(\eta)$
- Only the northern production occurs in equilibrium

$$\bar{t} < t < \check{t}$$



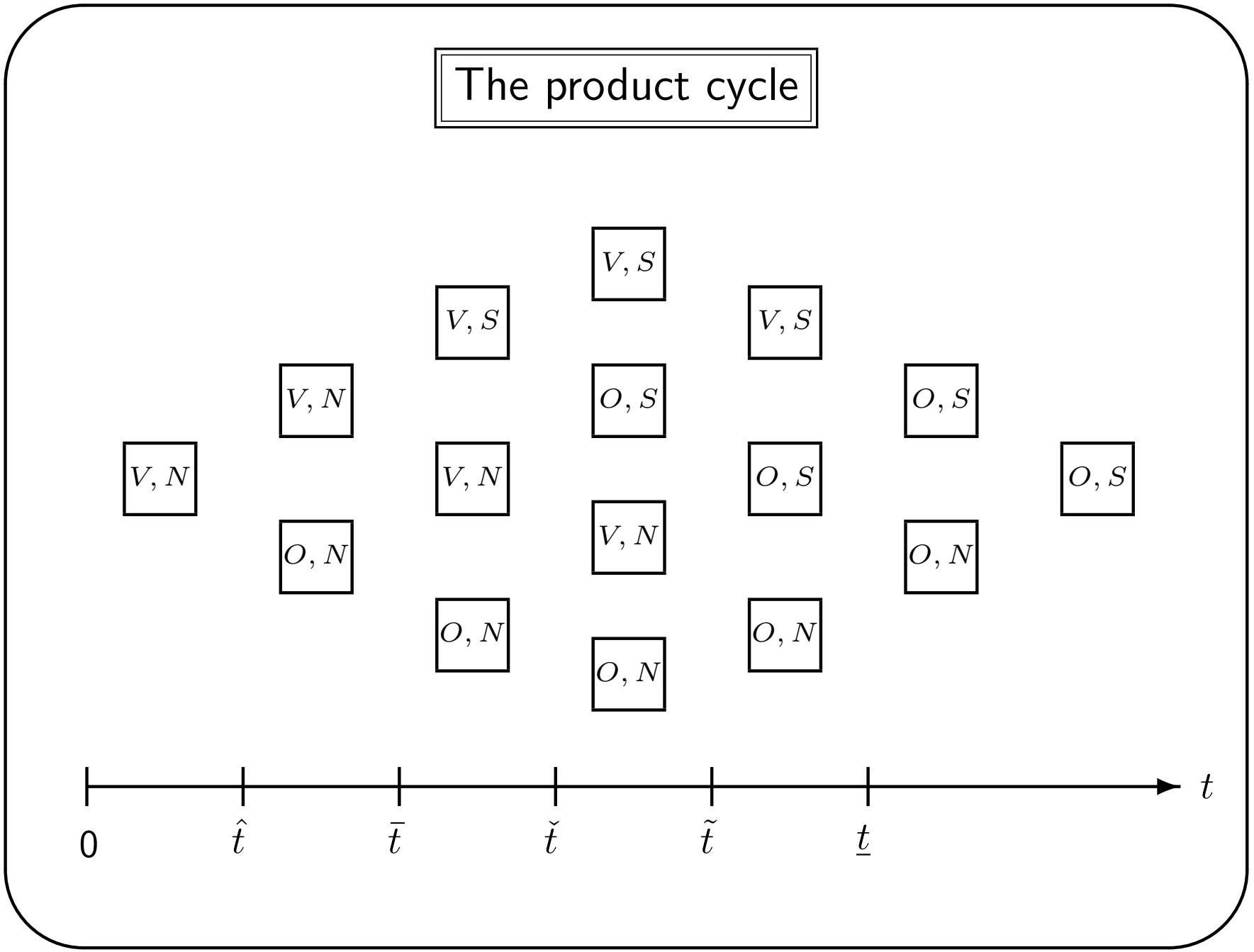

- The ranking of $\psi_k^\ell(\eta)$ becomes

  $$\psi_V^S(\eta) > \psi_V^N(\eta) > \psi_O^S(\eta) > \psi_O^N(\eta)$$

- FDI comes to emerge in equilibrium

- Productivity sorting:

  - Highest $\longrightarrow (V, S)$
  - Intermediate $\longrightarrow (V, N)$
  - Lowest $\longrightarrow (O, N)$


The product cycle
V, S
V, S
V, S
V, N
O, S
O, S
V, N
V, N
O, S
O, S
V, N
O, N
O, N
O, N
O, N
O, N
t
0
$\hat{t}$
$\bar{t}$
$\check{t}$
$\tilde{t}$
$\underline{t}$

- The shift to domestic outsourcing is earlier than the shift to FDI:
  - Empirical evidence reports that this is not always true
  - The strong legal protection in North is crucial for this shift
  - If $\delta^N = \delta^S$, this product cycle never occurs