Optimal Tariffs in the Melitz Model: A Sufficient Statistics Approach for Trade Policy*

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Abstract

This paper shows that the variable nature of the trade elasticity provides new policy implications for optimal tariffs. To achieve the goal, we develop a heterogeneous firm model with (*i*) a general productivity distribution so that the trade elasticity is bilateral-specific to country-pairs; (*ii*) no outside good so that the wage rate is endogenous; and (*iii*) import tariffs so that tariff revenue is one of the welfare components. In this general setting, we find that the optimal level of import tariffs is the same across different trade models with a constant trade elasticity, conditional on the two sufficient statistics for welfare—the domestic trade share and the trade elasticity. However, the equivalence of optimal tariffs across different trade models no longer holds when the trade elasticity differs across markets. Calibrating the model to US data, optimal tariffs with a variable trade elasticity are substantially lower than those with a constant trade elasticity. Moreover, using analytical solutions of comparative statics, the effect of country size on optimal tariffs is quantitatively much smaller than that of variable trade costs.

Keywords: Optimal tariffs, variable trade elasticity, trade liberalization, country size.
JEL Classification Numbers: F12, F13, F16

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1 Introduction

Gains from trade can be calculated by the only two sufficient statistics—the response of trade flows to changes in trade costs and the share of domestic expenditure—in a large class of trade models (Arkolakis et al., 2012). Stimulated by the theoretical results, recent empirical research has estimated the two sufficient statistics and substantiated significant heterogeneity in these measures that systematically vary with country characteristics. The first element, the "trade elasticity," tends to differ depending on whether country-pairs are proximate or distant, and large or small. For example, Bas et al. (2017) find that the trade elasticity is smaller for proximate country-pairs where the trade volume is already large. The second element, the "domestic trade share," also tends to be considerably affected by trade environments, in the sense that the domestic trade share is higher, the larger and the *less* open are countries.[1] These pieces of empirical evidence imply that trade liberalization and country size may have a different effect on the two sufficient statistics for welfare gains from trade. Thus we need to take account of cross-country differences in these statistics to correctly expect the impact of trade, such as reduction in trade costs and expansion in market size through trade agreements.

How does heterogeneity in the trade elasticity and the domestic trade share affect optimal trade policy? To address this key question, we develop an asymmetric-country version of the Melitz (2003) model with CES preferences and monopolistic competition. One of the drawbacks in this framework is that when productivity is Pareto distributed—one of the most commonly used productivity distributions in the literature, the trade elasticity is *unique* to any country-pairs, irrespective of country characteristics. Moreover, firms' markups are constant which, under firm heterogeneity, implies that country size has no effect on the domestic trade share via selection. To circumvent these limitations and provide more realistic policy implications, we make three main departures from existing work. First, we work with a general productivity distribution that makes the trade elasticity *bilateral-specific* to country-pairs. Second, we consider endogenous wages that restore the role of country size in the domestic trade share via selection. Finally, we analyze not only iceberg trade costs but also import tariffs that a government chooses so as to maximize welfare. These distinctions jointly help us to understand the different effect of competitive pressures on the two sufficient statistics for welfare and address its consequence for optimal trade policy in a unified single setting.

Our starting point is to note that not only does trade liberalization but also country size affects wages. Trade costs have been steadily declining over time by both technological improvements and trade negotiations. For example, Hummels (2007) finds that the measure of international air transport prices per ton has fallen more than ten times worldwide between 1955 and 2004 due largely to the adoption of jet engines; similarly continuous effort by the World Trade Organization (WTO) has decreased worldwide average tariffs from 8.6 percent to 3.2 percent between 1960 and 1995, greatly increasing wages of trading countries. On the other hand, the significance of changes in country size is best demonstrated with an example. Figure 1 displays the transition in population and GDP per capita as a measure of country size and wages respectively. Panel A shows the case of the United States, indicating a clear monotone relationship between population and GDP per capita. In contrast, Panel B shows the case of Japan where population is gradually declining due mainly to the low birthrate. According to the Cabinet Office of Japan, the population is expected to decrease from 124 million in 2020 to 97 million in 2050 and to 86 million in 2060. It is often said that gradient shrinking in its domestic market size together with heavy reliance on overseas demand could force Japan to see a steep decline in GDP per capita (Nikkei Asia, 2019). This shows that changes in both trade costs and country size were significant over the last decades, critically affecting wages and hence national welfare.

[1] For this trend in the domestic trade share, see Eaton and Kortum (2002, 2011) with aggregate data, and Bernard et al. (2007) and Mayer and Ottaviano (2008) with firm-level data.

Panel A. United States

Panel B. Japan

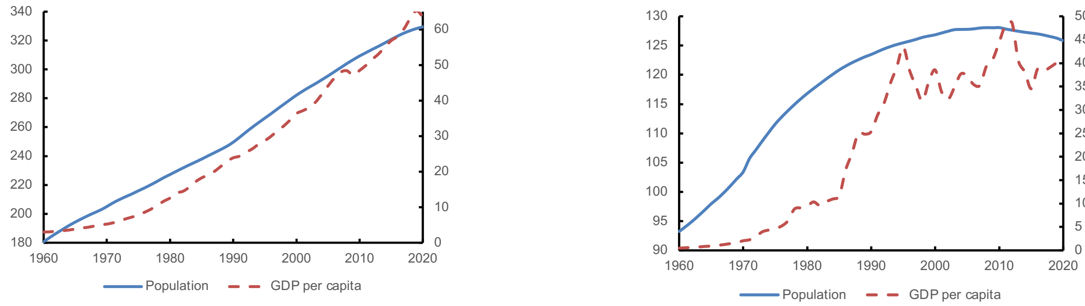

Figure 1: Population and GDP per capita during 1960–2020

Source: World Bank Data.
Note: The left (right) scale measures population in units of millions (GDP per capita in units of thousand US dollars).

We show that if the wage rate is endogenous, a large country entails *weak* domestic selection. To see why, consider reduction in trade costs which has two effects on expected profits. First, trade liberalization directly decreases expected profits by reducing markups on imports. Second, such liberalization indirectly increases expected profits by reducing wages, i.e., production costs from the viewpoint of firms, as trade liberalization leads to a rise in imports which is counteracted by a fall in wages (Demidova and Rodríguez-Clare, 2013). In contrast, expansion in country size has no direct effect on expected profits through markups under CES preferences and monopolistic competition, whereas it indirectly decreases expected profits by raising wages, as a large country has high wages when there are trade costs (Krugman, 1980). Thus, more efficient firms find it profitable to produce in another country with lower wages, but less efficient firms find it possible to survive in a country with higher wages, which causes weak domestic selection in a large country.

Weak domestic selection associated with increased wages may account for the shift in trade patterns that Japan experienced in the late 1990s. Using firm-level data on manufacturing for Japan, Fukao et al. (2008) explore how firms' productivity differences affected firms' turnover between 1990 and 2003. They find that the turnover rate is significantly higher for less productive firms, but nearly a half of the top 10 percent of the most productive firms also exit. This puzzling fact can be explained by increased wages in Japan, which induces these most productive firms to seek for cheaper labor in foreign markets such as China, while simultaneously allowing less productive firms to survive in the domestic market in Japan.

Our selection effect of country size contradicts the finding in Melitz and Ottaviano (2008). The key reason comes from a freely traded "outside" good in their model. Then, the wage rate is exogenous and the difference in country size allows for the home market effect on trade patterns such that a large (resp. small) country specializes in the differentiated (resp. outside) good sector. As a result, the larger is country size, the tougher is competition in the differentiated good sector, forcing the least efficient firms to exit. If an outside good is absent, however, the wage rate is endogenous and the difference in country size does not allow for the home market effect via changes in wages as seen above. Thus it is not surprising that the effect of country size on selection is different from Melitz and Ottaviano (2008).[2] While a large country accommodates inefficient firms in the domestic market, it can nonetheless enjoy welfare gains from its market size since a negative impact on domestic selection may be dominated by a positive impact on product variety.

[2]Our paper also differs from Melitz and Ottaviano (2008) in consumer preferences that generate constant or variable markups; however, the absence of an outside good can reverse their results even with variable markups (Demidova, 2017).

Given that endogenous wages create a different effect of trade liberalization and country size on selection, what can we say about its policy implications? In analyzing optimal trade policy, we show that the difference is crucial for the characterization of optimal tariffs, i.e., the welfare-maximizing tariffs that each country would impose without fearing retaliation. In the present model, optimal tariffs are inversely related to a trading partner's export supply elasticity, which is composed of the domestic trade share and the trade elasticity, as in existing models. In contrast to these models, however, trade liberalization and country size do not always lead to high optimal tariffs in our model. From a policy point of view, the effect of country size on optimal tariffs is of particular interest: a large country does not always benefit from high tariffs. Our model shows that although a large country can enjoy a terms-of-trade gain from setting tariffs as in the conventional optimal tariff theory, it also suffers from a welfare loss from weak domestic selection where tariffs accelerate this loss by protecting inefficient firms from foreign competition. With this tradeoff in mind, we find that whether the former benefit of tariffs dominates the latter cost depends on whether the trade elasticity is constant or variable. If the trade elasticity is variable and differs across markets as reported by recent empirical work,[3] optimal tariffs can decline with country size through endogenous changes in the trade elasticity.

To help better appreciate the policy result, following Chaney (2008), let us decompose the trade elasticity into the intensive margin elasticity and the extensive margin elasticity where the former refers to the elasticity of each incumbent firm's shipment whereas the latter refers to the elasticity of new entrants' shipment. Since the intensive margin elasticity is constant under CES preferences and monopolistic competition, the variable nature of the trade elasticity should come from the extensive margin elasticity, which in turn depends on the micro structure of the model. In the homogeneous firm model where all firms export, there is no adjustment margin from new firms' entry (i.e., the extensive margin elasticity is zero) and hence the trade elasticity is the same as the intensive margin elasticity. In the heterogeneous firm model where productivity is drawn from a Pareto distribution, the extensive margin elasticity is constant and so is the trade elasticity (Chaney, 2008). In these special cases, country size has no effect on the trade elasticity and optimal tariffs always increase with country size only through changes in the domestic trade share (Gros, 1987; Felbermayr et al., 2013). In more general cases, however, the theoretical result that the trade elasticity is constant does not hold; more importantly, empirical work has found that the trade elasticity substantially differs across country-pairs. In these cases, country size affects optimal tariffs not only through changes in the domestic trade share but also through changes in the trade elasticity. Due to this additional channel that most of previous work has not taken into account, optimal tariffs do not necessarily increase with country size.

In light of the characterization of optimal tariffs, we finally address quantitative implications of our findings by measuring a discrepancy in optimal tariffs that arises when the trade elasticity is assumed constant despite that the "true" trade elasticity is variable. Our model calibrated to US data indeed demonstrates that optimal tariffs with a variable trade elasticity are substantially lower than those with a constant trade elasticity. In our numerical exercise, levels of optimal tariffs with a variable trade elasticity are around two-thirds (smaller than a half) of those with a constant trade elasticity in the heterogeneous (homogeneous) firm model. The difference in optimal tariffs is primarily accounted for by the trade elasticity being implicitly assumed to be constant in the literature. Using the analytical solutions of comparative statics, we also find that the effect of country size on optimal tariffs is quantitatively much smaller than that of variable trade costs. This quantitative result is consistent with our theoretical result that when the trade elasticity differs across markets, the role of tariffs in improving welfare may be limited for a large country, identifying the potential importance in reconsidering policy implications.

[3] See, for example, Helpman et al. (2008), Novy (2013), Spearot (2013) and Bas et al. (2017). Our theoretical approach is closer to Helpman et al. (2008) and Bas et al. (2017) who rest on CES preferences and monopolistic competition to provide evidence.

A number of papers have explored welfare and policy implications in the homogeneous and heterogeneous firm models. Regarding welfare implications, Arkolakis et al. (2012) derive a simple formula that can capture welfare gains only by the domestic trade share and the trade elasticity. As this applies to a surprisingly large set of trade models, followup papers have examined extension/robustness of their welfare results. For example, Arkolakis et al. (2019) study general demand functions that yield variable markups, Felbermayr et al. (2015) introduce tariffs that raise government revenue, and Head et al. (2014) and Melitz and Redding (2015) consider a non-Pareto distribution that makes the trade elasticity variable. We show that the Arkolakis et al. (2012) welfare formula can be used to reconsider the conventional wisdom of optimal tariffs. In particular, conditional on the two sufficient statistics for welfare, the optimal level of import tariffs is the same across different trade models with a constant trade elasticity, but more generally it depends on the micro structure that makes the trade elasticity variable.[4] We also find that firm heterogeneity drawn outside a Pareto distribution can affect a welfare measurement as in Head et al. (2014) and Melitz and Redding (2015); however the scope of this paper differs from theirs since we analytically show a new optimal tariff formula with a variable trade elasticity and quantitatively investigate the effect of the two major sources of competitive pressures on optimal trade policy. Moreover, like original work by Melitz (2003), they mainly consider trade between symmetric countries and hence cannot distinguish bilateral and unilateral effects on optimal trade policy.

As for policy implications, there is a large literature of optimal tariffs. Gros (1987) derives optimal tariffs in the homogeneous firm model which is inversely related to the trade elasticity and the domestic trade share of a trading partner. Using Ossa (2011)'s framework featured with tariff-induced production relocation effects, Ossa (2014) provides a comprehensive analysis of optimal tariffs in a multi-sector, general-equilibrium model which nests the traditional (terms-of-trade), new trade (profit-shifting) and political-economy motives in the homogeneous firm model. These analyses of optimal tariffs are extended to the heterogeneous firm model by Demidova and Rodríguez-Clare (2009) for a small economy and Felbermayr et al. (2013) for a large economy. In so doing, Felbermayr et al. (2013) show that optimal tariffs are lower in the heterogeneous firm model than the homogeneous firm model, holding the domestic trade share equal. While existing work contributes to our understanding of optimal trade policy, one of the limitations is that the trade elasticity is constant in either the homogeneous or heterogeneous firm model. However, the existence of a constant trade elasticity is highly sensitive to parameter restrictions, and welfare changes can be mis-estimated when the "true" trade elasticity is variable (Melitz and Redding, 2015). In the context of trade policy, the optimal level of import tariffs can be mis-estimated when the same parameter restrictions are imposed. We highlight this key caveat not only by analytically characterizing optimal tariffs with a variable trade elasticity, but also by quantitatively measuring these magnitudes from our model and existing models calibrated to US data.

Recently, Costinot et al. (2020) offer a strict generalization of Gros (1987) in the homogeneous firm model, and Demidova and Rodríguez-Clare (2009) and Felbermayr et al. (2013) in the heterogeneous firm model with a Pareto distribution. They find that when tariffs are uniform across firms, optimal tariffs can be lower in the heterogeneous firm model than in the homogeneous firm model due to non-convexity of aggregate goods across domestic and foreign markets. In contrast, we show that the same result can arise due to variability of the trade elasticity across domestic and foreign markets. Although their new element in optimal tariffs is closely related to ours in the sense that both arise in the presence of selection, our element is relatively easy to measure from firm-level data which is in turn directly used for quantifying optimal tariffs. In that respect, we investigate a different but complementary channel in optimal trade policy.

[4] Our sufficient statistics approach for optimal trade policy is related to that by Lashkaripour (2021) in that such policy can be calculated by the welfare formula by Arkolakis et al. (2012). One of the critical differences is that his baseline analysis builds on the Ricardian setup of Eaton and Kortum (2002), which means that the trade elasticity is constant at the supply side parameter.

2 Setup

Basics. There are two countries indexed by i, j that use labor to produce differentiated goods in one sector. Country i is populated by a mass L_i of identical consumers whose preferences are

$$U_i = \left(\sum_{n=i,j} \int_{\omega \in \Omega_n} q_{ni}(\omega)^\rho d\omega \right)^{1/\rho}, \qquad 0 < \rho < 1,$$

where an elasticity of substitution between varieties is $\sigma = 1/(1-\rho) > 1$. Throughout this paper, we denote the exporting (importing) country by the first (second) subscript. Hence $q_{ji}(\omega)$ is an export quantity shipped from country j to country i of variety ω. Consumer utility maximization subject to budget constraint yields the demand of variety ω, which takes the following form:

$$q_{ji}(\omega) = R_i P_i^{\sigma-1} p_{ji}(\omega)^{-\sigma},$$

where $p_{ji}(\omega)$ is a price of variety ω, P_i is a price index associated with an aggregate good $Q_i \equiv U_i$ and R_i is aggregate expenditure. These aggregates satisfy $P_i Q_i = R_i$.

Firm behavior is similar to that modeled by Melitz (2003). Upon paying fixed entry costs f_i^e (measured in country i's labor units with wages w_i), a mass M_i^e of entrants draw productivity φ from a fixed distribution $G_i(\varphi)$ with support $(\varphi_{\min}, \varphi_{\max})$, where the upper bound is either finite ($\varphi_{\max} < \infty$) or infinite ($\varphi_{\max} = \infty$). If a firm from country j chooses to serve country i, it incurs variable trade costs $\theta_{ji} \geq 1$ (with $\theta_{jj} = 1$) and fixed trade costs f_{ji} (both measured in country j's labor units with wages w_j). Also, a government in each country imposes import tariffs on foreign varieties and the firm incurs ad valorem tariffs $\tau_{ji} = 1 + t_{ji}$, where $\tau_{ji} \geq 1$ (with $\tau_{jj} = 1$). Tariffs are imposed before each firm sets markups, that is, tariffs are modeled as cost shifters ignoring the aspect of demand shifters.[5] As a result, country i's government collects tariff revenue $(\tau_{ji} - 1)p_{ji}(\omega)/\tau_{ji}$ per unit, so that the firm receives $p_{ji}(\omega)/\tau_{ji}$ per unit.

Equilibrium Conditions. A firm from country j to country i with productivity φ incurs both marginal costs $\theta_{ji} w_j/\varphi$ and tariffs τ_{ji}. Given consumer preferences, firm profit maximization implies that the optimal pricing is to charge a constant markup $\sigma/(\sigma-1) = 1/\rho$ over these costs: $p_{ji}(\varphi) = \tau_{ji}\theta_{ji} w_j/\rho\varphi$. In the present setting, it is convenient to define firm revenue *net of tariffs* $r_{ji}(\varphi) \equiv p_{ji}(\varphi) q_{ji}(\varphi)/\tau_{ji}$ which yields firm variable profit $r_{ji}(\varphi)/\sigma$. From consumer demand and firm pricing, firm variable profit is strictly increasing in productivity. As each firm incurs a fixed cost $w_j f_{ji}$, there is a unique productivity cutoff at which a firm makes zero profits, namely $r_{ji}(\varphi_{ji}^*)/\sigma = w_j f_{ji}$. This is referred to as the **zero cutoff profit (ZCP) condition**:

$$B_i \tau_{ji}^{-\sigma} (\theta_{ji} w_j)^{1-\sigma} (\varphi_{ji}^*)^{\sigma-1} = w_j f_{ji}, \tag{1}$$

where

$$B_i \equiv \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} R_i P_i^{\sigma-1}$$

is the index of market demand. Note that (1) also pins down the domestic productivity cutoff φ_{jj}^* when $i = j$. We restrict attention to the case where selection into exporting occurs, $\varphi_{ji}^* > \varphi_{jj}^*$. From (1), this holds when trade costs are sufficiently large and country size is not too different. The latter implies that relative market demand B_i/B_j—proportional to relative country size measured by R_i/R_j—is not too large or too small.

[5]See Felbermayr et al. (2015) for the differences between cost shifters and demand shifters of tariffs.

Free entry requires that expected profits earned from all operating countries equal the fixed entry costs. Following Melitz (2003), let $J_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^*} \right)^{\sigma-1} - 1 \right] dG_i(\varphi)$. From the definition of φ_{ij}^* in light of (1), expected profits earned by firms from country i to country j are given by $w_i f_{ij} J_i(\varphi_{ij}^*)$. Since firms incur the fixed entry costs measured in country i's labor units $w_i f_i^e$, the **free entry (FE) condition** is expressed as

$$\sum_{n=i,j} f_{in} J_i(\varphi_{in}^*) = f_i^e. \tag{2}$$

The FE condition determines the market demands B_i, B_j by adjusting the price indices P_i, P_j in equilibrium, so that potential entrants make zero expected profits.

Labor is used for both entry and production, which must equal aggregate labor supply in the economy. Using (1) and (2), the **labor market clearing (LMC) condition** is expressed as (see Appendix A.1)

$$\frac{R_i - T_i}{w_i} = L_i,$$

where $T_i \equiv (\tau_{ji} - 1) R_{ji}$ is aggregate tariff revenue and R_{ji} is aggregate expenditure on country j's goods in country i. As usual, the LMC condition determines the wage rates w_i, w_j. Rewriting this as $R_i = w_i L_i + T_i$ implies that country i's wages are determined by the equality between aggregate expenditure R_i and aggregate labor income $w_i L_i$ plus aggregate tariff revenue T_i, as there are no pure profits under free entry.

To work on general equilibrium, we relate the LMC condition with the **trade balance (TB) condition**. While the TB condition requires $R_{ij} = R_{ji}$, it is equivalent to the LMC condition in that the two conditions lead to the same equality, $R_i = w_i L_i + T_i$ (see Appendix A.1). Hence we can use either condition to pin down wages in general equilibrium. Let $\lambda_{ji} \equiv \tau_{ji} R_{ji} / \sum_n \tau_{ni} R_{ni}$ denote the foreign trade share from country j's goods in country i inclusive of tariffs. Similarly, let $\tilde{\lambda}_{ji} \equiv R_{ji} / \sum_n R_{ni}$ denote the corresponding trade share net of tariffs. Solving λ_{ji} for R_{jj}/R_{ji} and substituting it into the definition of $\tilde{\lambda}_{ji}$,

$$\tilde{\lambda}_{ji} = \frac{\lambda_{ji}}{\tau_{ji}(1 - \lambda_{ji}) + \lambda_{ji}}.$$

Not surprisingly, $\tilde{\lambda}_{ji} = \lambda_{ji}$ when country i does not impose tariffs on foreign goods imported from country j ($\tau_{ji} = 1$). Using this share, aggregate expenditure spent on domestic and foreign goods are respectively given by $R_{ii} = \tilde{\lambda}_{ii} w_i L_i$ and $R_{ji} = \tilde{\lambda}_{ji} w_i L_i$. Moreover, using these aggregates in the LMC condition $R_i = w_i L_i + T_i$, we get a familiar expression of the TB condition that applies to the presence of tariffs:

$$w_i L_i = \sum_{n=i,j} \tilde{\lambda}_{in} w_n L_n. \tag{3}$$

Now, we are ready for characterizing levels of the key endogenous variables when countries have an option to set tariffs. For given levels of exogenous variables, **equilibrium in levels** is defined as a set of the vector $\{\varphi_{ii}^*, \varphi_{ij}^*, B_i, w_i\}$ which is jointly characterized by the system of eight equations in (1), (2), and (3) for i, j. By Walras's law, levels of wages in country j can be normalized to unity, $w_j = 1$. Once levels of these variables are determined, levels of other endogenous variables are written as a function of them. In particular, using the definition of B_i in (1), welfare per worker is expressed as follows (see Appendix A.2):

$$W_i = \left(\frac{L_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma-1}} (\mu_i)^{\frac{1}{\rho}} \rho \varphi_{ii}^*, \tag{4}$$

where $\mu_i \equiv R_i/w_iL_i$ is referred to as a "tariff multiplier" (Felbermayr et al., 2015) in the analysis below. The multiplier enters the welfare expression (4) because tariff revenue is assumed to be rebated back to consumers. Rewriting $\mu_i = 1 + (\tau_{ji} - 1)\tilde{\lambda}_{ji}$ from $R_i = w_iL_i + T_i$ and plugging $\tilde{\lambda}_{ji}$, we get

$$\mu_i = \frac{\tau_{ji}}{\tau_{ji}(1-\lambda_{ji}) + \lambda_{ji}}.$$

Obviously, $\mu_i = 1$ when country i does not impose tariffs on foreign goods.

3 Trade Liberalization

The previous section has defined equilibrium in *levels*. Based on that, the next two sections define equilibrium in *changes*. We start with examining the impact of trade barriers, holding other exogenous variables constant. Demidova and Rodríguez-Clare (2013) consider a welfare effect of asymmetric trade liberalization in the Melitz (2003) model, dispensing with the assumptions of a Pareto distribution and an outside good. They show in particular that unilateral reduction in trade barriers of either exporting or importing always increases welfare in a liberalizing country, which stands in sharp contrast to the presence of an outside good in the model with CES preferences (Demidova, 2008) and quadratic preferences (Melitz and Ottaviano, 2008). In this section, with the help of the hat algebra, we analytically show their results.[6]

Suppose that country i unilaterally reduces trade costs of importing from country j. While we focus mainly on the impact of variable trade costs θ_{ji} below, the impact of fixed trade costs f_{ji} and ad valorem tariffs τ_{ji} on the key equilibrium variables is similar. In contrast to variable and fixed trade costs, tariffs do not use up real resources and instead raise government revenue, which may give an incentive to manipulate the terms of trade. In Section 5, we will characterize welfare-maximizing optimal tariffs by taking account of this motive. In the circumstances, let a "hat" denote proportional changes in variables of interests (e.g., $\hat{x} \equiv dx/x$). Taking the log and differentiating the ZCP condition (1) with respect to θ_{ji},

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ji} = \sigma\hat{w}_j + (\sigma - 1)\hat{\theta}_{ji}. \tag{5}$$

Similarly, differentiating the FE condition (2) with respect to θ_{ji} and rearranging,

$$\hat{\varphi}^*_{ij} = -\alpha_i\hat{\varphi}^*_{ii}, \tag{6}$$

where

$$\alpha_i \equiv \frac{f_{ii}J_i'(\varphi^*_{ii})\varphi^*_{ii}}{f_{ij}J_i'(\varphi^*_{ij})\varphi^*_{ij}}.$$

Since $J_i(\varphi^*)$ is strictly decreasing in φ^*, α_i is positive. Then (6) means that changes in θ_{ji} always shift φ^*_{ii} and φ^*_{ij} in opposite directions. Moreover, exploiting the facts that $J_i(\varphi^*)$ relates to expected profits and that the TB condition requires $R_{ij} = R_{ji}$, we can show that α_i equals the ratio of aggregate expenditure spent on domestic and foreign goods in country i, i.e., R_{ii}/R_{ji} (see Appendix A.3). This in turn allows us to express the foreign trade shares and the tariff multiplier in terms of α_i:

$$\lambda_{ji} = \frac{\tau_{ji}}{\alpha_i + \tau_{ji}}, \quad \tilde{\lambda}_{ji} = \frac{1}{\alpha_i + 1}, \quad \mu_i = \frac{\alpha_i + \tau_{ji}}{\alpha_i + 1}.$$

[6] Though the results in this section are not entirely new, previous work has not provided the analytical solutions of the impact of trade costs under a general productivity distribution, which is shown to be useful in quantifying the comparative statics.

Finally, using $\tilde{\lambda}_{ji}$ introduced above, rewrite the TB condition (3) as $w_i L_i/(\alpha_i+1) = w_j L_j/(\alpha_j+1)$ where α_i is a function of $\varphi_{ii}^*, \varphi_{ij}^*$. Differentiating this equality with respect to θ_{ji} and using (6),

$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}_{ii}^* + \beta_j \hat{\varphi}_{jj}^*, \tag{7}$$

where

$$\beta_i \equiv \frac{\alpha_i}{\alpha_i+1}[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij})\alpha_i],$$

and $\gamma_{in} \equiv -d \ln \int_{\varphi_{in}^*}^{\varphi_{\max}} \varphi^{\sigma-1} dG_i(\varphi)/d \ln \varphi_{in}^*$ is the extensive margin elasticity that arises in firm heterogeneity.[7] (7) shows that changes in wages (by variable trade costs) are associated with changes in productivity cutoffs, which operates through two channels. First is the *intensive* margin: changes in θ_{ji} lead to an adjustment in each incumbent firm's shipment through changes in consumer demand over foreign goods with an elasticity of $\sigma-1$. Second is the *extensive* margin: changes in θ_{ji} lead to an adjustment in new entrants' shipment through changes in competitiveness of each market with an elasticity of γ_{in}. Changes through these two margins are given in β_i, which is also a function of $\varphi_{ii}^*, \varphi_{ij}^*$.

Now, we are ready for characterizing changes in the key endogenous variables. Just as (1), (2) and (3) are used to solve for equilibrium in levels, (5), (6) and (7) are also used to solve for equilibrium in changes. In the comparative statics examined here, for given changes in variable trade costs $\hat{\theta}_{ji}$, **equilibrium in changes** is defined as a set of the vector $\{\hat{\varphi}_{ii}^*, \hat{\varphi}_{ij}^*, \hat{B}_i, \hat{w}_i\}$ which is jointly characterized by the system of eight equations in (5), (6), and (7) for i, j. By Walras's law, changes in country j's wages can be normalized to zero, $\hat{w}_j = 0$. Once changes in these variables are determined, changes in other endogenous variables are written as a function of them. In particular, changes in welfare per worker are expressed as (see Appendix A.4)

$$\hat{W}_i = \left(\frac{(\tau_{ji}-1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}_{ii}^*. \tag{8}$$

This means that the domestic productivity cutoff φ_{ii}^* is a single sufficient statistic for welfare even with tariff revenue. Changes in θ_{ji} induce changes in welfare through changes in φ_{ii}^* that relate to resource reallocations à la Melitz (2003). In the presence of tariffs, such changes also induce changes in welfare through changes in the tariff multiplier μ_i that relate to tariff revenue rebated back to consumers. (8) shows however that welfare changes associated with these two effects are captured solely by changes in φ_{ii}^*.

It is possible to solve the system of eight equations ((5), (6), (7)) for eight unknowns ($\hat{\varphi}_{ii}^*, \hat{\varphi}_{ij}^*, \hat{B}_i, \hat{w}_i$ for i, j), where we have chosen labor in country j as the numéraire. Solving (5), (6) and (7) simultaneously yields the following relationships for equilibrium in changes:

$$\begin{aligned} \hat{\varphi}_{ii}^* &= -\frac{\rho(\beta_j+\rho)}{\Xi}\hat{\theta}_{ji}, \\ \hat{\varphi}_{jj}^* &= -\frac{\rho(\beta_i-\rho\alpha_i)}{\Xi}\hat{\theta}_{ji}, \\ \hat{w}_i &= \frac{\rho^2(\beta_i+\alpha_i\beta_j)}{\Xi}\hat{\theta}_{ji}, \end{aligned} \tag{9}$$

where $\beta_i - \rho\alpha_i > 0$ (from the definitions of α_i and β_i) and $\Xi \equiv \sum_n(\beta_n+\rho) - \sum_n(\beta_n - \rho\alpha_n) > 0$. From (9), reduction in θ_{ji} increases $\varphi_{ii}^*, \varphi_{jj}^*$ and decreases w_i. From (8), it then follows that welfare rises in country j as well as country i since a decline in w_i is smaller than a decline in P_i thereby increasing real wages w_i/P_i. Tariff revenue rebated back to consumers increases by raising μ_i, which additionally contributes to welfare.

[7] See Arkolakis et al. (2012, p.110) where unit labor requirements in their paper are the *inverse* of productivity in our paper.

Intuition behind the results is clearly seen by solving (5) and (6) first without (7):

$$\begin{aligned} \hat{\varphi}^*_{ii} &= \frac{1}{\alpha_i\alpha_j - 1}\hat{\theta}_{ji} - \frac{\alpha_j + 1}{\rho(\alpha_i\alpha_j - 1)}\hat{w}_i, \\ \hat{\varphi}^*_{jj} &= -\frac{\alpha_j}{\alpha_i\alpha_j - 1}\hat{\theta}_{ji} + \frac{\alpha_i + 1}{\rho(\alpha_i\alpha_j - 1)}\hat{w}_i, \end{aligned} \tag{10}$$

where $\alpha_i\alpha_j - 1 > 0$. In (10), the first term is the direct effect of variable trade costs and the second term is the indirect effect through changes in wages.[8] The direct effect decreases expected profits in a liberalizing country by reducing markups on imports, but increase expected profits in a non-liberalizing country by allowing firms to export more easily. As a result, reduction in θ_{ji} deters entry in country i and induces entry in country j, decreasing φ^*_{ii} and increasing φ^*_{jj}. Note that this effect exists even when wages are fixed by an outside good. From (8), these changes imply that unilateral trade liberalization decreases welfare in a liberalizing country but increases welfare in non-liberalizing country (Demidova, 2008; Melitz and Ottaviano, 2008).

If an outside good is absent, reduction in θ_{ji} leads to a rise in imports in country i which is counteracted by a fall in wages. The indirect effect increases expected profits in a liberalizing country by reducing production costs from the viewpoint of firms, but decreases expected profits in a non-liberalizing country by raising production costs there relative to a liberalizing country. Hence, reduction in w_i (by reduction in θ_{ji}) induces entry in country i but deters entry in country j, increasing φ^*_{ii} and decreasing φ^*_{jj}. While the indirect effect operates in opposite directions to the direct effect, (9) shows that both φ^*_{ii} and φ^*_{jj} rise as a result of reduction in θ_{ji}, which holds only when the indirect effect outweighs the direct effect for φ^*_{ii} but the converse is true for φ^*_{jj}. Then these changes imply that unilateral trade liberalization increases welfare in a liberalizing country as well as in a non-liberalizing country (Demidova and Rodríguez-Clare, 2013).

While we have focused on the impact of variable trade costs of importing θ_{ji}, the impact of *any* trade costs $(\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji})$ on productivity cutoffs is qualitatively similar (see Appendix A.5). In case of variable trade costs of exporting θ_{ij}, for example, we get

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\rho(\beta_j - \rho\alpha_j)}{\Xi}\hat{\theta}_{ij}, \\ \hat{\varphi}^*_{jj} &= -\frac{\rho(\beta_i + \rho)}{\Xi}\hat{\theta}_{ij}, \\ \hat{w}_i &= -\frac{\rho^2(\beta_j + \alpha_j\beta_i)}{\Xi}\hat{\theta}_{ij}. \end{aligned}$$

Hence, reduction in export costs θ_{ij} also increases the domestic productivity cutoff in both countries as above. The only difference is that reduction in *import* costs θ_{ji} *decreases* w_i, whereas reduction in *export* costs θ_{ij} *increases* w_i. The difference in wage changes reflects the fact that reduction in θ_{ij} leads to a rise in exports in country i (or equivalently a rise in imports in country j) which must be counteracted by a rise in wages. The same claim applies to fixed trade costs and tariffs.

Finally, note that, starting from a symmetric situation, welfare gains from unilateral trade liberalization are always greater in a liberalizing country than in a non-liberalizing country. Consider the effect of variable trade costs of importing θ_{ji}. Evaluating (9) at $\alpha_i = \alpha_j$ and $\beta_i = \beta_j$ reveals that $|\hat{\varphi}^*_{ii}| > |\hat{\varphi}^*_{jj}|$, which implies that $\hat{W}_i > \hat{W}_j$ from (8). Thus, reduction in θ_{ji} leads to greater welfare gains in country i than in country j. The result holds for variable trade costs of exporting θ_{ij} in the sense that, starting from a symmetric situation, reduction in θ_{ij} leads to greater welfare gains in country j than country i.

[8] To be precise, changes in wages are changes in country i's relative wages since country j's wages are normalized to unity.

Proposition 1 *Unilateral trade liberalization has the following effects:*

(i) *The wage rate falls in a liberalizing country.*

(ii) *The domestic productivity cutoff rises in both liberalizing and non-liberalizing countries. As a result, the domestic trade share falls in both countries.*

(iii) *Unilateral trade liberalization is always welfare-enhancing for both countries. Starting from a symmetric situation, the welfare effect is always greater in a liberalizing country than in a non-liberalizing country.*

Proposition 1 is essentially the same as the finding in Demidova and Rodríguez-Clare (2013).[9] They find, without resorting to a specific productivity distribution and introducing an outside good, that endogenous wages can reverse the impact of asymmetric trade liberalization on welfare in a liberalizing country due to a failure of the home market effect on the trade patterns. One of the differences is that they graphically show their results with a simple figure, while we analytically show our results with the hat algebra. More important is our tractability in studying the impact of another competitive measure, country size, which can be examined in a parallel manner with trade liberalization without parameterizing a productivity distribution (Section 4). Furthermore, our analytical solutions of comparative statics allow us to address the quantitative impact of unilateral changes in these competitive pressures on optimal trade policy (Sections 5 and 6).

4 Country Size

We next consider the impact of country size, holding other exogenous variables constant, which also has been extensively explored in the literature. Melitz and Ottaviano (2008) are the first to show that a country with larger size entails higher productivity and welfare through tougher competition in the domestic market. Due to the existence of an outside good incorporated in their model, however, they find that trade liberalization and country size have an opposite impact on welfare in a country of origin: a unilaterally liberalizing country is worse off by relocating entry from a liberalizing country to a non-liberalizing country. We show that, in the absence of an outside good, endogenous wages can also reverse the impact of country size on selection under any productivity distribution function: a large country faces weak domestic selection so that it accommodates inefficient firms in the domestic market, which stands in sharp contrast to the finding in Melitz and Ottaviano (2008). Although this channel via selection negatively affects welfare, a large country can nonetheless enjoy welfare gains from its market size since a negative effect on domestic selection may be dominated by a positive effect on product variety.

Suppose that country i unilaterally expands market size L_i. Denoting proportional changes in variables of interests by a "hat" again, and taking the log and differentiating the ZCP condition (1) with respect to L_i,

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ji} = \sigma \hat{w}_j. \tag{11}$$

Changes in the FE condition are the same as (6), changes in the TB condition (3) with respect to L_i are

$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}^*_{ii} + \beta_j \hat{\varphi}^*_{jj} - \hat{L}_i, \tag{12}$$

[9] The results also relate to Felbermayr et al. (2013), though their analysis is less general than ours in the sense that it relies on a Pareto distribution. The restriction is shown to have important consequences for policy implications.

where the definitions of α_i and β_i are exactly the same as those in Section 3. In contrast to changes in θ_{ji}, however, changes in country size have no direct impact on productivity cutoffs in (11) while such changes have a direct impact on wages in (12).

The definition of equilibrium in changes with respect to country size is also similar to that in Section 3: for given changes in country size $\hat{L}_i$, equilibrium in changes is defined as a set of the vector $\{\hat{\varphi}^*_{ii}, \hat{\varphi}^*_{ij}, \hat{B}_i, \hat{w}_i\}$ which is jointly characterized by (6), (11) and (12) for i, j. The only important difference is that changes in welfare per worker must be modified as country size directly enters the welfare expression (4). Consequently, welfare changes that correspond to (8) are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}^*_{ii} + \frac{\hat{L}_i}{\sigma - 1}. \tag{13}$$

Hence, we need to take account of changes in both φ^*_{ii} and L_i to evaluate the welfare impact.

As in unilateral trade liberalization, we can explicitly solve the system of equations in (6), (11) and (12). Solving these eight equations for the eight unknowns, we get

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\rho(\alpha_j + 1)}{\Xi} \hat{L}_i, \\ \hat{\varphi}^*_{jj} &= \frac{\rho(\alpha_i + 1)}{\Xi} \hat{L}_i, \\ \hat{w}_i &= \frac{\rho^2(\alpha_i \alpha_j - 1)}{\Xi} \hat{L}_i. \end{aligned} \tag{14}$$

(14) shows that expansion in L_i decreases φ^*_{ii} but increases φ^*_{jj} and w_i. From (13), it then follows that welfare always rises in country j, but can rise or fall in country i depending on the extent to which expansion in market size decreases the domestic productivity cutoff there.

Intuition is again clearly explained by solving (6) and (11) first without (12):

$$\begin{aligned} \hat{\varphi}^*_{ii} &= -\frac{\alpha_j + 1}{\rho(\alpha_i \alpha_j - 1)} \hat{w}_i, \\ \hat{\varphi}^*_{jj} &= \frac{\alpha_i + 1}{\rho(\alpha_i \alpha_j - 1)} \hat{w}_i. \end{aligned} \tag{15}$$

Simple comparison between (10) and (15) immediately reveals that the direct effect of country size is absent in this case due to the peculiar and restrictive property of CES preferences and monopolistic competition, and there is only the indirect effect through changes in wages. Hence, if wages are exogenously fixed by an outside good, (15) shows that country size has no impact on the domestic productivity cutoff. Because expansion in L_i does not induce entry or exit, (13) implies that unilateral market expansion increases welfare in an expanding country due solely to increased product variety, as is standard with a heterogeneous firm model (Melitz, 2003), let alone a homogeneous firm model (Krugman, 1980).

If an outside good is absent, expansion in L_i leads to high wages in country i when there are trade costs. The indirect effect decreases expected profits in an expanding country by raising production costs from the viewpoint of firms, but increases expected profits in a non-expanding country by lowering production costs there relative to an expanding country. Hence, expansion in w_i (by expansion in L_i) deters entry in country i but induces entry in country j, decreasing φ^*_{ii} and increasing φ^*_{jj} as shown in (15). It is worth stressing that the negative effect on aggregate productivity relates to the home market effect on wages (Krugman, 1980).[10]

[10] The negative impact is absent in Krugman (1980) as aggregate productivity is exogenously given.

This causes higher marginal cost and lower profitability in an expanding country, leading to less competitive pressures on firms and makes it possible for less productive firms to survive there. Intuition also explains why market expansion in one country affects aggregate productivity in another country, which does not arise in the presence of an outside good (Melitz and Ottaviano, 2008).

It remains to show the impact of country size on welfare in an expanding country. From (13), the impact depends on the magnitude of decline in φ_{ii}^* caused by expansion in L_i, which respectively reflect the effect of weak domestic selection and increased product variety. To see which effect dominates, it is useful to express changes in welfare in terms of changes in φ_{ii}^* only (see Appendix A.6):

$$\hat{W}_i = \frac{1}{\sigma-1}\left((\sigma-1)(\beta_i+\rho) - \frac{\sigma\beta_i}{\mu_i} - (\beta_j - \rho\alpha_j)\left(\frac{\alpha_i+1}{\alpha_j+1}\right)\right)\hat{\varphi}_{ii}^*. \tag{16}$$

Since expansion in L_i decreases φ_{ii}^*, (16) means that the country benefits from market expansion if the value in the brackets is negative. Unfortunately this is not always the case, and we cannot say for sure that unilateral market expansion generates welfare gains in this setting. It is possible to show, however, that starting from a symmetric situation ($\alpha_i = \alpha_j$, $\beta_i = \beta_j$) and free trade ($\mu_i = 1$), such expansion unambiguously improves welfare in both expanding and non-expanding countries.

Proposition 2 *Unilateral market expansion has the following effects:*

(*i*) *The wage rate rises in an expanding country.*

(*ii*) *The domestic productivity cutoff falls in an expanding country but rises in a non-expanding country. As a result, the domestic trade share rises in an expanding country but falls in a non-expanding country.*

(*iii*) *Starting from a symmetric situation and free trade, unilateral market expansion is welfare-enhancing for both expanding and non-expanding countries.*

The result in Proposition 2 has a noticeable difference from that in the existing literature. In an influential study on allocation efficiency with general consumer preferences, Dhingra and Morrow (2019) find that market expansion provides welfare gains when consumer preferences are "aligned," so that demand shifts alter private and social markups in the same directions. The result suggests that market expansion always improves welfare under CES preferences, which is not true in our model. As shown by Dhingra and Morrow (2019), one of the sufficient conditions for welfare gains is that market expansion does not have a negative impact on productivity. This condition is not satisfied here because market expansion entails weak domestic selection that works to decline productivity in an expanding country. Hence, market expansion does not always lead to welfare gains due to distortions from weak domestic selection in our setting, whereas distortions stem from variable markups in their setting.[11]

5 Trade Policy

So far, we have examined the impact of exogenous changes in the two competitive measures on key endogenous variables without specifying a productivity distribution function and relying on an outside good. In this section, we show that the generality is important for the characterization of a country's optimal trade policy.

[11] Felbermayr and Jung (2018) also show that a larger country tends to have a weaker domestic selection effect but their analysis of country size is confined to a Pareto distribution.

Suppose that a government sets the tariff rate to maximize welfare. This section focuses on characterizing optimal tariffs for country i, i.e., the tariffs country i would impose without fearing retaliation from country j. Appendix B provides the analysis of Nash tariffs, i.e., the tariffs each country would impose by taking account of retaliation from another country.

Consider the effect of country i's import tariffs τ_{ji} on welfare, holding country j's import tariffs τ_{ij} fixed. In country j, the effect of τ_{ji} is essentially the same as that of variable trade costs θ_{ji} in the sense that changes in welfare with respect to τ_{ji} are uniquely determined by changes in the domestic productivity cutoff φ^*_{jj}. From Proposition 1, we already know that the cutoff decreases with tariffs τ_{ji} and country j's welfare falls. In country i, on the other hand, there is an additional effect of τ_{ji} on welfare: imposition of τ_{ji} improves the terms of trade for country i, which operates through changes in the tariff multiplier μ_i. From this channel, changes in country i's welfare corresponding to (8) and (13) are expressed as

$$\hat{W}_i = \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho} \frac{\beta_i}{\alpha_i} + 1 \right) \hat{\varphi}^*_{ii} + \frac{\lambda_{ji}}{\rho} \hat{\tau}_{ji}.$$

The first term is a welfare loss from tariffs due to protection of inefficient firms from foreign competition, and the second term is a welfare gain from tariffs due to improvement in the terms of trade. Upon rearrangement, welfare changes associated with these two effects are captured solely by changes in φ^*_{ii} (see Appendix A.7):

$$\hat{W}_i = \frac{\lambda_{ji}(\beta_i - \rho\alpha_i)}{\rho} \left(\frac{\beta_j - \rho\alpha_j}{\beta_j + \rho} - \frac{1}{\tau_{ji}} \right) \hat{\varphi}^*_{ii}. \tag{17}$$

Let us first consider the effect of tariffs on country i's welfare in the neighborhood of free trade at $\tau_{ji} = 1$. Recall from Proposition 1 that φ^*_{ii} also decreases with τ_{ji}. Setting $\tau_{ji} = 1$ in (17) implies that small increase in τ_{ji} from free trade unambiguously improves country i's welfare (which comes at the expense of country j) and thus the welfare-maximizing optimal tariffs are strictly positive for country i. It is also possible to show that, starting from a symmetric situation, country i's gain from tariffs cannot compensate for country j's loss, and hence the effect of τ_{ji} on world welfare is always negative.

Before moving to characterizing country i's optimal tariffs, it is useful to relate the expression in (17) with that in the existing literature. Using λ_{ii} and μ_i in terms of α_i, we can alternatively express (17) as

$$\hat{W}_i = -\frac{\alpha_i}{\beta_i} \hat{\lambda}_{ii} + \left(\frac{\beta_i - \rho\alpha_i}{\rho\beta_i} \right) \hat{\mu}_i. \tag{18}$$

Welfare changes in (18) encompass the results in Arkolakis et al. (2012) without tariff revenue and those in Felbermayr et al. (2015) with tariff revenue for the Melitz (2003) model under a Pareto distribution, which is by far one of the most commonly used distributions in the literature. While this distributional assumption is known to provide a reasonable approximation for the firm size distribution, it entails some specific limitations. In particular, if productivity is Pareto distributed with a shape parameter k, the extensive margin elasticity is constant at $\gamma_{ii} = \gamma_{ij} \equiv \gamma = k - (\sigma - 1)$, meaning that the effect of firm entry and exit due to trade costs is of the same magnitude between domestic and foreign markets. Moreover, substituting γ_{ii}, γ_{ij} into β_i introduced in Section 3, we find that β_i/α_i equals $\sigma - 1 + \gamma$, i.e., the trade elasticity initially shown by Chaney (2008) under a Pareto distribution. Since the extensive margin elasticity is constant, the above trade elasticity is also constant across different markets. Denoting this unique trade elasticity by $\varepsilon \equiv \sigma - 1 + \gamma$, (18) is expressed as

$$\hat{W}_i = -\frac{1}{\varepsilon} \hat{\lambda}_{ii} + \left(1 + \frac{\eta}{\varepsilon} \right) \hat{\mu}_i,$$

where $\eta \equiv \frac{k}{\sigma-1}(1+\frac{1-\sigma}{k}) > 0$. The above expression shows that welfare changes can be captured solely by the two sufficient statistics λ_{ii} and ε without tariff revenue as indicated by the first term (Arkolakis et al., 2012), but their welfare formula requires qualification with tariff revenue if tariffs act as cost shifters as indicated by the second term (Felbermayr et al., 2015).

The results however depend critically on the assumption that the trade elasticity is unique across markets, as stressed by Melitz and Redding (2015). By definition, the extensive margin elasticity differs across markets under a general productivity distribution, and hence the trade elasticity is bilateral-specific to country-pairs i, j. Denoting this variable trade elasticity by $\varepsilon_{ij} \equiv \sigma - 1 + \gamma_{ij}$, we further express (18) as

$$\hat{W}_i = \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} \left(\hat{M}_i^e - \hat{\lambda}_{ii} \right) + \left(\frac{1}{\rho} - \frac{1}{\varepsilon_{ij} + \gamma_{ii} - \gamma_{ij}} \right) \hat{\mu}_i. \tag{19}$$

This expression is a counterpart to that in Melitz and Redding (2015, equation (33)), albeit that we derive welfare changes by tariffs that raise government revenue. Observe that besides the domestic trade share λ_{ii} and the trade elasticity ε_{ij}, welfare changes also depend on the extensive margin elasticity differential between domestic and export markets $\gamma_{ii} - \gamma_{ij}$, which arises whenever the trade elasticity differs across markets. They argue that, if there is the differential, the domestic trade share and the trade elasticity are no longer sufficient statistics for welfare, and welfare changes can be substantially mis-estimated if the trade elasticity is assumed constant despite that the "true" elasticity is variable. (19) shows that this critique applies to welfare changes associated with tariffs. For example, if the extensive margin is more elastic in the export market than in the domestic market ($\gamma_{ii} - \gamma_{ij} < 0$), welfare changes are smaller than those without this differential ($\gamma_{ii} - \gamma_{ij} = 0$). Recent empirical work documents that the trade elasticity indeed substantially differs across country-pairs, supporting their welfare result.[12]

We now turn to characterizing optimal tariffs for country i. As in most of previous work in the trade policy literature, we use the first-order condition of welfare maximization by assuming the sufficiency to be satisfied. Then setting $\hat{W}_i = 0$ in (17) and solving for τ_{ji} yields the following expression for optimal tariffs:

$$\tau_{ji}^* = 1 + \underbrace{\frac{\rho}{\frac{\alpha_j}{\alpha_j+1}\left(\frac{\beta_j}{\alpha_j} - \rho\right)}}_{t_{ji}^*} = \frac{\beta_j + \rho}{\beta_j - \rho\alpha_j} > 1.$$

Moreover, noting that $\tilde{\lambda}_{jj} = \alpha_j/(\alpha_j + 1)$ and rewriting the definition of β_j in terms of the trade elasticity ε_{ji}, we find that the optimal tariffs are implicitly characterized as a function of the key observable moments:

$$t_{ji}^* = \frac{\rho}{\tilde{\lambda}_{jj}\left(\varepsilon_{ji} + (\gamma_{jj} - \gamma_{ji})(1 - \tilde{\lambda}_{jj}) - \rho\right)}. \tag{20}$$

(20) shows that the optimal tariffs for country i are inversely related to country j's export supply elasticity, which is composed of the domestic trade share and the trade elasticity, as in existing models. One of the crucial differences in this model, however, is that the extensive margin elasticity differential enters the expression of country j's export supply elasticity, which reflects the aspect that the trade elasticity is not necessarily unique across different markets.

[12] Maintaining CES preferences and monopolistic competition so that the intensive margin is constant, Helpman et al. (2008) find that there is substantial variation in the trade elasticity across country-pairs due to the extensive margin. In a similar vein, Bas et al. (2017) show that the extensive margin varying with country-pairs plays a key role in quantifying the trade elasticity. These pieces of evidence suggest the existence of the extensive margin elasticity differential.

The optimal tariff formula (20) can be regarded as a generalization of some of the well-known results in the trade policy literature. If productivity is Pareto distributed with a shape parameter k, the extensive margin elasticity is constant at $\gamma_{jj} = \gamma_{ji} = \gamma = k - (\sigma - 1)$ and the trade elasticity is constant at $\varepsilon_{ji} = \varepsilon = \sigma - 1 + \gamma$ as described above. Since $\varepsilon = k$ in that case, (20) reduces to

$$t_{ji}^{*} = \frac{\rho}{\tilde{\lambda}_{jj}(k - \rho)}. \tag{21}$$

This expression is exactly the same as the optimal tariffs shown by Felbermayr et al. (2013) in a heterogeneous firm model à la Melitz (2003) in which firms draw productivity from a Pareto distribution. Furthermore, it is also possible to consider a homogeneous firm model as a special case of a heterogeneous firm model in which firms draw productivity of either zero or constant from a degenerated distribution (Melitz and Redding, 2015). If trade costs are sufficiently low so that all homogeneous firms export in this class of the model, we can easily show that the extensive margin elasticity is constant at $\gamma_{jj} = \gamma_{ji} = \gamma = 0$ and the trade elasticity is constant at $\varepsilon_{ji} = \varepsilon = \sigma - 1$. In that case, thus, (20) reduces to

$$t_{ji}^{*} = \frac{1}{\tilde{\lambda}_{jj}(\sigma - 1)}. \tag{22}$$

This expression is exactly the same as the optimal tariffs shown by Gros (1987) in a homogeneous firm model à la Krugman (1980).

At this standpoint, the optimal tariff formula (20) poses two caveats. First, we cannot always say that the optimal tariffs are smaller in the heterogeneous firm model than in the homogeneous firm model. Just as the different trade models yield the different domestic trade shares $\tilde{\lambda}_{jj}$, these models also yield the different trade elasticities ε_{ji}. This means that the optimal tariffs in the different trade models are not directly comparable without controlling for the difference in the trade elasticity. Our formula is useful for shedding light on this point. Plugging (20) in $\gamma_{jj} - \gamma_{ji} = 0$ that holds in the heterogeneous model with a Pareto distribution and the homogeneous firm model with a degenerated distribution, we find that conditional on the two empirically observable moments above, the optimal tariffs are the same between the different trade models. The result is, of course, obtained by applying the welfare formula by Arkolakis et al. (2012) to our optimal tariff formula: conditional on the two sufficient statistics for welfare, changes in welfare associated with tariffs are the same; consequently, levels of the optimal tariffs are also the same.

Second, the equivalence of the optimal tariffs across the different trade models holds only if the extensive margin elasticity differential is zero ($\gamma_{jj} - \gamma_{ji} = 0$). If the condition is violated, however, the optimal tariffs are different even after controlling for the two sufficient statistics for welfare. Consider, for example, the case in which the extensive margin is more elastic in the export market than in the domestic market ($\gamma_{jj} - \gamma_{ji} < 0$). As seen in (19), welfare changes associated with tariffs are smaller than those without the differential. Since the welfare-maximizing optimal tariffs are strictly positive, this implies in the trade policy context that a government faces a smaller welfare loss from tariffs and has a more incentive to impose higher tariffs. Indeed, (20) shows that levels of the optimal tariffs are higher for $\gamma_{jj} - \gamma_{ji} < 0$ than for $\gamma_{jj} - \gamma_{ji} = 0$, conditional on the two sufficient statistics. This arises because a government does not take account of the difference in the impact of tariffs on firm entry and exit across markets. The converse is true for another case ($\gamma_{jj} - \gamma_{ji} > 0$), in that levels of the optimal tariffs are lower than those in the absence of this differential. In these more general cases, the domestic trade share and the trade elasticity are no longer sufficient statistics not only for welfare as in Melitz and Redding (2015), but also for optimal trade policy.

Proposition 3 *Conditional on the domestic trade share and the trade elasticity, levels of the optimal tariffs have the following properties:*

(*i*) *If the extensive margin elasticity is the same between domestic and export markets, levels of the optimal tariffs are the same across the different trade models.*

(*ii*) *If the extensive margin is more (less) elastic in the export market than in the domestic market, levels of the optimal tariffs are higher (lower) than those in the absence of this differential.*

In Proposition 3, we compare the optimal tariffs across the different trade models, holding *both* the domestic trade share and the trade elasticity equal that endogenously arise in the respective model. If the optimal tariffs are compared without such conditioning, the proposition no longer holds. The optimal tariffs in (20), (21) and (22) depend on the domestic trade share, which is a function of tariffs and hence is not always the same level. The fact that the optimal tariffs are implicitly characterized means that we cannot solve for the optimal tariffs in closed forms as in existing work (Gros, 1987; Felbermayr et al., 2013). To avoid this difficulty, Felbermayr et al. (2013) compare the optimal tariffs in the heterogeneous firm model and the homogeneous firm model, holding *only* the domestic trade share equal. Recently, Costinot et al. (2020) show that the optimal tariffs can be lowered under a non-Pareto distribution (relative to those under a Pareto distribution). Although they also stress the role of a general productivity distribution in characterizing the optimal tariffs as in our paper, the optimal tariffs are compared under the same condition as that in Felbermayr et al. (2013). Unfortunately, we cannot adopt their conditioning since not only is the domestic trade share but also the trade elasticity and the extensive margin elasticity differential are a function of tariffs. Thus we cannot figure out which optimal tariffs are lowest among (20), (21) and (22) without conditioning some variables of the models. For this reason, we use numerical solutions in Section 6 in order to investigate whether the effect of a variable trade elasticity on optimal trade policy is of quantitatively significant magnitude.

Next, we examine the impact of trade costs and country size on the optimal tariffs. Consider the optimal tariffs with $\gamma_{jj} - \gamma_{ji} = 0$ in (21) and (22), in which case changes in exogenous variables affect country i's optimal tariffs τ_{ji}^* only through changes in the domestic trade share $\tilde{\lambda}_{jj}$. Proposition 1 says that reduction in any trade costs increases the domestic productivity cutoff φ_{jj}^* which decreases the domestic trade share $\tilde{\lambda}_{jj}$. On the other hand, Proposition 2 says that expansion in country i's size increases φ_{jj}^* which decreases $\tilde{\lambda}_{jj}$. The comparative statics suggest that country i's optimal tariffs for are higher, the lower are any trade costs between the two countries or the larger is country i's size. These properties of the optimal tariffs are exactly the same as those with a constant trade elasticity in the literature; see Gros (1987) for the homogeneous firm model and Felbermayr et al. (2013) for the heterogeneous firm model. Consider next the optimal tariffs with $\gamma_{jj} - \gamma_{ji} \neq 0$ in (20), in which case changes in exogenous variables affect the optimal tariffs not only through changes in the domestic trade share $\tilde{\lambda}_{jj}$ but also through changes in the trade elasticity ε_{ji}. Due to *endogenous* changes in the trade elasticity, the aforementioned properties of the optimal tariffs are not necessarily satisfied, even though the comparative statics results in Propositions 1 and 2 continue to hold.

The additional channel for the optimal tariffs can be shown more formally by making clear the relationship between the extensive margin elasticity differential and the trade elasticity. Applying the comparative statics in Propositions 1 and 2, if the differential exists, the trade elasticity is not constant and differs across markets. In case of variable trade costs θ_{ji}, for example, we have the following relationship (see Appendix A.8):

$$\gamma_{jj} - \gamma_{ji} \lesseqqgtr 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{d\theta_{ji}} \gtreqqless 0.$$

If the differential is negative and the extensive margin is more elastic in the export market than in the domestic market, reduction in variable trade costs decreases the trade elasticity. This accords with evidence that the trade elasticity is small for proximate country-pairs where the trade volume is already large (Bas et al., 2017). If the differential is positive, such reduction increases the trade elasticity. Only when there is no differential which holds under a Pareto distribution, is the trade elasticity invariant to changes in variable trade costs and unique across different markets.[13]

It is easily shown that changes in any exogenous variables have an additional effect on the optimal tariffs. Consider reduction in variable trade costs θ_{ji}. If the differential is negative ($\gamma_{jj} - \gamma_{ji} < 0$), such reduction decreases the trade elasticity ($\frac{d\varepsilon_{ji}}{d\theta_{ji}} > 0$) as well as the domestic trade share in country j ($\frac{d\tilde{\lambda}_{jj}}{d\theta_{ji}} > 0$). Due to an extra adjustment through ε_{ji} that is absent in the optimal tariffs (21) and (22), the impact of variable trade costs on the optimal tariffs (20) is reinforced. If the differential is positive, the converse is true in the sense that the impact on the optimal tariffs is attenuated. Only when there is no differential, is the trade elasticity constant and reduction in θ_{ji} affects the optimal tariffs only through decreases in the domestic trade share. These highlight a possibility that the effect of variable trade costs on the optimal tariffs can be substantially mis-estimated if the trade elasticity is assumed constant despite that the "true" elasticity is variable. In other words, there can be a discrepancy in the optimal tariffs not only in terms of levels but also in terms of changes associated with exogenous shocks (see Appendix A.9).

Proposition 4 *Reduction in trade costs between the two countries and expansion in country i's size lead to the following changes in the optimal tariffs for country i:*

(i) *If the extensive margin elasticity is the same between the domestic and export markets, they increase the optimal tariffs only through decreases in the domestic trade share.*

(ii) *If the extensive margin is more (less) elastic in the export market than in the domestic market, they reinforce (attenuate) the impact on the optimal tariffs through decreases (increases) in the trade elasticity.*

One of the interesting results in Proposition 4 arises when the extensive margin is less elastic in the export market than in the domestic market ($\gamma_{jj} - \gamma_{ji} > 0$). In this case, the model predicts that the optimal tariffs for country i are lower, the lower are trade costs and the larger is country i's size. From a policy point of view, the effect of country size is of particular interest. When a government in a large country sets the tariff rate, it can enjoy a terms-of-trade gain by setting tariffs, just like the conventional optimal tariff theory. However, in the presence of firm heterogeneity, a large country suffers from weak domestic selection which negatively affects welfare by allowing inefficient firms to survive there. With this selection effect, the imposition of tariffs accelerates the welfare loss from protecting inefficient firms against foreign competition. Taken together, the optimal tariffs are *decreasing* in country size only if the welfare loss from protecting inefficient firms by tariffs is stronger than the welfare gain from improving the terms-of-trade by tariffs, which occurs under the condition that $\gamma_{jj} - \gamma_{ji} > 0$ in this model. In Section 6, we show that, even if $\gamma_{jj} - \gamma_{ji} \leq 0$ so that the optimal tariffs are strictly increasing in country size as in the existing studies, the impact of country size on the optimal tariffs is quantitatively very limited relative to that of variable trade costs. These results highlight one of the main policy implications from our analysis: when the trade elasticity differs across markets, a large country would not necessarily enjoy large welfare gains from setting high tariffs.

[13]Strictly speaking, we need to impose a restriction on a productivity distribution such that the extensive margin elasticity is a monotonic function in the productivity cutoff for this result. Then the sign of the differential does not switch with changes in any key exogenous variables under a given distribution (so long as selection into exporting is satisfied).

6 Quantitative Relevance

This section explores the quantitative relevance of our theoretical results. Using standard values of the model's parameters in analytical solutions, we numerically compare the optimal tariffs across the different trade models. Appendix C offers a detailed discussion of procedures and parameter values in the quantitative exercise below, drawing on the working paper version of this paper (Ara, 2021).

Calibration. To introduce a variable trade elasticity in the analysis, we employ a *bounded* Pareto distribution. Specifically, when productivity φ is Pareto distributed with a shape parameter k with support $(\varphi_{\min}, \varphi_{\max})$, the distribution is given by the following functional form:[14]

$$G_i(\varphi) = \frac{1 - \left(\frac{\varphi_{\min}}{\varphi}\right)^k}{1 - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}.$$

Notice that when the upper bound is infinite ($\varphi_{\max} = \infty$), this collapses to an *unbounded* Pareto distribution that is often used in the literature, in which case the extensive margin elasticity is the same across markets. However, when the upper bound is finite ($\varphi_{\max} < \infty$), the extensive margin elasticity differs across markets. Thus, the latter is apt for (20) while the former is apt for (21) or (22). Specifying the productivity distribution and using the parameter values from the existing literature, we are able to uniquely determine values of the domestic and export productivity cutoffs. These values in turn pin down values of the three key moments of optimal tariffs $\varepsilon_{ji}, \gamma_{jj} - \gamma_{ji}, \tilde{\lambda}_{jj}$ that appear in (20), (21) and (22) in an initial equilibrium.

Our interest is in addressing how the optimal tariffs with a variable trade elasticity, (20), are quantitatively different from those with a constant trade elasticity, (21) or (22), for given levels of exogenous variables. The problem is that all of the optimal tariffs depend on the domestic trade share which is a function of tariffs, and hence we cannot directly compare them without conditioning on some equilibrium variables. For this reason, we compare (20), (21) and (22) holding values of productivity cutoffs equal across the different trade models in an initial equilibrium. With such conditioning, numerical solutions greatly help make the comparison.

From the analytical solutions of comparative statics in Sections 3 and 4, we can also study the quantitative impact of exogenous variables on the optimal tariffs. Below, we focus on the case where country i unilaterally changes variable trade costs (θ_{ji}) and market size (L_i), and explore their effects on country i's optimal tariffs (t^*_{ji}). Proposition 4 suggests that reduction in θ_{ji} and expansion in L_i have *qualitatively* similar effects on t^*_{ji}. Nevertheless, the numerical exercise allows us to investigate how these exogenous changes have *quantitatively* different effects on the optimal tariffs.

The formula in (20), (21) and (22) applies to the optimal tariffs set by country i on imports from country j, requiring the key moments in country j. This suggests that, when choosing standard values of the parameters based on estimates from US data, we should treat the United States as country j in the numerical illustration. In other words, the optimal tariffs we quantify are those *faced* by the United States. We do not try to quantify the optimal tariffs *chosen* by the United States, as the parameter values of other countries are hard to find in the existing empirical literature relative to those of the United States. We follow Felbermayr et al. (2013) in assuming that the two countries differ in their tariff rate but are otherwise identical in an initial equilibrium. For simplicity, we treat country i as the rest of world and consider a situation where country i optimally sets the tariff rate taking country j's tariff rate as given.

[14] See, for example, Feenstra (2017). Using this parameterization, Helpman et al. (2008) develop a gravity equation model that generates a variable trade elasticity.

Panel A. Variable trade costs

Panel B. Country size

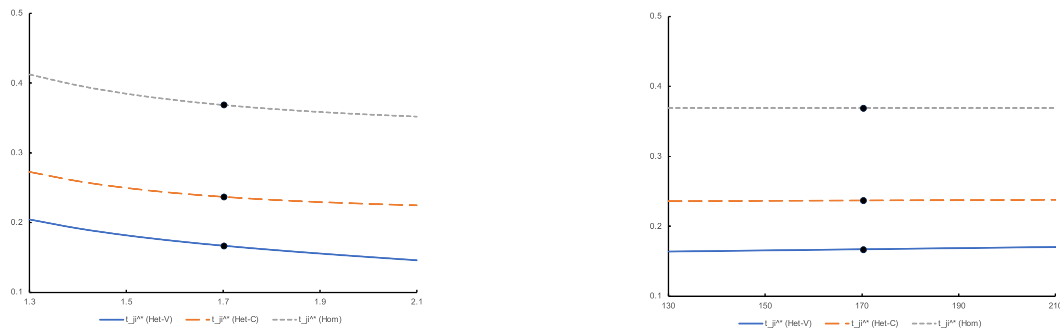

Figure 2: Optimal tariffs across different trade models

Note: In an initial equilibrium, $\theta_{ij} = \theta_{ji} = 1.7$ and $L_j = L_j = 170$. See Appendix C for a discussion of other parameter values.

Levels of Optimal Tariffs. Figure 2 shows quantitative comparison of the optimal tariffs across the different trade models. Panel A is the case of variable trade costs, while Panel B is the case of country size. In Figure 2, the solid, dashed and dotted curves represent the optimal tariffs in (20), (21) and (22), where the dots denote the optimal tariffs in an initial equilibrium which are 16.6 percent, 23.6 percent and 36.7 percent, respectively. The numerical comparison indicates that a variable trade elasticity lowers the optimal tariffs substantially: levels of the optimal tariffs with a variable trade elasticity are around two-thirds (smaller than a half) of those with a constant trade elasticity in the heterogeneous (homogeneous) firm model. The results illustrate the quantitative relevance of Proposition 3: levels of the optimal tariffs are quantitatively quite different across the different trade models, conditional on the key endogenous variables.

To understand the point, compare the optimal tariffs in (21) and (22). In estimating (22), we consider an extended homogeneous firm model where firms draw productivity of either zero or constant from a degenerated distribution (Melitz and Redding, 2015). Since productivity cutoffs are equal across the different trade models, values of all equilibrium variables (including the probability of entry and exporting) are also the same so that the two models generate the same aggregate variables (including the domestic trade share $\tilde{\lambda}_{jj}$) in an initial equilibrium. As a result, the only difference between (21) and (22) is the underlying parameter values, which implies that the optimal tariffs are lower in the heterogeneous firm model than in the homogeneous firm model, holding the domestic trade share equal. In fact, levels of the optimal tariffs in our numerical exercise are of comparable magnitude to those in the existing literature.[15]

Next, compare the optimal tariffs in (20) and (21). In estimating (20), we consider a finite upper bound where the extensive margin elasticity γ_{ji} differs across markets. Since this is the same across markets in (21), the key difference is whether the extensive margin elasticity is variable or not, which has two critical effects on optimal tariffs. First, the trade elasticity $\varepsilon_{ji} = \sigma - 1 + \gamma_{ji}$ is endogenously greater when the upper bound is finite, lowering (20) relative to (21). Second, the extensive margin elasticity differential $\gamma_{jj} - \gamma_{ji}$ is negative when the upper bound is finite, raising (20) relative to (21), as seen in Section 5. In our numerical exercise where the domestic trade share $\tilde{\lambda}_{jj}$ is large enough, (20) implies that the latter is dominated by the former. Hence the optimal tariffs are lower in the heterogeneous firm model with a variable trade elasticity than in that with a constant trade elasticity.

[15] In the heterogeneous firm model with an unbounded Pareto distribution, Felbermayr et al. (2013) find that the optimal tariffs are 26.4 percent. Levels of the optimal tariffs are not the same, as we choose the parameter values in Melitz and Redding (2015).

Changes in Optimal Tariffs. It is possible to explore changes in the optimal tariffs with respect to variable trade costs and country size. Our analytical solutions of comparative statics reveal that these two exogenous variables have different effects on productivity cutoffs. Exploiting this feature, we can address how unilateral changes in θ_{ji} and L_i have quantitatively different effects on the optimal tariffs. For expositional purposes, we restrict attention here to changes in (20). For example, 17.6 percent reduction in variable trade costs (from $\theta_{ji} = 1.7$ to $\theta_{ji} = 1.4$) increases country i's optimal tariffs by 14.7 percent (from $t_{ji}^* = 0.166$ to $t_{ji}^* = 0.191$). However, 17.6 percent expansion in market size (from $L_i = 170$ to $L_i = 200$) increases the optimal tariffs by 1.5 percent (from $t_{ji}^* = 0.166$ to $t_{ji}^* = 0.169$). The results show the quantitative implications of Proposition 4: the impact of two exogenous variables on the optimal tariffs is quantitatively quite different.

Intuition behind the result is explained by noting that country size has no direct effect on productivity cutoffs under CES preferences and monopolistic competition. (10) shows that variable trade costs have not only the direct effect but also the indirect effect through changes in wages; however, (15) shows that country size has the indirect effect only. Our analytical solutions of comparative statics further show that the indirect effect is of the same magnitude in (10) and (15) evaluated at a symmetric situation. It then follows that unilateral changes in variable trade costs have a larger effect on the optimal tariffs than those in country size, due to the direct effect that is missing in the latter.

The quantitative exercise also confirms our policy implications that a large country does not always benefit from high tariffs in our general model setting. Though a large country can benefit from terms-of-trade gains, it also suffers from weak domestic selection whereby tariffs exacerbate this selection effect even further by protecting inefficient firms. Under a bounded Pareto distribution that yields a negative differential, the benefit of tariffs is greater than the cost of tariffs and hence country i's optimal tariffs are strictly increasing with its size, as in existing work. However, a government needs to strike a balance between welfare gains associated with the terms-of-trade improvement and welfare losses associated with the weak domestic selection effect. As a result of this tradeoff, country size has a quantitatively limited effect on optimal tariffs relative to variable trade costs. We hope that our result highlights the potential importance of reconsidering policy implications in the presence of firm heterogeneity with a variable trade elasticity.

Role of Generality. Now we are able to explain the role of our generality in deriving the policy implications. In the Introduction, we noted that our model has the three distinctive features: (*i*) the trade elasticity differs across markets; (*ii*) the wage rate is endogenous; and (*iii*) a government sets the tariff rate. Clearly, if we drop (*iii*), the optimal tariffs cannot be derived, implying that nuanced policy implications in this paper come from (*i*) and (*ii*).[16] If we drop (*i*), levels of the optimal tariffs are lower in (20) than in (21) as illustrated in Figure 2. Changes in the optimal tariffs are also critically affected by (*i*). While our analytical solutions show that all of our results (especially the weak domestic selection effect) hold without (*i*), the impact of exogenous variables on the optimal tariffs is much weaker without (*i*). For example, market expansion from $L_i = 170$ to $L_i = 200$ increases the optimal tariffs by merely 0.3 percent in (21), which is 1.5 percent in (20). This reflects the result of Proposition 4 that the trade elasticity endogenously reacts to any exogenous shocks in (20), while that remains constant in (21). On the other hand, if we drop (*ii*), the indirect effect through changes in wages disappears, which in turn affects changes in the optimal tariffs with respect to exogenous variables through changes in productivity cutoffs, as explained above. In a nutshell, the generality of our model setting is useful in quantifying the optimal tariffs in terms of both levels and changes.

[16] As pointed out by an anonymous referee, a government might set other import barriers. The optimal level of such barriers would satisfy the properties in Propositions 3 and 4, but we consider only import tariffs in this paper to highlight our novelty of the optimal tariffs relative to existing work.

7 Conclusion

This paper presents a heterogeneous firm model of trade to study optimal tariffs with a variable trade elasticity. To provide a better understanding of the impact of trade liberalization and country size on optimal trade policy, we consider a general setting where the trade elasticity is bilateral-specific to country-pairs and the wage rate is endogenously determined. Our key contributions to the literature are summarized as follows. The optimal level of import tariffs is inversely related to the two empirically observable moments—the domestic trade share and the trade elasticity—where the second integrant is either constant or variable depending on the micro structure of the model. If the trade elasticity is constant and the same across markets as assumed in previous work, the optimal level of import tariffs is the same between different trade models, holding both the domestic trade share and trade elasticity equal. However, if the trade elasticity is variable and differs across markets as reported by empirical work, levels of optimal tariffs are mis-estimated due to the variable nature of the trade elasticity. The same claim applies to changes in optimal tariffs associated with trade costs and country size, in the sense that the effects of these exogenous variables on optimal tariffs depend on the micro structure that makes the trade elasticity variable.

We also explore the quantitative relevance of our theoretical results. Calibrating the model to US aggregate and firm-level data, we find that the optimal tariffs with a variable trade elasticity are significantly lower than those with a constant trade elasticity. Changes in optimal tariffs to exogenous shocks are quantitatively quite different. Our numerical solutions show that levels of optimal tariffs with a variable trade elasticity are around two-thirds (smaller than a half) of those with a constant trade elasticity in the heterogeneous (homogeneous) firm model, holding key endogenous variables the same across different trade models. Despite that, however, levels of optimal tariffs predicted by our model—16.6 percent—are much higher than levels of actual tariffs observed in the real world—3.2 percent—as reviewed in the Introduction. In that sense, deriving the welfare-maximizing optimal tariffs is useful in appreciating the role played by WTO in reducing worldwide tariffs and thereby ensuring the gains from trade liberalization, even though such counterfactual non-cooperative tariffs are not permitted in reality. We hope that our results help to reconsider policy implications in the presence of firm heterogeneity with a variable trade elasticity.

Nevertheless, much remains to be done. On the theory side, the variable nature of the trade elasticity comes from the extensive margin which is made possible by departing from a Pareto distribution that is commonly used in the literature. However, it might come from the intensive margin that relates to the firm-level elasticity. To correctly examine the variability of the trade elasticity in trade policy evaluations, it is necessary to drop CES preferences with constant markups and instead use general preferences with variable markups that differ across firms. We expect that a variable trade elasticity would play a more critical role in optimal trade policy in that case. On the quantitative side, on the other hand, we have employed a bounded Pareto distribution to quantify optimal tariffs in terms of levels and changes. While the distribution allows optimal tariffs to increase with country size, this may not be true. For example, Naito (2019) finds a significantly negative relationship between GDP and tariffs across countries, meaning that larger countries tend to set lower tariffs. To quantify such optimal tariffs, we need to replace a (bounded or unbounded) Pareto distribution with another one where the extensive margin is less elastic in the export market than in the domestic market; however we are uncertain about which firm productivity distributions yield this outcome and whether the resulting quantification is able to provide a good fit for aggregate and firm-level data. We leave these theoretical and quantitative extensions and their implications for optimal trade policy to future work.

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Appendices (Not for Publication unless Requested)

A Proofs

A.1 Proof of LMC and TB Conditions

We first show that the LMC condition is given by

$$L_i = \frac{R_i - T_i}{w_i}.$$

Aggregate labor in country i's economy is given by $L_i = L_i^e + L_i^p$, where L_i^e and L_i^p denote aggregate labor used for entry and production respectively. The LMC for entry requires $L_i^e = M_i^e f_i^e$. Recalling that $r_{ij}(\varphi) = \frac{p_{ij}(\varphi)q_{ij}(\varphi)}{\tau_{ji}}$ and using (1), (2) and the definition of $J_i(\varphi^*)$ in Section 2.1, we get

$$L_i^e = \frac{M_i^e}{w_i} \sum_{n=i,j} \left\{ \frac{1}{\sigma} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) - \left[1 - G_i(\varphi_{in}^*)\right] w_i f_{in} \right\}.$$

On the other hand, with a linear cost function, the LMC condition for production requires

$$L_i^p = M_i^e \sum_{n=i,j} \int_{\varphi_{in}^*}^{\varphi_{\max}} \left(f_{in} + \frac{\theta_{in} q_{in}(\varphi)}{\varphi} \right) dG_i(\varphi).$$

Noting that firm pricing rule generates the relationship $q_{ij}(\varphi) = \frac{\tau_{ij} r_{ij}(\varphi)}{p_{ij}(\varphi)} = \frac{\rho\varphi r_{ij}(\varphi)}{\theta_{ij} w_i}$, we get

$$L_i^p = \frac{M_i^e}{w_i} \sum_{n=i,j} \left\{ \left[1 - G_i(\varphi_{in}^*)\right] w_i f_{in} + \frac{\sigma - 1}{\sigma} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) \right\}.$$

Summing up aggregate labor used for entry and production,

$$\begin{aligned} L_i &= \frac{M_i^e}{w_i} \sum_{n=i,j} \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi) \\ &= \frac{\sum_n R_{in}}{w_i}, \end{aligned}$$

where $R_{in} = M_i^e \int_{\varphi_{in}^*}^{\varphi_{\max}} r_{in}(\varphi) dG_i(\varphi)$ is aggregate revenue (or expenditure) of goods from country i to country $n = i, j$ net of tariffs. The result follows from $R_i = \sum_n \tau_{ni} R_{ni}$ and $R_{ij} = R_{ji}$.

Next, we show that the LMC condition is equivalent with the TB condition. On the one hand, aggregate labor income in country i consists of revenues by domestic firms and exporting firms of country i net of tariffs, $w_i L_i = \sum_n R_{in}$. On the other hand, aggregate expenditure in country i consists of expenditures on domestic goods and foreign goods inclusive of tariffs $R_i = \sum_n \tau_{ni} R_{ni}$. From these, the TB condition, $R_{ij} = R_{ji}$, is

$$\underbrace{R_{ii} + R_{ij}}_{w_i L_i} = \underbrace{R_{ii} + \tau_{ji} R_{ji}}_{R_i} - \underbrace{(\tau_{ji} - 1) R_{ji}}_{T_i}.$$

Hence, the TB condition is equivalent with the LMC condition, in the sense that both conditions induce the same equality, $R_i = w_i L_i + T_i$.

A.2 Proof of Welfare

We show the derivation of (4). Welfare per worker is given by

$$\begin{aligned} W_i &\equiv \frac{U_i}{L_i} \\ &= \frac{R_i}{L_i P_i} \\ &= \frac{\mu_i w_i}{P_i} \end{aligned}$$

where the second equality follows from defining an aggregate good $Q_i \equiv U_i$ that satisfies $P_i Q_i = R_i$, and the third equality follows from noting that $R_i = \mu_i w_i L_i$ (from the definition of the tariff multiplier μ_i). Further, substituting $R_i = \mu_i w_i L_i$, aggregate market demand is expressed as

$$B_i = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} \mu_i w_i L_i P_i^{\sigma-1}.$$

Substituting this into (1) that pins down φ_{ii}^* and rearranging, the real wage rate is

$$\frac{w_i}{P_i} = \left(\frac{\mu_i L_i}{\sigma f_{ii}}\right)^{\frac{1}{\sigma-1}} \rho \varphi_{ii}^*,$$

which shows that the real wage rate depends not only on the domestic productivity cutoff φ_{ii}^*, but also the tariff multiplier μ_i. This implies that the real wage rate is the same as that in a standard Melitz model without tariff revenue ($\mu_i = 1$); see for example Demidova and Rodríguez-Clare (2013). Finally, substituting w_i/P_i into above W_i establishes the result.

A.3 Proof of α_i

We first show that α_i in (6) satisfies

$$\begin{aligned} \alpha_i &\equiv \frac{f_{ii} J_i'(\varphi_{ii}^*) \varphi_{ii}^*}{f_{ij} J_i'(\varphi_{ij}^*) \varphi_{ij}^*} \\ &= \frac{f_{ii} (\varphi_{ii}^*)^{1-\sigma} V_i(\varphi_{ii}^*)}{f_{ij} (\varphi_{ij}^*)^{1-\sigma} V_i(\varphi_{ij}^*)}, \end{aligned} \tag{A.1}$$

where $V_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \varphi^{\sigma-1} dG_i(\varphi)$ is a decreasing function of φ^*. To show the equality in (A.1), differentiating $J_i(\varphi^*) \equiv \int_{\varphi^*}^{\varphi_{\max}} \left[\left(\frac{\varphi}{\varphi^*}\right)^{\sigma-1} - 1\right] dG_i(\varphi)$ with respect to φ^*,

$$J_i'(\varphi^*) = -\left(\frac{\sigma-1}{\varphi^*}\right)[J_i(\varphi^*) + 1 - G_i(\varphi^*)].$$

Moreover, from the functional forms of $J_i(\varphi^*)$ and $V_i(\varphi^*)$, we get the following equality:

$$J_i(\varphi^*) + 1 - G_i(\varphi^*) = (\varphi^*)^{1-\sigma} V_i(\varphi^*).$$

Finally, substituting this into $J_i'(\varphi^*)$ gives us the result.

Next, we show several properties of α_i.

- The first property is that $\alpha_i \alpha_j - 1 > 0$. To show this, it follows from (1) that

$$\left(\frac{\varphi_{ij}^*}{\varphi_{ii}^*} \right)^{\sigma-1} = \frac{\tau_{ij}^{\sigma} \theta_{ij}^{\sigma-1} f_{ij}}{f_{ii}} \frac{B_i}{B_j}. \tag{A.2}$$

  Substituting this equality into $\alpha_i \alpha_j$ that satisfies (A.1),

$$\alpha_i \alpha_j = (\tau_{ij}\tau_{ji})^{\sigma} (\theta_{ij}\theta_{ji})^{\sigma-1} \left(\frac{V_i(\varphi_{ii}^*) V_j(\varphi_{jj}^*)}{V_i(\varphi_{ij}^*) V_j(\varphi_{ji}^*)} \right) > 1.$$

  The inequality follows from $\varphi_{ij}^* > \varphi_{ii}^*$ and noting that $V_i(\varphi^*)$ is strictly decreasing in φ^*.

- The second property is that $\alpha_i = R_{ii}/R_{ij}$. Using (1), $R_{ij} = M_i^e \int_{\varphi_{ij}^*}^{\varphi_{\max}} r_{ij}(\varphi) dG_i(\varphi)$ is given by

$$R_{ij} = M_i^e \sigma w_i f_{ij} (\varphi_{ij}^*)^{1-\sigma} V_i(\varphi_{ij}^*). \tag{A.3}$$

  The result follows from substituting (A.3) into the equality of (A.1).

- The third property is that $\lambda_{ji}, \tilde{\lambda}_{ji}$ and μ_i are written in terms of α_i. By definition,

$$\begin{aligned} \lambda_{ji} &= \frac{\tau_{ji} R_{ji}}{R_{ii} + \tau_{ji} R_{ji}} = \frac{\tau_{ji} R_{ij}}{R_{ii} + \tau_{ji} R_{ij}} = \frac{\tau_{ji}}{\alpha_i + \tau_{ji}}, \\ \tilde{\lambda}_{ji} &= \frac{\lambda_{ji}}{\tau_{ji}(1-\lambda_{ji}) + \lambda_{ji}} = \frac{1}{\alpha_i + 1}, \\ \mu_i &= \frac{\tau_{ji}}{\tau_{ji}(1-\lambda_{ji}) + \lambda_{ji}} = \frac{\alpha_i + \tau_{ji}}{\alpha_i + 1}. \end{aligned} \tag{A.4}$$

  This follows from the second property and the TB condition.

A.4 Proof of β_i

We show the derivation of (8). Taking the log and differentiating W_i in (4) with respect to θ_{ji},

$$\hat{W}_i = \frac{1}{\rho} \hat{\mu}_i + \hat{\varphi}_{ii}^*.$$

To express $\hat{\mu}_i$ in terms of $\hat{\varphi}_{ii}^*$, taking the log and differentiating μ_i in (A.4) with respect to θ_{ji},

$$\begin{aligned} \hat{\mu}_i &= -\left(\frac{(\tau_{ji} - 1)\alpha_i}{(\alpha_i + \tau_{ji})(\alpha_i + 1)} \right) \hat{\alpha}_i \\ &= -\left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\alpha_i + 1} \right) \hat{\alpha}_i, \end{aligned}$$

where the second equality comes from $\lambda_{ii} = \alpha_i/(\alpha_i + \tau_{ji})$ in (A.4). Further, taking the log and differentiating α_i in (A.1) with respect to θ_{ji},

$$\begin{aligned} \hat{\alpha}_i &= -[\sigma - 1 + \gamma_{ii} + (\sigma - 1 + \gamma_{ij})\alpha_i] \hat{\varphi}_{ii}^* \\ &= -\left(\frac{(\alpha_i + 1)\beta_i}{\alpha_i} \right) \hat{\varphi}_{ii}^*, \end{aligned}$$

where the second equality comes from the definition of β_i. Expressing $\hat{\mu}_i$ in terms of $\hat{\varphi}_{ii}^*$ gives us the result.

A.5 Proof of Proposition 1

We first show the derivation of (9). From (5), (6), and (7), it follows that

$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ii} = \sigma \hat{w}_i, \tag{A.5}$$
$$\hat{B}_j + (\sigma - 1)\hat{\varphi}^*_{jj} = \sigma \hat{w}_j, \tag{A.6}$$
$$\hat{B}_j + (\sigma - 1)\hat{\varphi}^*_{ij} = \sigma \hat{w}_i, \tag{A.7}$$
$$\hat{B}_i + (\sigma - 1)\hat{\varphi}^*_{ji} = \sigma \hat{w}_j + (\sigma - 1)\hat{\theta}_{ji}, \tag{A.8}$$
$$\hat{\varphi}^*_{ij} = -\alpha_i \hat{\varphi}^*_{ii}, \tag{A.9}$$
$$\hat{\varphi}^*_{ji} = -\alpha_j \hat{\varphi}^*_{jj}, \tag{A.10}$$
$$\hat{w}_i - \hat{w}_j = -\beta_i \hat{\varphi}^*_{ii} + \beta_j \hat{\varphi}^*_{jj}. \tag{A.11}$$

Note that (A.5)-(A.11) are the system of seven equations with seven unknowns where we have chosen $w_j = 1$ and hence $\hat{w}_j = 0$. From (A.5), (A.8), (A.9), (A.11) and (A.6), (A.7), (A.10), (A.11) respectively,

$$\begin{aligned}(\rho + \beta_i)\hat{\varphi}^*_{ii} - (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj} &= -\rho\hat{\theta}_{ji},\\ -(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + (\beta_j + \rho)\hat{\varphi}^*_{jj} &= 0,\end{aligned}$$

where

$$\beta_i - \rho\alpha_i = \frac{\alpha_i}{\alpha_i + 1}[\sigma - 1 - \rho + \gamma_{ii} + (\sigma - 1 - \rho + \gamma_{ij})\alpha_i] > 0.$$

Solving for $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{jj}$ and subsequently substituting them into (A.11) yields (9). Then,

$$\frac{d\varphi^*_{ii}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{jj}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ij}}{d\theta_{ji}} > 0,\ \frac{d\varphi^*_{ji}}{d\theta_{ji}} > 0,\ \frac{dB_i}{d\theta_{ji}} > 0,\ \frac{dB_j}{d\theta_{ji}} > 0,\ \frac{dw_i}{d\theta_{ji}} > 0.$$

Further, from (8), we have that $dP_i/d\theta_{ji} > 0$ and $dP_j/d\theta_{ji} > 0$. In contrast, if w_i is exogenous,

$$\frac{d\varphi^*_{ii}}{d\theta_{ji}} > 0,\ \frac{d\varphi^*_{jj}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ij}}{d\theta_{ji}} < 0,\ \frac{d\varphi^*_{ji}}{d\theta_{ji}} > 0,\ \frac{dB_i}{d\theta_{ji}} < 0,\ \frac{dB_j}{d\theta_{ji}} > 0,\ \frac{dw_i}{d\theta_{ji}} = 0,$$

and, from (8), we have that $dP_i/d\theta_{ji} < 0$ and $dP_j/d\theta_{ji} > 0$.

Next, we show the impacts of fixed trade costs and tariffs. Following similar steps, the impacts of f_{ji} are

$$\begin{aligned}\hat{\varphi}^*_{ii} &= -\frac{\beta_j + \rho}{\sigma\Xi}\hat{f}_{ji},\\ \hat{\varphi}^*_{jj} &= -\frac{\beta_i - \rho\alpha_i}{\sigma\Xi}\hat{f}_{ji},\\ \hat{w}_i &= \frac{\rho(\beta_i + \alpha_i\beta_j)}{\sigma\Xi}\hat{f}_{ji},\end{aligned} \tag{A.12}$$

and those for f_{ij}:

$$\begin{aligned}\hat{\varphi}^*_{ii} &= -\frac{\beta_j - \rho\alpha_j}{\sigma\Xi}\hat{f}_{ij},\\ \hat{\varphi}^*_{jj} &= -\frac{\beta_i + \rho}{\sigma\Xi}\hat{f}_{ij},\\ \hat{w}_i &= -\frac{\rho(\beta_j + \alpha_j\beta_i)}{\sigma\Xi}\hat{f}_{ij}.\end{aligned}$$

and those for τ_{ji}:

$$\begin{aligned}\hat{\varphi}_{ii}^* &= -\frac{\beta_j+\rho}{\Xi}\hat{\tau}_{ji},\\ \hat{\varphi}_{jj}^* &= -\frac{\beta_i-\rho\alpha_i}{\Xi}\hat{\tau}_{ji},\\ \hat{w}_i &= \frac{\rho(\beta_i+\alpha_i\beta_j)}{\Xi}\hat{\tau}_{ji},\end{aligned}\tag{A.13}$$

and those for τ_{ij}:

$$\begin{aligned}\hat{\varphi}_{ii}^* &= -\frac{\beta_j-\rho\alpha_j}{\Xi}\hat{\tau}_{ij},\\ \hat{\varphi}_{jj}^* &= -\frac{\beta_i+\rho}{\Xi}\hat{\tau}_{ij},\\ \hat{w}_i &= -\frac{\rho(\beta_j+\alpha_j\beta_i)}{\Xi}\hat{\tau}_{ij}.\end{aligned}$$

Hence, reduction in any trade costs on exports and imports raises φ_{ii}^* and φ_{jj}^*, but starting from a symmetric situation (i.e., $\alpha_i=\alpha_j$ and $\beta_i=\beta_j$), the effect of trade liberalization is always greater in a liberalizing country than in a non-liberalizing country. Only the difference is that reduction in *import* costs $\theta_{ji}, f_{ji}, \tau_{ji}$ *reduces* w_i, whereas reduction in *export* costs $\theta_{ij}, f_{ij}, \tau_{ij}$ *raises* w_i.

A.6 Proof of Proposition 2

We first show the derivation of (14). From (11) and (12), it follows that

$$\hat{B}_i+(\sigma-1)\hat{\varphi}_{ii}^*=\sigma\hat{w}_i,\tag{A.14}$$

$$\hat{B}_j+(\sigma-1)\hat{\varphi}_{jj}^*=\sigma\hat{w}_j,\tag{A.15}$$

$$\hat{B}_j+(\sigma-1)\hat{\varphi}_{ij}^*=\sigma\hat{w}_i,\tag{A.16}$$

$$\hat{B}_i+(\sigma-1)\hat{\varphi}_{ji}^*=\sigma\hat{w}_j,\tag{A.17}$$

$$\hat{w}_i-\hat{w}_j=-\beta_i\hat{\varphi}_{ii}^*+\beta_j\hat{\varphi}_{jj}^*-\hat{L}_i.\tag{A.18}$$

Note that (A.14)-(A.18) are the system of seven equations with seven unknowns where we have chosen $w_j=1$ and hence $\hat{w}_j=1$. From (A.9), (A.14), (A.17), (A.18) and (A.10), (A.15), (A.16), (A.18) respectively,

$$\begin{aligned}(\beta_i+\rho)\hat{\varphi}_{ii}^*-(\beta_j-\rho\alpha_j)\hat{\varphi}_{jj}^* &= -\hat{L}_i,\\ -(\beta_i-\rho\alpha_i)\hat{\varphi}_{ii}^*+(\beta_j+\rho)\hat{\varphi}_{jj}^* &= \hat{L}_i.\end{aligned}$$

Solving for $\hat{\varphi}_{ii}^*$ and $\hat{\varphi}_{jj}^*$ and subsequently substituting them into (A.18) yields (14).

Next, we show the derivation of (16). Substituting $\hat{L}_i=-(\beta_i+\rho)\hat{\varphi}_{ii}^*+(\beta_j-\rho\alpha_j)\hat{\varphi}_{jj}^*$ above into (13),

$$\begin{aligned}\hat{W}_i &= \left(\frac{(\tau_{ji}-1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i}+1\right)\hat{\varphi}_{ii}^*+\frac{1}{\sigma-1}(-(\beta_i+\rho)\hat{\varphi}_{ii}^*+(\beta_j-\rho\alpha_j)\hat{\varphi}_{jj}^*)\\ &= \frac{1}{\rho}\left((1-\lambda_{ii})\beta_i-\lambda_{ii}\frac{\beta_i}{\alpha_i}+\rho-\frac{\beta_i+\rho}{\sigma}\right)\hat{\varphi}_{ii}^*+\frac{1}{\sigma-1}(\beta_j-\rho\alpha_j)\hat{\varphi}_{jj}^*\\ &= \frac{1}{\sigma-1}\left((\sigma-1)(\beta_i+\rho)-\sigma\beta_i\left(\frac{\alpha_i+1}{\alpha_i+\tau_{ji}}\right)-(\beta_j-\rho\alpha_j)\left(\frac{\alpha_i+1}{\alpha_j+1}\right)\right)\hat{\varphi}_{ii}^*,\end{aligned}$$

where the second equality comes from rewriting $\lambda_{ii} = \alpha_i/(\alpha_i + \tau_{ji})$ in (A.4) and the third equality comes from rewriting the first two relationships in (14) as

$$\hat{\varphi}^*_{jj} = -\left(\frac{\alpha_i + 1}{\alpha_j + 1}\right)\hat{\varphi}^*_{ii}.$$

Finally, we show that starting from a symmetric situation and free trade, market expansion unambiguously improves welfare for country i. Evaluating (16) at $\alpha_i = \alpha_j, \beta_i = \beta_j$ and $\mu_i = 1$,

$$\hat{W}_i = -\frac{1}{\sigma - 1}\left(\beta_i - (\sigma - 1)\rho + (\beta_i - \rho\alpha_i)\right)\hat{\varphi}^*_{ii},$$

where $\beta_i - (\sigma - 1)\rho > 0$. The desired result follows from $\hat{\varphi}^*_{ii} < 0$. Together with (6) and (11),

$$\frac{d\varphi^*_{ii}}{dL_i} < 0,\ \frac{d\varphi^*_{jj}}{dL_i} > 0,\ \frac{d\varphi^*_{ij}}{dL_i} > 0,\ \frac{d\varphi^*_{ji}}{dL_i} < 0,\ \frac{dB_i}{dL_i} > 0,\ \frac{dB_j}{dL_i} < 0,\ \frac{dw_i}{dL_i} > 0.$$

Further, from (13), we have that $dP_i/dL_i < 0$ and $dP_j/dL_i < 0$. In contrast, if w_i is exogenous,

$$\frac{d\varphi^*_{ii}}{dL_i} = 0,\ \frac{d\varphi^*_{jj}}{dL_i} = 0,\ \frac{d\varphi^*_{ij}}{dL_i} = 0,\ \frac{d\varphi^*_{ji}}{dL_i} = 0,\ \frac{dB_i}{dL_i} = 0,\ \frac{dB_j}{dL_i} = 0,\ \frac{dw_i}{dL_i} = 0,$$

and, from (13), $dP_i/dL_i < 0$ and $dP_j/dL_i = 0$.

A.7 Proof of Proposition 3

We first show the derivation of (17). Taking the log and differentiating W_i in (4) with respect to τ_{ji},

$$\begin{aligned}\hat{W}_i &= \frac{1}{\rho}(\tau_{ji} - 1)\left(\frac{\alpha_i}{\alpha_i + \tau_{ji}}\right)\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii} + \frac{1}{\rho}\left(\frac{\tau_{ji}}{\alpha_i + \tau_{ji}}\right)\hat{\tau}_{ji} + \hat{\varphi}^*_{ii} \\ &= \left(\frac{(\tau_{ji} - 1)\lambda_{ii}}{\rho}\frac{\beta_i}{\alpha_i} + 1\right)\hat{\varphi}^*_{ii} + \frac{1}{\rho}\lambda_{ji}\hat{\tau}_{ji},\end{aligned}$$

where the second equality follows from $\lambda_{ii} = \alpha_i/(\alpha_i + \tau_{ji})$ and $\lambda_{ji} = \tau_{ji}/(\alpha_i + \tau_{ji})$ from (A.4). Compared to (8), there is an additional term that captures changes in tariff revenue raised by changes in τ_{ji}. Taking the log and differentiating (1) with respect to τ_{ji} gives the counterparts to (A.5) and (A.8). Cancelling $\hat{B}_i$ out from these and using (6) and (7) that hold for changes in τ_{ji},

$$\hat{\tau}_{ji} = -(\beta_i + \rho)\hat{\varphi}^*_{ii} + (\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}.$$

Further, noting that $\lambda_{ji} = 1 - \lambda_{ii}$ and substituting $\hat{\tau}_{ji}$ derived above,

$$\hat{W}_i = -\frac{1}{\rho}\frac{\lambda_{ii}}{\alpha_i}(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + \frac{1}{\rho}\lambda_{ji}(\beta_j - \rho\alpha_j)\hat{\varphi}^*_{jj}. \tag{A.19}$$

Since increase in tariffs decreases φ^*_{ii} and φ^*_{jj}, (A.19) shows that tariffs in country i have a positive (negative) impact on welfare in country i by increasing (decreasing) the consumption of domestic (imported) varieties. In fact, $\hat{\varphi}^*_{ii}$ and $\hat{\varphi}^*_{jj}$ have the following relationship from (A.13):

$$\hat{\varphi}^*_{jj} = \left(\frac{\beta_i - \rho\alpha_i}{\beta_j + \rho}\right)\hat{\varphi}^*_{ii}.$$

Substituting this into (A.19) and rearranging,

$$\hat{W}_i = \frac{\beta_i - \rho\alpha_i}{\rho}\left(-\frac{\lambda_{ii}}{\alpha_i} + \frac{\lambda_{ji}(\beta_j - \rho\alpha_j)}{\beta_j + \rho}\right)\hat{\varphi}^*_{ii}.$$

Further, substituting $\lambda_{ii}/\alpha_i = \lambda_{ji}/\tau_{ji}$ from (A.4) into the above, we obtain the expression in (17).

Next, we show that starting from a symmetric situation, country i's gain from tariffs cannot compensate country j's loss. In country j that faces tariffs by country i, the effect of τ_{ji} is essentially the same as that of θ_{ji}, and changes in welfare per worker with respect to τ_{ji} are expressed as

$$\hat{W}_j = \left(\frac{(\tau_{ij} - 1)\lambda_{jj}}{\rho}\frac{\beta_j}{\alpha_j} + 1\right)\hat{\varphi}^*_{jj}.$$

Adding $\hat{W}_i$ in (A.19) and this,

$$\begin{aligned}\hat{W}_i + \hat{W}_j &= -\frac{1}{\rho}\frac{\lambda_{ii}}{\alpha_i}(\beta_i - \rho\alpha_i)\hat{\varphi}^*_{ii} + \left(\frac{(\tau_{ji} - 1)\lambda_{jj}}{\rho}\frac{\beta_j}{\alpha_j} + 1 + \frac{\lambda_{ji}}{\rho}(\beta_j - \rho\alpha_j)\right)\hat{\varphi}^*_{jj} \\ &= \frac{\beta_i - \rho\alpha_i}{\rho\Xi}\left(\frac{\beta_j + \rho}{\alpha_i + \tau_{ji}} - \frac{(\tau_{ji} - 1)\beta_j}{\alpha_j + \tau_{ij}} - \rho - \frac{\tau_{ji}(\beta_j - \rho\alpha_j)}{\alpha_i + \tau_{ji}}\right)\hat{\tau}_{ji},\end{aligned}$$

where the second equality follows from using (A.4) and (A.13). Notice that the first term is positive and the others are negative in the brackets, and thus changes in total welfare are in general ambiguous, as in changes in country i's welfare. However, evaluating at a symmetric situation where $\alpha_i = \alpha_j$, $\beta_i = \beta_j$ and $\tau_{ij} = \tau_{ji}$,

$$\hat{W}_i + \hat{W}_j = -\frac{\beta_i - \rho\alpha_i}{\rho\Xi}\left(\frac{(\tau_{ji} - 1)(\beta_i + \rho + \beta_i - \rho\alpha_i)}{\alpha_i + \tau_{ji}}\right)\hat{\tau}_{ji},$$

where the value in the brackets is positive from observing that $\tau_{ji} - 1 \geq 0$. This establishes the desired result.

Finally, we show the derivation of (18) and (19). Taking the log and differentiating W_i with respect to τ_{ji}, welfare changes can be simply expressed as

$$\hat{W}_i = \frac{\hat{\mu}_i}{\rho} + \hat{\varphi}^*_{ii},$$

which is the same as those by θ_{ji}. To show that changes can be expressed in terms of changes in λ_{ii} and μ_i, we notice that $\lambda_{ii} \times \mu_i = \alpha_i/(\alpha_i + 1)$ from (A.4). Taking the log and differentiating this with respect to τ_{ji},

$$\hat{\lambda}_{ii} + \hat{\mu}_i = -\frac{\beta_i}{\alpha_i}\hat{\varphi}^*_{ii}. \tag{A.20}$$

Solving for $\hat{\varphi}^*_{ii}$ and substituting it into the welfare changes gives us the expression in (18). Regarding (19), from the definition of β_i and $\tilde{\lambda}_{ji} = 1 - \tilde{\lambda}_{ii}$, β_i/α_i is given by

$$\frac{\beta_i}{\alpha_i} = \varepsilon_{ij} + (\gamma_{ii} - \gamma_{ij})(1 - \tilde{\lambda}_{ii}),$$

where $\varepsilon_{ij} \equiv \sigma - 1 + \gamma_{ij}$. Using the general expression of β_i/α_i, let us further express (18) as

$$\hat{W}_i = -\left(\frac{\alpha_i + 1}{\varepsilon_{ij}(\alpha_i + 1) + \gamma_{ii} - \gamma_{ij}}\right)\hat{\lambda}_{ii} + \left(\frac{1}{\rho} - \frac{\alpha_i + 1}{\varepsilon_{ij}(\alpha_i + 1) + \gamma_{ii} - \gamma_{ij}}\right)\hat{\mu}_i.$$

After rearranging, this can be rewritten as

$$\hat{W}_i = -\left(\frac{1}{\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij}}\right)\hat{\lambda}_{ii} - \left(\frac{\alpha_i(\gamma_{ii}-\gamma_{ij})}{(\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij})((\alpha_i+1)\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij})}\right)\hat{\lambda}_{ii}$$
$$+\left(\frac{1}{\rho} - \frac{1}{\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij}} - \frac{\alpha_i(\gamma_{ii}-\gamma_{ij})}{(\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij})((\alpha_i+1)\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij})}\right)\hat{\mu}_i.$$

Further applying (A.3) to the LMC condition,

$$L_i = M_i^e \sigma \sum_{n=i,j} f_{in}(\varphi_{in}^*)^{1-\sigma} V_i(\varphi_{in}^*).$$

Taking the log and differentiating this equality with respect to θ_{ij} and using (6),

$$\hat{M}_i^e = \frac{\alpha_i}{\alpha_i+1}(\gamma_{ii}-\gamma_{ij})\hat{\varphi}_{ii}^*.$$

Solving this for $\hat{\varphi}_{ii}^*$ that holds for changes in τ_{ji} and substituting this and β_i/α_i into (A.20),

$$\hat{\lambda}_{ii} = -\left(\frac{(\alpha_j+1)\varepsilon_{ij}+\gamma_{ii}-\gamma_{ij}}{\alpha_i(\gamma_{ii}-\gamma_{ij})}\right)\hat{M}_i^e - \hat{\mu}_i.$$

Substituting this into the second $\hat{\lambda}_{ii}$ above yields the expression $\hat{W}_i$ in (19), which becomes the same as that in Melitz and Redding (2015) without tariff revenue ($\hat{\mu}_i = 0$).

A.8 Proof of γ_{jn}

We first show that, if the extensive margin elasticity differential is negative (positive), the trade elasticity is increasing (decreasing) in trade costs. Let $\phi \in \{\theta_{ij}, \theta_{ji}, f_{ij}, f_{ji}, \tau_{ij}, \tau_{ji}\}$ denote a set of trade costs between countries. From the definition of γ_{jn}, re-express this as a function of the productivity cutoff φ_{jn}^* for $n = i, j$:

$$\gamma_j(\varphi_{jn}^*) \equiv -\frac{d\ln V_j(\varphi_{jn}^*)}{d\ln \varphi_{jn}^*}.$$

If $\gamma_j(\varphi_{jn}^*)$ is strictly increasing (decreasing) in the productivity cutoff φ_{jn}^*, the differential is negative (positive) so long as selection into the export market is satisfied:

$$\gamma_j'(\varphi_{jn}^*) \gtrless 0 \quad \Longrightarrow \quad \gamma_{jj} - \gamma_{ji} \lessgtr 0.$$

Thus, if the extensive margin elasticity $\gamma_{jn}^* = \gamma_j(\varphi_{jn}^*)$ is a monotonic function in the productivity cutoff φ_{jn}^*, the sign of the differential is the same for a given productivity distribution $G_j(\varphi)$. Moreover, differentiating $\varepsilon_{ji} = \sigma - 1 + \gamma_{ji}$ with respect to ϕ defined above,

$$\frac{d\varepsilon_{ji}}{d\phi} = \gamma_j'(\varphi_{ji}^*)\frac{d\varphi_{ji}^*}{d\phi}.$$

Since $\frac{d\varphi_{ji}^*}{d\phi} > 0$ from Proposition 1, so long as $\gamma_j(\varphi_{ji}^*)$ is a monotonic function of φ_{jn}^*,

$$\gamma_j'(\varphi_{jn}^*) \gtrless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{d\phi} \gtrless 0. \tag{A.21}$$

Next, we show that, if the differential is negative (positive), the trade elasticity is decreasing (increasing) in country i's market size, while the converse is true for country j's market size. Differentiating ε_{ji} with respect to L_i and L_j respectively and noting that $\frac{d\varphi^*_{ji}}{dL_i} < 0$ and $\frac{d\varphi^*_{ji}}{dL_j} > 0$ from Proposition 2,

$$\gamma_j'(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{dL_i} \lesseqgtr 0,$$
$$\gamma_j'(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d\varepsilon_{ji}}{dL_j} \gtreqless 0.$$

A.9 Proof of Proposition 4

We first show that, if the differential is negative (positive), reduction in trade costs has the impact on the optimal tariffs t^*_{ji} not only by decreasing the domestic trade share $\tilde{\lambda}_{jj}$ but also by decreasing (increasing) the trade elasticity ε_{ji}. The optimal tariffs (20) are rewritten as

$$t^*_{ji} = \frac{\rho}{\tilde{\lambda}_{jj}\left(\frac{\beta_j}{\alpha_j} - \rho\right)},$$

where reduction in trade costs always decreases $\tilde{\lambda}_{jj}$ irrespective of the sign of $\gamma_{jj} - \gamma_{ji}$ from Proposition 1. Thus, it suffices to show that, if $\gamma_{jj} - \gamma_{ji}$ is negative (positive), β_j/α_j decreases (increases) with ϕ. For that purpose, rewrite the definition of β_j in Section 3 as

$$\frac{\beta_j}{\alpha_j} = \varepsilon_{ji} + \frac{\gamma_{jj} - \gamma_{ji}}{\alpha_j + 1}.$$

Differentiating this with respect to ϕ,

$$\begin{aligned}\frac{d(\beta_j/\alpha_j)}{d\phi} &= \gamma_j'(\varphi^*_{ji})\frac{d\varphi^*_{ji}}{d\phi} + \frac{-\gamma_j'(\varphi^*_{ji})\frac{d\varphi^*_{ji}}{d\phi}(\alpha_j+1) - (\gamma_{ji} - \gamma_{jj})\frac{d\alpha_j}{d\phi}}{(\alpha_j+1)^2} \\ &= \frac{\alpha_j}{\alpha_j+1}\left(\frac{d\varepsilon_{ji}}{d\phi} - \left(\frac{\gamma_{jj} - \gamma_{ji}}{\alpha_j(\alpha_j+1)}\right)\frac{d\alpha_j}{d\phi}\right).\end{aligned}$$

Using (A.21) and noting that $\frac{d\alpha_j}{d\phi} > 0$,

$$\gamma_j'(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{d\phi} \gtreqless 0. \tag{A.22}$$

Next, we show that market size has a similar impact on t^*_{ji}. From the impact of market size on $\tilde{\lambda}_{jj}$ from Proposition 2, it suffices to show the impact of L_i, L_j on β_j/α_j. Differentiating β_j/α_j above with respect to L_i and L_j respectively and noting that $\frac{d\varphi^*_{ji}}{dL_i} < 0$ and $\frac{d\varphi^*_{ji}}{dL_j} > 0$ from Proposition 2,

$$\gamma_j'(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{dL_i} \lesseqgtr 0,$$
$$\gamma_j'(\varphi^*_{jn}) \gtreqless 0 \quad \Longrightarrow \quad \frac{d(\beta_j/\alpha_j)}{dL_j} \gtreqless 0.$$

B Nash Tariffs

Suppose that each country sets a tariff rate to maximize respective welfare. We now allow country i to choose tariffs taking into account country j's retaliation against country i's tariffs and derive Nash tariffs.

Proposition 1 shows that increase in country j's tariffs τ_{ij} always decreases the domestic trade share $\tilde{\lambda}_{jj}$. In addition, Proposition 4 shows that when $\gamma_{jj} - \gamma_{ji} < 0$, this increase also decreases the trade elasticity ε_{ji}. These jointly mean that the best response function is downward-sloping so that tariffs are strategic substitutes for one another. If $\gamma_{jj} - \gamma_{ji} > 0$ and increase in ε_{ji} is greater than decrease in $\tilde{\lambda}_{jj}$, country i's optimal tariffs are increasing in country j's tariffs. In this case, the best response functions are upward-sloping and the optimal tariffs are strategic complements for one another. As usual, the Nash tariffs τ^*_{ji}, τ^*_{ij} are determined at which the best response functions intersect in the (τ_{ji}, τ_{ij}) space, but the variable nature of the trade elasticity alters the equilibrium properties of such tariffs. Further, the Nash tariffs are bounded from above and below. If trade costs are so high that no firm exports from country i, the domestic trade share in country j approaches to unity ($\tilde{\lambda}_{jj} = 1$). Using this fact in (20), it follows immediately that the lower bound is

$$\underline{\tau}^*_{ji} = 1 + \frac{\rho\alpha_j}{\beta_j - \rho\alpha_j} = \frac{\beta_j}{\beta_j - \rho\alpha_j}.$$

Note that this bound also represents country i's optimal tariffs when country i is treated as a small economy (relative to country j). In particular, from $\beta_j/\alpha_j = k$ under a Pareto distribution, it reduces to $\underline{\tau}^*_{ji} = k/(k-\rho)$ which is the optimal tariffs in a small economy (Demidova and Rodríguez-Clare, 2009).

On the other hand, if trade costs are so low that all operating firms export from country j ($\varphi^*_{jj} = \varphi^*_{ji}$), we have $\alpha_j = f_{jj}/f_{ji}$ (from the definition of α_j) and $\gamma_{jj} = \gamma_{ji}$ (from the definition of γ_{jn}) and hence $\beta_j/\alpha_j = \varepsilon_{ji}$. Using these and $\tilde{\lambda}_{jj} = \alpha_j/(\alpha_j + 1)$ in (20), the upper bound is

$$\overline{\tau}^*_{ji} = 1 + \frac{\rho\left(1 + \frac{f_{jj}}{f_{ji}}\right)}{\frac{f_{jj}}{f_{ji}}\left(\varepsilon_{ji} - \rho\right)} = \frac{\varepsilon_{ji} + \rho\frac{f_{ji}}{f_{jj}}}{\varepsilon_{ji} - \rho}.$$

Note that both $\underline{\tau}^*_{ji}$ and $\overline{\tau}^*_{ji}$ are variable and endogenously react to exogenous shocks.

To better appreciate the equilibrium properties of the Nash tariffs, we follow Felbermayr et al. (2013) in assuming that the two countries are symmetric and choose their tariffs non-cooperatively. In Nash equilibrium, these countries impose the same optimal tariffs $\tau^*_{ij} = \tau^*_{ji} \equiv \tau^*$ where wages are equalized between them $w_i = w_j \equiv w = 1$. Exploiting the symmetry, let us further define

$$\begin{gathered}\theta_{ij} = \theta_{ji} \equiv \theta, \quad f_{ii} = f_{jj} \equiv f_d, \quad f_{ij} = f_{ji} \equiv f_x, \quad L_i = L_j \equiv L, \quad \varphi^*_{ii} = \varphi^*_{jj} \equiv \varphi^*_d, \quad \varphi^*_{ij} = \varphi^*_{ji} \equiv \varphi^*_x, \\ \tilde{\lambda}_{ii} = \tilde{\lambda}_{jj} \equiv \tilde{\lambda}, \quad \varepsilon_{ij} = \varepsilon_{ji} \equiv \varepsilon, \quad \gamma_{ii} = \gamma_{jj} \equiv \gamma_d, \quad \gamma_{ij} = \gamma_{ji} \equiv \gamma_x, \quad \alpha_i = \alpha_j \equiv \alpha, \quad \beta_i = \beta_j \equiv \beta.\end{gathered}$$

Then, finding the Nash tariffs is equivalent to finding a solution to the fixed point problem $\tau = f(\tau)$ in (20) where the dependence of $f(\tau)$ on θ, f_x and L is understood:

$$f(\tau) = 1 + \frac{\rho}{\tilde{\lambda}\left(\varepsilon - (\gamma_d - \gamma_x)(1 - \tilde{\lambda}) - \rho\right)}.$$

Since all of the key endogenous variables (i.e., $\tilde{\lambda}$, ε, $\gamma_d - \gamma_x$) are a function of tariffs, the fixed point problem only implicitly characterizes the Nash tariffs as in the optimal tariffs in the previous subsection. Nevertheless, we can discuss the following equilibrium properties of the Nash tariffs.

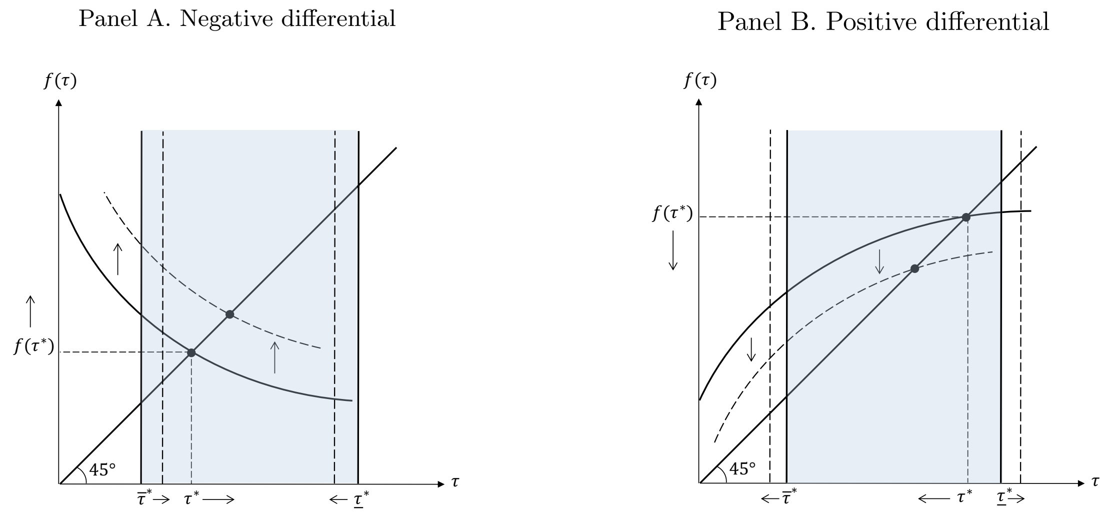

Figure 3: Effect of trade liberalization on Nash tariffs

From Propositions 1 and 4, we get the following equilibrium properties of the Nash tariffs. If $\gamma_d - \gamma_x < 0$, both the domestic trade share $\tilde{\lambda}$ and trade elasticity ε increases with τ. In this case, $f(\tau)$ is strictly decreasing in τ, reflecting that tariffs are strategic substitutes. In contrast, if $\gamma_d - \gamma_x > 0$, $\tilde{\lambda}$ increases with τ but ε decreases with it. In this case, $f(\tau)$ is strictly increasing in τ if reduction in τ leads to increase in ε relatively more than decrease in $\tilde{\lambda}$, reflecting that tariffs are strategic complements. Figure 3 depicts a 45-degree line plus a $f(\tau)$ curve for two possible cases: tariffs are strategic substitutes in Panel A and tariffs are strategic complements in Panel B. In either panel, the Nash tariffs are found at which a 45-degree line and a $f(\tau)$ curve intersect. Such tariffs lie within the shaded area in the figure where the lower and upper bounds are respectively denoted by $\underline{\tau}^*_{ij} = \underline{\tau}^*_{ji} \equiv \underline{\tau}^*$ and $\overline{\tau}^*_{ij} = \overline{\tau}^*_{ji} \equiv \overline{\tau}^*$.

Consider first the impact of trade liberalization at the symmetric situation. Reduction in trade costs (both variable θ and fixed f_x) always decreases the domestic trade share $\tilde{\lambda}$. At the same time, such reduction can affect the trade elasticity ε, depending on the sign of the differential $\gamma_d - \gamma_x$. If the differential is negative, reduction in θ also decreases the trade elasticity ε. In this case, the $f(\tau)$ curve shifts up and the Nash tariffs τ^* become higher. Moreover, the gap between the upper and lower bounds becomes narrower (i.e., the Nash tariffs tend to converge) as a result of such reduction in Panel A. If the differential is positive, the converse is true in Panel B. Finally, if the differential is zero, such reduction has no impact on the trade elasticity and the Nash tariffs become higher only through a decline in the domestic trade share, whereby the two bounds are unaffected. While the impact of trade costs on the Nash tariffs are similar to that on the optimal tariffs, changes in the key variables arise on a different scale between *bilateral* reduction in trade costs (examined here) and *unilateral* reduction in these costs (examined in Section 3). In the case of variable trade costs θ, solving the system of three equations ((5), (6)) at the symmetric situation for three unknowns $(\hat{\varphi}^*_d, \hat{\varphi}^*_x, \hat{B})$,

$$\hat{\varphi}^*_d = -\frac{1}{\alpha+1}\hat{\theta}, \quad \hat{\varphi}^*_x = \frac{\alpha}{\alpha+1}\hat{\theta}.$$

Comparing this and (9) reveals that reduction in variable trade costs has different impacts on the cutoffs, even if these changes are evaluated at the symmetric situation, reflecting the fact that both countries reduce variable trade costs here while only country i reduces such costs in (9).

Consider next the impact of market size at the symmetric situation. Bilateral expansion in market size (L) has no impact on the domestic trade share, because market size has no effect on the productivity cutoffs with the equalized wage, i.e., $\hat{w}_i = \hat{w}_j = 0$ (see (15)). Noting that the extensive margin elasticities are a function of these cutoffs, this also means that market size has no effect on the trade elasticity and the extensive margin elasticity differential. Consequently, the $f(\tau)$ curve does not shift at all, so that the Nash tariffs and the two bounds also remain unchanged. In contrast to trade liberalization above, this impact of market size necessarily holds irrespective of the sign of the differential.

One of the key upshots of our argument is that the optimal trade policy can be substantially mis-estimated even in an environment in which countries choose tariffs non-cooperatively. This is of particular importance for assessment of the optimal trade policy in globalization where reductions in transportation or communication costs are significant. The model shows that, whenever the trade elasticity differs across markets, there is an additional channel through which trade costs endogenously affect the optimal trade policy. In fact, recent work using firm-level data has identified the empirical relevance of this aspect. For example, estimating trade flows in their generalized gravity equation, Helpman et al. (2008) find substantial variation in the trade elasticity with respect to observable trade costs (proxied by distance) between country-pairs, which indicates that the trade elasticity is not unique in reality. Calibrating their heterogeneous firm model into US firm-level data, Melitz and Redding (2015) also show that missing the variable nature of the trade elasticity can give rise to a quantitatively large discrepancy between the predicted and "true" welfare gains from trade liberalization. In the context of trade policy, these insights imply that the Nash tariffs can be mis-estimated if the governments fail to take account of the micro structure that makes the trade elasticity variable in their policy making. To the best of our knowledge, however, it is not widely known whether the variable nature of the trade elasticity can also generate a quantitatively large discrepancy between the predicted and "true" optimal trade policy, which critically affect welfare.

Proposition 5 *Evaluating at a symmetric situation, the Nash tariffs have the following equilibrium properties:*

(i) *If the extensive margin elasticity is the same between the domestic and export markets, reduction in trade costs increases the Nash tariffs only through decreases in the domestic trade share.*

(ii) *If the extensive margin is more (less) elastic in the export market than in the domestic market, they reinforce (attenuate) the impact on the Nash tariffs through decreases (increases) in the trade elasticity.*

(iii) *Regardless of the sign of the extensive margin elasticity differential, market size has no impact on the Nash tariffs.*

We close this section by noting the quantitative relevance of the above results. Using the parameter values in the optimal tariffs and applying our analytical solutions to the Nash tariffs, it is also possible to examine the quantitative relevance of Nash tariffs albeit at the limited situation where the two countries are symmetric. As the wage effect disappears in that case, country size has no impact on the Nash tariffs, regardless of the trade elasticity is constant or variable. As shown above, even if the wage effect is absent, trade costs have an impact on the productivity cutoffs and hence the Nash tariffs, which can be different between constant and variable trade elasticities. Given that, as in the optimal tariffs examined in Section 6, we can expect that the impact of trade costs on the Nash tariffs is much stronger with a variable trade elasticity than with a constant trade elasticity, because the trade elasticity is endogenously responsible to changes in trade costs, generating larger changes in the Nash tariffs for a variable trade elasticity.

C Numerical Solutions

Our calibration procedures closely follow Melitz and Redding (2015). We first consider the heterogeneous firm models with variable and constant trade elasticities and compare the optimal tariffs in (20) and (21). Then we consider the heterogeneous and homogeneous firm models with a constant trade elasticity and compare the optimal tariffs in (21) and (22). Following Felbermayr et al. (2013), the two countries are assumed to differ in their tariff rate but are otherwise identical in an initial equilibrium where all exogenous variables are the same in both cases. For simplicity, we use the short-hand notations introduced in Appendix B (e.g., $\theta_{ij} = \theta_{ji} \equiv \theta$).

Comparison between (20) and (21). We choose the elasticity of substitution between varieties $\sigma = 4$ and hence $\rho = 0.75$. We set the shape parameter of a Pareto distribution $k = 4.25$, the scale parameter $\varphi_{\min} = 1$, and the upper bound either $\varphi_{\max} = 2.85$ in (20) or $\varphi_{\max} = \infty$ in (21) .

We follow Melitz and Redding (2015) in calibrating trade costs to match the average fraction of exports in firm sales in US manufacturing (which is 0.14 as reported by Bernard et al. (2007)). In contrast to their study that matches this number to variable trade costs only, we also consider tariffs and hence $\frac{\tau^{-\sigma}\theta^{1-\sigma}}{1+\tau^{-\sigma}\theta^{1-\sigma}} = 0.14$. We set τ equal to 1.045 which matches the world applied tariff rate (weighted mean, all products in 2002), where the world tariff rate is obtained from the World Bank Data for the same year as Bernard et al. (2007). Together with $\sigma = 4$, this implies $\theta = 1.7$. Regarding fixed costs, we set $f_d = f_e = 1$ while $f_x = 0.535$ for bounded Pareto and $f_x = 0.545$ for unbounded Pareto; see Melitz and Redding (2015) for detailed discussions. Regarding country size, we set $L = 170$ to make the effect of θ and L easily comparable.

Using these parameter values and specifications of the distribution, we can uniquely determine values of equilibrium variables. In our numerical exercise, we do this by solving two equations. One equation is the share of firms that export in each country, which is given as $\chi \equiv [1 - G(\varphi_x^*)]/[1 - G(\varphi_d^*)]$. Under a bounded Pareto distribution, this share is expressed in terms of φ_d^*, φ_x^* along with distributional parameters:

$$\chi = \frac{\left(\frac{\varphi_{\min}}{\varphi_x^*}\right)^k - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}{\left(\frac{\varphi_{\min}}{\varphi_d^*}\right)^k - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^k}.$$

Another equation for the unknowns φ_d^*, φ_x^* is selection into the export market. Evaluating the ZCP condition in (1) at the symmetric situation, selection into exporting in (A.2) implies

$$\left(\frac{\varphi_x^*}{\varphi_d^*}\right)^{\sigma-1} = \frac{\tau^\sigma \theta^{\sigma-1} f_x}{f_d}.$$

Solving these two relationships for the two unknowns, φ_d^*, φ_x^*, the former is expressed as

$$(\varphi_d^*)^{-k} = \frac{\varphi_{\max}^k (1-\chi)}{\tau^{-\frac{k\sigma}{\sigma-1}} \theta^{-k} \left(\frac{f_x}{f_d}\right)^{-\frac{k}{\sigma-1}} - \chi}.$$

The average share of firms that export in US manufacturing is 0.18 (Bernard et al., 2007) and hence $\chi = 0.18$. Further, plugging the calibrated parameter values yields values of two unknowns in an initial equilibrium under a bounded Pareto distribution: $\varphi_d^* = 1.16$, $\varphi_x^* = 1.70$. Note that values of these two cutoffs are not uniquely determined under a unbounded Pareto distribution with $\varphi_{\max} = \infty$. Once these values are determined, values of other key endogenous variables are automatically pinned down, as shown in the main text.

We compare (20) and (21) holding φ_d^*, φ_x^* determined above equal across different models, where $w = 1$ in the initial equilibrium. The key endogenous variables in these optimal tariff formulas are

$$\varepsilon_x = \sigma - 1 + \gamma_x,$$
$$\gamma_n = (k - (\sigma - 1)) \frac{\left(\frac{\varphi_{\min}}{\varphi_n^*}\right)^{k-(\sigma-1)}}{\left(\frac{\varphi_{\min}}{\varphi_n^*}\right)^{k-(\sigma-1)} - \left(\frac{\varphi_{\min}}{\varphi_{\max}}\right)^{k-(\sigma-1)}},$$
$$\tilde{\lambda} = \frac{\alpha}{\alpha + 1},$$

where $n = d, x$. Observe that, even if values of productivity cutoffs are the same in the initial equilibrium, values of the three moments are different between bounded and unbounded Pareto distributions. For example, the trade elasticity ε_x is k under an unbounded Pareto distribution with $\varphi_{\max} = \infty$, while it is greater than k under a bounded Pareto distribution with $\varphi_{\max} < \infty$. Similarly, the extensive margin elasticity differential $\gamma_d - \gamma_x$ is zero for $\varphi_{\max} = \infty$, while it is negative for $\varphi_{\max} < \infty$ so long as selection into exporting is satisfied $(\varphi_x^* > \varphi_d^*)$. Finally, the domestic trade share $\tilde{\lambda}$ differs between these two cases, since (A.1) and (A.2) imply

$$\alpha = \tau^\sigma \theta^{\sigma-1} \frac{V(\varphi_d^*)}{V(\varphi_x^*)}.$$

Applying a Pareto distribution to $V(\varphi^*)$ introduced in Appendix A.3, we get

$$V(\varphi^*) = \frac{k\varphi_{\min}^k}{k - (\sigma - 1)} \left((\varphi^*)^{-(k-(\sigma-1))} - \varphi_{\max}^{-(k-(\sigma-1))} \right),$$

which takes different values, depending on whether $\varphi_{\max} = \infty$ or $\varphi_{\max} < \infty$. Taken together, we can quantify the three key moments of (20) and (21). It is clear that different values of the three key moments give rise to different values of the optimal tariffs in the initial equilibrium, given as the dots in Figure 2.

Further, the analytical solutions of comparative statics outcomes in Sections 3 and 4 allow us to address the quantitative impact of unilateral changes in trade costs and market size on the optimal tariffs. As we examine the effect of unilateral changes in the exogenous variables, we must depart from the symmetric situation in the initial equilibrium for the comparative statics. For this reason, the country subscripts i, j are re-attached to relevant variables below and examine the effect of unilateral changes in exogenous variables from the initial equilibrium. Consider the effect of θ_{ji} where country i unilaterally changes variable trade costs of importing from country j. Evaluating (9) at the symmetric situation $\alpha_i = \alpha_j = \alpha$, $\beta_i = \beta_j = \beta$ and using (6), changes in the productivity cutoffs in country j from the initial equilibrium are

$$\hat{\varphi}_{jj}^* = -\frac{\rho(\beta - \rho\alpha)}{\Xi} \hat{\theta}_{ji},$$
$$\hat{\varphi}_{ji}^* = \frac{\rho\alpha(\beta - \rho\alpha)}{\Xi} \hat{\theta}_{ji}.$$

Thus, starting from the symmetric situation, 1 percent reduction in θ_{ji} leads to $\frac{\rho(\beta-\rho\alpha)}{\Xi}$ percent increase in φ_{jj}^* and $\frac{\rho\alpha(\beta-\rho\alpha)}{\Xi}$ percent decrease in φ_{ji}^* respectively. Using the calibrated values in the initial equilibrium, we can compute changes in φ_{jj}^* and φ_{ji}^* from the initial equilibrium. These changes are then used to compute changes in the three key moments of optimal tariffs for changes in the optimal tariffs. Note that, depending on whether $\varphi_{\max} = \infty$ or $\varphi_{\max} < \infty$, not only is α but also β and Ξ take different values, and so do $\hat{\varphi}_{jj}^*, \hat{\varphi}_{ji}^*$. This generates different changes in optimal tariffs in (20) or (21), given as the curves in Figure 2.

Comparison between (21) and (22). We keep the parameters in the heterogeneous firm model the same as for an unbounded Pareto distribution and so does (21). As for (22), we choose a degenerate distribution in the homogeneous firm model so that these two models generate the same aggregate variables in the initial equilibrium. Let $\tilde{\varphi}_d^*$ and $\tilde{\varphi}_x^*$ denote (exogenous) domestic and export productivity cutoffs in the homogenous firm model. To meaningfully compare the two different models, we choose values of productivity cutoffs so that $\varphi_d^* = \tilde{\varphi}_d^*$ and $\varphi_x^* = \tilde{\varphi}_x^*$ in the initial equilibrium. Under the condition, the aggregate equilibrium outcomes, including the share of firms that export χ and the domestic trade share $\tilde{\lambda}$, are the same in the initial equilibrium (Melitz and Redding, 2015). Despite that, levels of the optimal tariffs are different between the models in the initial equilibrium, because the trade elasticity ε_x consists of the the intensive margin elasticity $\sigma - 1$ and the extensive margin elasticity γ in (21), while it consists only of the intensive margin elasticity in (22). As the domestic trade share $\tilde{\lambda}$ is the same between these models, this difference implies that as long as average firm size is finite under a Pareto distribution so that $k > \sigma - 1$, the optimal tariffs are lower for (21) than for (22) in the initial equilibrium, given as the dots in Figure 2.

Further, changes in the optimal tariffs are different, since the productivity cutoffs endogenously respond to changes in exogenous variables in the heterogeneous firm model, while they are constant in the homogenous firm model. This difference implies that the heterogeneous firm model has an additional adjustment margin that is absent in the homogenous firm model (Melitz and Redding, 2015). In our policy context, the difference implies that the optimal tariffs in the heterogeneous firm model react to changes in exogenous variables more sharply than those in the homogenous firm model. Hence, changes in the optimal tariffs are greater for (21) than for (22) from the initial equilibrium, given as the curves in Figure 2.