Input Tariff in Oligopoly: Entry, Heterogeneity, and Demand Curvature*

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Abstract

How does an increase in tariff on intermediate input affect different margins of trade and what in turn are consequences for optimal tariff? We address this question in a setting with vertical specialization where oligopolistic, downstream Home firms procure input from perfectly competitive, Foreign upstream firms. Our key focus is to understand how Home optimal tariff departs from the competitive benchmark (inverse of foreign export supply elasticity). While underproduction in oligopoly puts a downward pressure on tariff, welfare improvement arising from rationalization (in presence of entry) and possible reallocation (in presence of cost heterogeneity) can put an upward pressure on tariff. Hence, in general, optimal tariff can be higher or lower than the competitive benchmark.

Keywords: Optimal tariff, input trade, oligopoly, free entry, cost heterogeneity
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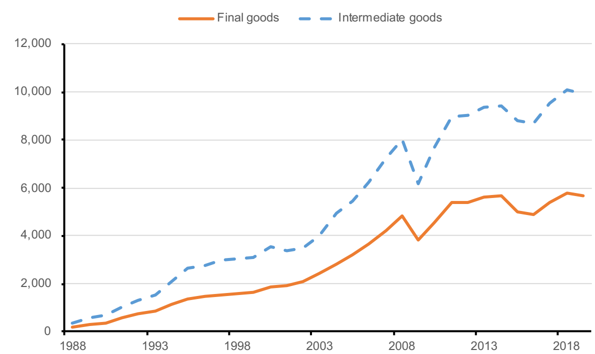
1 Introduction

Trade in the modern economy is largely in car parts, not cars (Krugman, 2018). A significant fraction of world trade—nearly two-thirds according to the estimates of Johnson and Noguera (2012)—is in intermediate goods. A reduction in tariff across the globe has led to tremendous growth in intermediate goods trade prompted by vertical specialization across countries (Hummels et al., 2001; Yi, 2003; Hanson et al., 2005). While final goods trade has also increased, intermediate goods trade has grown at a faster rate over the last three decades (see Figure 1). Increased importance of intermediate goods trade has led to development of new trade models that embed vertical specialization. Recently, attention has turned to trade policy in such multi-layered production settings. See, for example, Caliendo et al. (2023), Antràs et al. (2023), and Lashkaripour and Lugovskyy (2023). This paper aims to offer new insights to this growing literature.

Figure 1: World imports, 1988–2019



Source: Authors' own compilation based on the World Integrated Trade Solution (WITS) database.

Notes: We divide the world import value by applying the Broad Economic Categories (BEC) classification which distinguishes between intermediate, capital, and consumption goods. Intermediate goods are defined as any imports of intermediate and capital goods.

How do tariff reductions on intermediate goods imports affect the number of importers (extensive margin), average volume of imports per firm (intensive margin), and social welfare, and what, in turn, are the implications for optimal tariff? We address this question in an oligopoly environment with free entry where firms in one country (say Home) specialize in final goods and import intermediate goods from abroad (say Foreign). Interplay between demand features (e.g., curvature of demand, elasticity of slope) and entry environment (e.g., exogenous versus endogenous market structure) play an important role in determining these effects. Mrázová and Neary (2017) demonstrate how demand-side conditions shape comparative statics results in presence of monopoly and monopolistic competition. In our oligopoly setting where strategic interactions among large firms are explicitly modeled, demand-side conditions like slope and curvature play a critical role too. Our focus however is on the interaction: how the supply-side environment interacts with the demand-side features in determining the effect of tariff on imports and welfare. As we discuss below, the nuanced nature of this interaction is best reflected in our findings in intensive margin and optimal tariff.

When the market structure is exogenously given, intensive margin always increases with a reduction in tariff irrespective of the demand function. This demand-invariant nature of the result goes away when the market structure is determined endogenously. With free entry, intensive margin increases *if and only if* the demand function is strictly convex. For constant-elasticity demand—a prominent, strictly convex demand function—intensive margin increases as tariff declines. For linear demand, however, intensive margin is invariant to tariff as incentive to import more due to lower tariff is exactly offset by incentive to import less due to cannibalization arising from new entry induced by tariff cuts. Thus, the response of intensive margin to tariff depends on both demand conditions and market structure.

Like intensive margin, optimal tariff also displays a demand-invariant feature when the market structure is exogenous: irrespective of the demand function, optimal tariff is strictly lower than the competitive benchmark (inverse of export supply elasticity). Imposing tariff on intermediate input improves terms-of-trade (i.e., reduces input price) and increases welfare at Home. Counteracting this welfare gain from terms-of-trade improvement is a welfare loss from exacerbating underproduction in oligopoly. This lowers optimal tariff below the traditional competitive benchmark of inverse of export supply elasticity. As Lashkaripour and Lugovskyy (2023) note, tariff plays two roles in a second-best world like ours where subsidy on consumption and production is unavailable: tariff improves terms-of-trade and corrects misallocation. In an oligopolistic final good sector, this misallocation manifests in the form of underproduction which is partially corrected by imposing tariff at a lower level (than the competitive benchmark). In fact, optimal tariff could be negative if the final good sector is too concentrated. However, this result—tariff on intermediate input is lower than the competitive benchmark—does not necessarily hold once entry is allowed.

If the market structure is endogenous, an additional distortion/misallocation is present: *excessive entry*—too many firms enter in a free-entry equilibrium in a homogeneous products oligopoly (Mankiw and Whinston, 1986). Tariff exacerbates the welfare loss from underproduction but also mitigates the welfare loss from excessive entry, and hence we have to take into account this additional welfare improving effect in characterizing optimal tariff. In our model, intensive margin plays a key role in determining which forces in oligopoly—underproduction and excessive entry—dominate. We find that optimal tariff is strictly higher (lower) than the competitive benchmark if and only if intensive margin increases (decreases) with tariff, which holds for strictly concave (convex) demand. For linear demand where intensive margin is invariant to tariff, the above two forces exactly offset one another, and optimal tariff equals the competitive benchmark as it would be under perfect competition.

Discussions in Sections 2 and 3 focus on symmetric environments where all firms are identical. We introduce cost heterogeneity in Section 4, which generates two new features. First, as intensive margin differs across firms, the response of intensive margin to tariff also differs across firms: some firms increase their imports while others cut back their imports in response to increased tariff. Second, as import shares expand for some firms but shrink for others, tariff could affect production efficiency due to reallocation from inefficient firms to efficient firms. Once again, intensive margin plays a key role. We show that optimal tariff is strictly lower than the competitive benchmark if and only if weighted intensive margin—where output shares act as weights—decreases with tariff. This condition holds for linear as well as all convex demand functions. Strictly concave demand functions also satisfy this condition when the Herfindahl index is high.

A few remarks regarding our modeling choices are in order. First, we focus primarily on market interactions (rather than bilateral negotiations) as a large fraction of intermediate goods is internationally traded in markets among anonymous final goods producers and intermediate goods suppliers. Second, market-based interactions guide us to consider homogeneous (rather than differentiated) intermediate goods as homogeneous intermediate goods tend to be exchanged through markets whereas differentiated intermediate goods tend to be exchanged through non-market mechanisms (Nunn, 2007). Third, our approach is useful to shed new light on increasing market concentration in globalization, as documented by Head and Spencer (2017), which cannot be dealt with a monopolistically competitive framework where firms are small relative to markets. Fourth, we assume that a

Home downstream sector is oligopolistic and Home firms produce a homogeneous final good. In doing so, we acknowledge the empirical regularity that a small fraction of firms dominates international trade and follow the literature in trade theory where a differentiated good is typically paired with monopolistic competition and a homogeneous good with oligopolistic competition.[1] As shown in Section 5.1, our analysis can be easily extended to accommodate product differentiation. Nevertheless, we mainly consider a homogeneous-product setting as it provides a nuanced effect of tariff via entry, which in turn influences the sign (and magnitude) of optimal tariff in an important way. Finally, in order to highlight rich interactions between supply and demand conditions, we make several simplifying assumptions, including no Home intermediate goods producers, no Foreign consumers, and no policy instruments (other than tariff). At the end of Section 3.1, we discuss how the presence of Home intermediate goods producers and Foreign consumers would not qualitatively change our results. We also discuss the role of multiple instruments in Section 5.2.

Our paper is naturally related to an early literature on vertical oligopoly and trade policy (e.g., Ishikawa and Spencer, 1999). This literature focuses on strategic interactions among firms in vertically-related markets and examines the effect of export subsidy on intermediate goods and social welfare, taking the market structure as typically exogenous. In contrast, the market structure is endogenous in our setting which in turn affects optimal trade policy through entry/exit (extensive margin) induced by tariff changes. In single-stage oligopoly models, Horstmann and Markusen (1986), Venables (1985), and recently, Bagwell and Staiger (2012) and Etro (2011) all find that entry/exit considerations can alter optimal trade policy. For example, Bagwell and Staiger (2012) show that restraining export subsidy—an often used feature of trade agreements—can potentially benefit consumers when free entry is invoked in linear oligopoly. Our work complements this body of work by introducing multiple stages of production and general demand which generates a rich set of possibilities.

Blanchard (2007) provides an early analysis of trade policy with multiple production stages. In the presence of vertical foreign direct investment (FDI), she finds that governments of source countries have an incentive to improve market access for imported inputs from foreign affiliates of source country firms.[2] Our primary focus is, however, on foreign outsourcing rather than FDI. Antràs and Staiger (2012), Ornelas and Turner (2008, 2012), and Grossman et al. (2023) consider trade policy in the context of foreign outsourcing of intermediate goods. Antràs and Staiger (2012) and Ornelas and Turner (2008, 2012) study trade agreements and organizational structure respectively, whereas Grossman et al. (2023) study the effect of unanticipated tariff on intermediate goods in a setting with firm-to-firm supply relationships. At the heart of these papers is non-market interactions where the input price is determined by bilateral bargaining. This paper instead highlights market interactions where the input price is determined at the market-clearing level. As for non-market versus market interactions, the suitability of modeling approaches depends critically on a particular application (e.g., a particular industry or a particular question). Inderst (2010) notes that non-market interactions subject to contractual incompleteness might be more applicable in tight bilateral oligopoly with a differentiated product, whereas market interactions with uniform contractual terms are more applicable for a homogeneous product.[3]

We contribute to the literature on optimal tariff under imperfect competition. In monopolistic competition models with homogeneous firms, Gros (1987) and Flam and Helpman (1987) show that optimal tariff is strictly positive even in a small open economy. Venables (1987) and Ossa (2011) find that import tariff can induce welfare-inducing entry at Home by prompting consumers to switch from imported varieties to Home varieties. In models of monopolistic competition with heterogeneous firms, Demidova and Rodríguez-Clare (2009) are the

[1]See Neary (2016) for a comprehensive treatment of oligopoly in general equilibrium where firms are large in their own markets but small in the economy as a whole.

[2]See Blanchard and Matschke (2015) for empirical support for this argument.

[3]As mentioned earlier in the Introduction, we consider product differentiation in Section 5.1.

first one to study optimal tariff in a small economy, whereas Felbermayr et al. (2013) extend the analysis to a large economy. Despite a negative impact of productivity, both papers find that import tariff improves welfare. Costinot et al. (2020) provide a strict generalization of previous results, showing that optimal tariff can be lower (and even be negative) with more general settings, but they do not examine input-output linkages.

Presence of input-output linkages can lead to low optimal tariff in a second-best world. Caliendo et al. (2023) offers a second-best argument for low optimal tariff in the presence of roundabout production. In absence of other instruments, tariff is suitably adjusted downward to mitigate the welfare loss from double marginalization that arise in roundabout production structure. Antràs et al. (2023) provide a rationale for lower tariff on inputs (compared to final goods) on efficiency grounds in presence of scale economies. Unilateral tariff on either final goods or inputs enlarges each sector and raises welfare; however, input tariff raises final goods producers' costs, limiting their potential benefits and resulting in lower input tariff. In a fairly general multi-sector, multi-country setting, Lashkaripour and Lugovskyy (2023) show that optimal tariff is independent of input-output linkages, but only when optimal production subsidy is available.[4]

Rich in trade structure and vertical linkages, most of the above papers assume either perfect competition or monopolistic competition with CES preferences.[5] In monopolistic competition (with CES preferences), firm-level output is typically optimal and the free-entry number of firms is insufficient. In contrast, in a homogeneous product, free-entry oligopoly, firm-level output is always insufficient and too many firms enter in equilibrium. The nature of output and entry misallocation differs across monopolistic competition and oligopoly and so does the nature of optimal tariff. By focusing only on one vertically related sector, we sacrifice some of the richness examined by previous work. However, our focus (albeit narrow) allows us to describe a comprehensive account of the impact of tariff for an important class of imperfect competition settings: free-entry oligopoly with general demand. Given the fact that strategic interdependence among large firms has been increasing in globalization, we believe that our paper can fill this important gap in the literature.

As mentioned earlier, the importance of demand conditions—in particular elasticity of demand and elasticity of slope of demand—has been succinctly shown by Mrázová and Neary (2017). While elasticity is crucial for determining price levels, understanding change in price levels requires characterizing how elasticity changes, which depends on both elasticity of demand as well as its slope. Our work differs from theirs in that we mainly work with demand for inputs instead of demand for final goods. This difference does not feature prominently, however, as our production technology allows for smooth translation of final demand to input demand. Despite working with the input demand curve, all of our characterizations can be expressed in terms of curvature of final demand under exogenous and endogenous market structure, irrespective of whether cost heterogeneity is present or not. In addition, our work offers a new insight into optimal trade policy with input-output linkages, which is not examined in their work.

The plan of the paper is as follows. Section 2 presents the basic model and examines the impact of tariff on intensive and extensive margin in exogenous and endogenous market structure. Analyzing the welfare effect of tariff, Section 3 turn to characterizing optimal tariff in respective market structure. Section 4 derives optimal input tariff in presence of cost heterogeneity. Section 5 relaxes two crucial assumptions made in Sections 2-4: (i) all firms produce and sell homogeneous products, and (ii) tariff is the only policy instrument for a government. Finally, Section 6 concludes.

[4]See also Blanchard et al. (2017). In a perfectly competitive setting, they find that governments set lower tariff on goods where global value chain linkages are stronger. While their finding is similar to ours in spirit, the value added approach in their paper is more appropriate for tariff on final goods than tariff on inputs.

[5]Antràs et al. (2023) note that monopolistic competition with CES preferences is isomorphic to perfect competition with external economies to scale.

2 Model

We develop a model of vertical specialization where firms from one country (say Home) produce a final good by using an intermediate good imported from abroad (say Foreign). Home government imposes ad-valorem tariff on the intermediate good. Taking the input price and the tariff rate as given, Home firms compete in the final good market as Cournot competitors. First, we provide some details of our model (Section 2.1) and analyze a short-run setting where the number of Home firms is fixed (Section 2.2). Subsequently, we allow for entry and exit of firms which leads to endogenous determination of market structure (Section 2.3). Throughout Section 2, we focus on the effect of tariff. We examine how reduction in tariff affects aggregate imports, firm-level imports (intensive margin), and the number of importers (extensive margin) of the intermediate good. Equipped with these findings, in Section 3, we turn to characterizing welfare-maximizing optimal tariff for both settings—with and without free entry.

2.1 Basics

Home comprises a unit mass of identical individuals each with $\bar{L}$ units of labor endowment. One unit of labor is required for producing one unit of a numeraire good y which is freely traded and produced under perfect competition. As the price of y is 1 and each individual has $\bar{L}$ units of labor, the wage rate and labor income are unity and $\bar{L}$ respectively.

Each individual consumes a numeraire y and a final good Q. Preferences are given by a quasi-linear utility function $U = U(Q) + y$ where $U(0) = 0, U'(Q) > 0$, and $U''(Q) < 0$ for $Q > 0$. Assuming $\bar{L}$ to be high enough, utility maximization subject to budget constraint generates an inverse demand function: $P = P(Q)(\equiv U'(Q))$, where P is the price of final good. We assume that $P(Q)$ is twice continuously differentiable, $P'(Q) < 0$, and the following inequality holds for all $Q > 0$:

$$2P'(Q) + QP''(Q) < 0.$$

A large class of demand functions—including linear, constant-elasticity, and all logconcave demand functions—satisfy the inequality. Loosely speaking, it means that industry marginal revenue $(QP(Q))'$ is downward sloping. The condition can be alternatively expressed as in terms of elasticity of slope of demand:

$$\eta(Q) \equiv \frac{QP''(Q)}{P'(Q)} > -2. \tag{1}$$

Mrázová and Neary (2017) use this condition extensively in their work on demand structure and firm behavior. This captures the curvature of demand in that demand is concave (convex) if and only if η is positive (negative). Thus (1) rules out demand functions that are too convex.

On the production side, there is a large number of identical Home firms which produce the final good Q in the downstream sector. One unit of the intermediate good is required to produce one unit of the final good. There are no intermediate good producers at Home. Home firms import the intermediate good and transform it into the final good without incurring any additional costs. In contrast, the upstream sector comprises a large number of perfectly competitive, price-taking Foreign firms producing the intermediate good under a strictly convex cost function. This gives rise to upward sloping supply for intermediate imports:

$$r = h(X), \tag{2}$$

where r is the per-unit price of intermediate good X. We assume that $h'(X) > 0$ for all $X > 0$ and $0 \leq h(0) < \lim_{Q\to 0} P(Q)$ which ensures an interior equilibrium. Finally, a Home government imposes ad-valorem tariff on intermediate imports. Let t denote the ad-valorem tariff rate chosen by the Home government. Tariff revenues are rebated back to Home consumers.[6]

Throughout this section, we take the tariff rate as given and mainly focus on the positive analysis of tariff. In Section 2.2, we begin with the short-run analysis where the number of Home firms is fixed. In Section 2.3, we turn to the long-run analysis where the number of Home firms is endogenously determined via free entry, in which case each Home firm incurs the entry cost K prior to entering before producing the final good.

2.2 Short-Run Analysis

Let M denote the number of Home firms producing the final good. Given the input price r, ad-valorem tariff t and one-to-one production technology, each Home firm's unit cost is given by $r(1+t)$. In Cournot competition, Home firm i chooses q_i to maximize its profits $\pi_i \equiv \left(P\left(q_i + \sum_{j\neq i}^{M} q_j\right) - r(1+t)\right) q_i$ taking other firms' behavior as given. Solving the first-order conditions corresponding to Home firms' profit maximization problem yields the symmetric equilibrium quantity for each Home firm $q_1 = q_2 = ... = q_M \equiv q(> 0)$ such that

$$q = -\frac{P(Q) - r(1+t)}{P'(Q)}, \tag{3}$$

where Q uniquely solves the following:

$$MP(Q) + QP'(Q) - Mr(1+t) = 0. \tag{4}$$

One unit of the final good requires one unit of the intermediate good. Thus aggregate output produced in the downstream sector, Q, must equal aggregate output produced in the upstream sector, X. Substituting Q with X in (4) and rearranging yields the inverse demand function for intermediate imports:

$$r = \frac{P(X) + \frac{XP'(X)}{M}}{1+t} \equiv g(X, M, t). \tag{5}$$

Treating the number of firms as a continuous variable below and using (1), the demand for intermediate input in (5) satisfies the following inequalities:

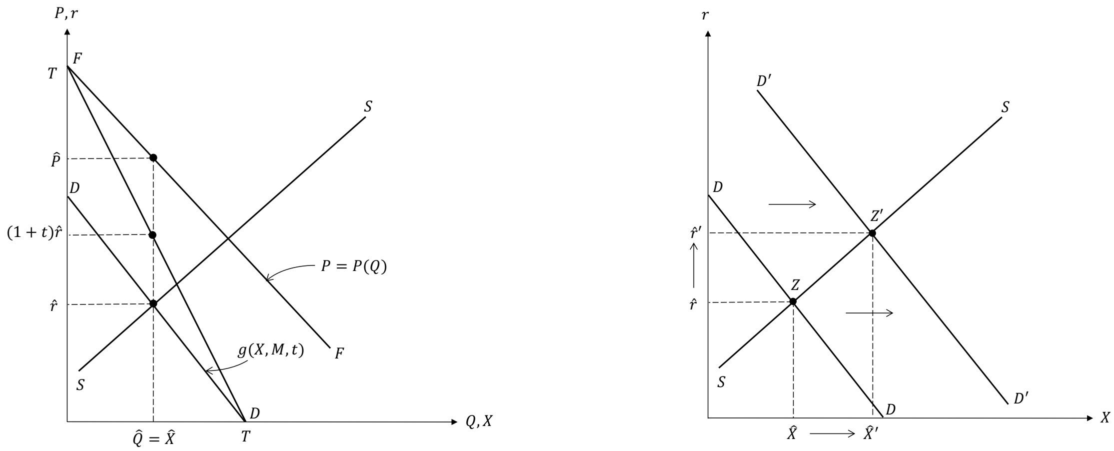
$$g_X = \frac{P'(X)(M + 1 + \eta(X))}{[M(1+t)]^2} < 0, \quad g_M = -\frac{XP'(X)}{M^2(1+t)} > 0, \quad g_t = -\frac{g(X, M, t)}{1+t} < 0.$$

From the signs of the partials, it follows immediately that: (i) the demand for intermediate imports is downward-sloping; (ii) an increase in the number of importers at Home raises the demand for intermediate imports; and (iii) a reduction in ad-valorem tariff acts as a reduction in unit costs and boosts up the demand for intermediate imports.

Equating demand and supply of intermediate imports given by (5) and (2) respectively yields the equilibrium values of aggregate intermediate imports and input price determined at the market-clearing level. In this paper, we use "hat" to denote equilibrium values of variable for an arbitrary tariff rate t. Thus, $\hat{X}$ and $\hat{r}$ respectively

[6]Our results hold for specific tariff as well. These two forms of tariff are equivalent in a framework like ours where the upstream sector is perfectly competitive.

Figure 2: Equilibrium with fixed number of firms



(a) (b)

denote aggregate imports and input price in equilibrium. Clearly, these equilibrium values satisfy

$$\hat{r} = h(\hat{X}) = g(\hat{X}, M, t). \tag{6}$$

Once these variables are determined, other equilibrium variables of the model are also determined. In particular, our production technology implies that each Home firm imports $\hat{x} = \frac{\hat{X}}{M}$ units of the intermediate good to produce $\hat{q} = \frac{\hat{Q}}{M}$ units of the final good.

Figure 2(a) illustrates the short-run equilibrium. FF and DD are the demand functions for the final good and the intermediate good respectively. TT plots $(1+t)g(X, M, t) = P(X) + \frac{XP'(X)}{M}$ against X, while SS plots the supply function $r = h(X)$. The intersection between DD and SS gives the equilibrium values $(\hat{X}, \hat{r})$. The tariff-inclusive input price $\hat{r}(1+t)$ and the final good price $\hat{P}$ are then read off TT and FF respectively at $X = \hat{X}$.

Proposition 1 describes how these equilibrium outcomes vary with tariff and the number of importers (see Appendix A.1.1 for proof).

Proposition 1 *For a given number of importers (M), a reduction in a tariff rate (t) raises aggregate imports ($\hat{X}$) and the firm-level import of the intermediate good ($\hat{x}$). For a given tariff rate t, an increase in the number of importers raises aggregate imports but lowers firm-level imports of the intermediate good. Input price ($\hat{r}$) increases with both a reduction in the tariff rate and an increase in the number of importers.*

Figure 2(b) illustrates demand and supply for intermediate imports. As noted earlier, both a reduction in tariff rate t and an increase in the number of importers M lead to higher demand for intermediate imports at a given r, shifting the demand curve DD outwards to $D'D'$. The supply curve SS remains unchanged. Hence, the equilibrium point moves from Z to Z', whereby both aggregate volume of imports and input price increase. While not stated in Proposition 1, the effect of changes in t or M on the final good market are immediate from the effect on $\hat{X}$. Whenever $\hat{X}$ increases, aggregate production of the final good $\hat{Q}(=\hat{X})$ increases, whereas the final good price $\hat{P} = P(\hat{Q})$ declines. Taken together, both lower tariff and increased competition lead to larger aggregate output and lower price in the final good market.

What about the effect on firm-level imports, $\hat{x} = \frac{\hat{X}}{M}$? As aggregate imports increase with tariff reduction and M is fixed, firm-level imports increase too. To examine the effect an increase in M on $\hat{x}$, decompose $\frac{d\hat{x}}{dM}$ as

$$\frac{d\hat{x}}{dM} = \frac{\partial \hat{x}}{\partial M} + \frac{\partial \hat{x}}{\partial r}\frac{d\hat{r}}{dM}.$$

If quantities are strategic substitutes, entry in oligopoly leads to a business-stealing (or cannibalization) effect: increased competition reduces each incumbent firm's output. The lower the production of the final good, the lower the imports of the intermediate good and thus $\frac{\partial \hat{x}}{\partial M} < 0$. In our setting, as an increase in M also induces an increase in the input price, tougher competition reduces the demand for intermediate good even further and thus $\frac{d\hat{x}}{dM} < \frac{\partial \hat{x}}{\partial M} < 0$.

To summarize, when the market structure is exogenous, the effect of tariff reduction, all three—input price, aggregate imports, firm-level imports (intensive margin)—increase with a reduction in tariff. However, what would happen when the market structure is endogenous so that the number of firms is responsible to a reduction in tariff? Below, we develop the long-run analysis with free entry of firms and show that the effect of tariff on intensive margin becomes ambiguous as a tariff reduction not only reduces cost for each importer but also leads to higher number of importers. Cost reduction raises firm-level imports while increased competition lowers it. In such a setting, the demand curvature play an important role in resolving this ambiguity.

2.3 Long-Run Analysis

One of the key features of the long-run analysis is that the number of importers, M, is endogenously determined by free entry. To avoid notational clutter, we continue to use "hat" to denote equilibrium values of variable for an arbitrary tariff rate t. For example, $\hat{X}$ and $\hat{x}$ respectively denote aggregate imports and firm-level imports. The only new notation is $\hat{M}$—the number of importers—which is endogenously determined and hence reacts to tariff change.

In the long-run setting, Home firms first incur entry cost $K (> 0)$ to enter the downstream sector. Assuming the entry cost is sufficiently low enough so that at least one Home firm engages in the final good production, the free-entry number of Home firms is determined implicitly by the value of M that satisfies the zero-profits condition: $(P(Q) - r(1+t))q - K = 0$, where q and Q are defined in (3) and (4) respectively. Rewriting (3) as $P(Q) - r(1+t) = -P'(Q)q$ and replacing q with $\frac{Q}{M}$, we can express the zero-profits condition as

$$-\frac{P'(Q)Q^2}{M^2} - K = 0. \tag{7}$$

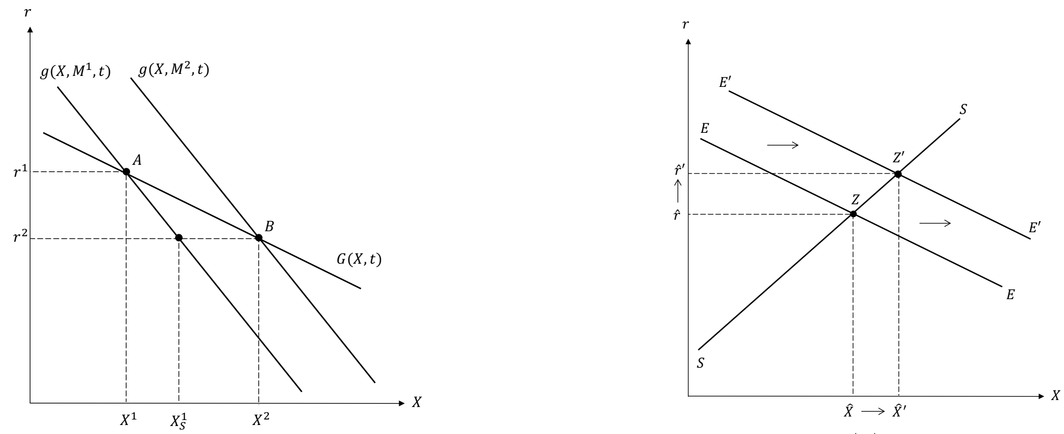
Moreover, differentiating (7) and applying (1), we find that there is a positive monotone association between aggregate output Q and the number of firms M in free-entry equilibrium:

$$\frac{dM}{dQ} = -\frac{q(2P'(Q) + QP''(Q))}{2K} > 0.$$

Exploiting this equilibrium relationship, let $M(Q)$ denote the unique value of M that is compatible with the free entry equilibrium level of Q. Though we can express each Home firm's Cournot output q as (3) as before, aggregate output Q now uniquely solves

$$M(Q)P(Q) + QP'(Q) - M(Q)r(1+t) = 0. \tag{8}$$

Figure 3: Equilibrium with free entry



(a) (b)

As in the short run, (1) guarantees uniqueness of Q in the long run (see Appendix A.1.2). Note that (8) is effectively the same as (4) except for one substitution: $M(Q)$ in place of M. This substitution preserves the analogy with the short-run analysis and simplifies the derivation of the aggregate demand curve for intermediate imports in the presence of entry. Substituting Q with X into (8) and rearranging, we immediately obtain the inverse demand function for intermediate imports:

$$r = \frac{P(X) + \frac{XP'(X)}{M(X)}}{1+t} = g(X, M(X), t) \equiv G(X, t). \tag{9}$$

As before, (9) is the inverse demand function for intermediate imports; however, the number of firms is subsumed into aggregate output due to (8). We refer to $G(X, t)$ in as entry-augmented demand for intermediate imports, which satisfies in light of (1) the following inequalities:

$$G_X = g_X + g_M M'(X) = \frac{P'(X)[2M(X) + \eta(X)]}{2M(X)(1+t)} < 0, \quad G_t = -\frac{G(X,t)}{1+t} < 0.$$

These partials show that (i) entry-augmented demand for intermediate imports is downward-sloping ($G_X < 0$); (ii) it increases with a reduction in tariff ($G_t < 0$).

Figure 3(a) illustrates the relationship between the long-run entry-augmented demand curve $G(X, t)$ and the short-run demand curves $g(X, M, t)$. Suppose that the input price is initially r^1. Let X^1 and M^1 denote aggregate demand and the number of importers that enter the downstream sector corresponding to $r = r_1$. Point A lies on the short-run demand curve D^1D^1 as depicted by $g(X, M^1, t)$. As the input price decreases to r_2, aggregate demand increases to X_S^1 if the number of importers is fixed at M^1. With free entry, however, as the input price decreases, entry becomes more profitable, and the number of importers increases to $M^2 (> M^1)$. The relevant short-run demand curve D^2D^2 is given by $g(X, M^2, t)$ which lies to the right of D^1D^1. Entry increases aggregate demand from X_S^1 to X^2. Therefore, the entry-augmented demand for intermediate imports $G(X, t)$ passes through points A and B.

Equating the demand and supply of intermediate imports—(9) and (2)—yields the equilibrium values $(\hat{X}, \hat{r})$:

$$\hat{r} = h(\hat{X}) = G(\hat{X}, t).$$

The equilibrium is depicted in Figure 3(b) where EE denotes the long run demand for the intermediate good $G(X,t)$, SS denotes the supply function $r = h(X)$, and the intersection point Z denotes the equilibrium: $(\hat{X}, \hat{r})$. Once these variables are determined, other equilibrium variables are also determined. Each Home firm imports $\hat{x} = \frac{\hat{X}}{\hat{M}}$ units of the intermediate good to produce $\hat{q} = \frac{\hat{Q}}{\hat{M}}$ units of the final good. Proposition 2 describes how these equilibrium outcomes vary with tariff reductions under free entry (see Appendix A.1.3 for proof).

Proposition 2 *A reduction in ad-valorem tariff rate t raises aggregate imports ($\hat{X}$), the number of importers ($\hat{M}$) and the input price ($\hat{r}$). Firm-level imports ($\hat{x}$) increase with tariff reduction if and only if the demand function is strictly convex.*

The effect of tariff reduction on $\hat{X}$ and $\hat{r}$ is immediate from Figure 3(b). Lower tariff leads to higher aggregate demand for intermediate imports at a given r. The entry-augmented demand curve EE shifts outward to $E'E'$ in Figure 3(b). The supply curve SS remains unchanged. As the equilibrium point moves from Z to Z', both aggregate imports $\hat{X}$ and input price $\hat{r}$ increase with a reduction in tariff. As aggregate imports $\hat{X}(= \hat{M}\hat{x})$ increase with lower tariff, at least one of the two—$\hat{M}$ and $\hat{x}$—must rise. Consider first the effect on the number of importers $\hat{M}$. Differentiating (7) with respect to t and rearranging,

$$\frac{d\hat{M}}{dt} = -\frac{P'(\hat{X})\hat{x}(1 + \frac{\hat{\eta}}{2})}{K}\frac{d\hat{X}}{dt} < 0, \tag{10}$$

where $\hat{\eta} = \eta(\hat{X})$ is equilibrium elasticity of slope of demand defined in (1). The inequality in (10) follows from noting that $d\hat{X}/dt < 0$ and our demand condition given in (1). If the demand function is not too convex, thus, the number of importers increases with reduction in tariff.

In contrast, the effect of tariff reduction on firm-level imports $\hat{x}$ critically depends on demand curvature. Recall that when M is fixed, aggregate imports $\hat{X}$ and firm-level imports $\hat{x}(= \frac{\hat{X}}{M})$ move in the same direction. That is not necessarily the case under free entry. Rewriting the zero-profits condition (7) as $-P'(\hat{X})\hat{x}^2 - K = 0$ and differentiating, the effect of tariff on intensive margin is expressed as

$$2\hat{x}\frac{d\hat{x}}{dt} = \frac{KP''(\hat{X})}{[P'(\hat{X})]^2}\frac{d\hat{X}}{dt}, \tag{11}$$

which implies that firm-level imports and aggregate imports move in the same direction if and only if the inverse demand function is strictly concave, i.e., $P''(.) > 0$. Intuitively, lower tariff prompts each Home firm to scale up production, which in turn raises the demand for intermediate good. However, entry (induced by lower tariff) prompts existing incumbent firms to produce less and import less (due to increased competition). (11) suggests that these two opposing effects exactly cancel out one another for linear demand with $P''(.) = 0$. For strictly convex demand with $P''(.) < 0$, the intensive margin increases with a reduction in tariff while the opposite is true for strictly concave demand.

Here is a summary of our long-run findings. Both input price and aggregate imports are decreasing in tariff irrespective of the entry environment. However, as the number of importers endogenously decreases with tariff, firm-level imports may increase or decrease with tariff. In contrast to the short run where firms always cut back their imports in response to higher tariff, firm-level imports decline in response to higher tariff, which occurs if and only if the demand is strictly convex. As we show below, the effect of tariff on firm-level imports plays an important role in determining whether optimal tariff in our setting falls below or stays above the competitive benchmark: inverse of export supply elasticity.

3 Welfare and Optimal Tariff

The traditional terms-of-trade motive for tariff—which manifests in the form of input price reduction—exists for both short-run and long-run settings. Counteracting this in the short run is underproduction in oligopoly, which always makes optimal tariff lower compared to what it would be in a perfectly competitive downstream sector: inverse of export supply elasticity. In the long run, however, optimal tariff could be higher since, in a homogeneous-product setting like ours, tariff improves welfare by mitigating excess entry in the downstream sector. In this section, we first show that a necessary and sufficient condition for optimal tariff to be lower than the competitive benchmark is that firm-level imports (i.e., intensive margin) decline with tariff or equivalently tariff lowers the scale of production at the firm level. Subsequently we characterize optimal tariff for the two settings examined in Section 2. We show that negative optimal tariff arises irrespective of demand curvature in the short run as intensive margin always decreases with tariff. In contrast, this possibility depends strongly on demand curvature and arises only for strictly convex demand in the long run.

Let us start with the welfare expression. Quasilinear preferences together with the assumption that all tariff revenues are rebated back to consumers allow us to express Home welfare as

$$W \equiv \underbrace{\left[\int_0^{\hat{Q}} P(y)dy - P(\hat{Q})\hat{Q}\right]}_{\text{Consumer surplus}} + \underbrace{\left[P(\hat{Q})\hat{Q} - \hat{r}(1+t)\hat{X} - \hat{M}K\right]}_{\text{Home profits}} + \underbrace{\hat{r}t\hat{X}}_{\text{Tariff revenues}} + \underbrace{\bar{L}}_{\text{Labor income}},$$

where all variables are evaluated at equilibrium values for an arbitrary tariff rate t just as in Section 2. Clearly a government chooses the tariff rate to maximize Home welfare. Canceling out the common terms in welfare and subsequently differentiating it with respect to t,

$$\frac{dW}{dt} = \left[\left(\hat{P} - \hat{r}(1+t)\right)\hat{x} - K\right]\frac{d\hat{M}}{dt} + \hat{M}\left(\hat{P} - \hat{r}(1+t)\right)\frac{d\hat{x}}{dt} + \hat{r}\left(t - \frac{1}{e_s}\right)\frac{d\hat{X}}{dt}, \tag{12}$$

where $e_s \equiv \frac{\hat{r}}{\hat{X}h'(\hat{X})}$ is elasticity of export supply of intermediate imports.

While (12) holds irrespective of market structure, it can be simplified further. On the one hand, the number of Home firms M is fixed in the short run, and hence $\frac{d\hat{M}}{dt} = 0$ in (12). On the other hand, Home profits dissipate due to free entry in the long run, and hence $(\hat{P} - \hat{r}(1+t))\hat{x} - K = 0$ in (12). As $((\hat{P} - \hat{r}(1+t))\hat{x} - K)\frac{d\hat{M}}{dt} = 0$ in both short run and long run, the following equality must hold at the welfare maximizing value of t:

$$M\left(\hat{P} - \hat{r}(1+t)\right)\frac{d\hat{x}}{dt} + \hat{r}\left(t - \frac{1}{e_s}\right)\frac{d\hat{X}}{dt} = 0.$$

Oligopolistic market structure always involves pricing above marginal cost, i.e., $\hat{P} - \hat{r}(1+t) > 0$. Furthermore, as seen in the last section, aggregate output always decreases with tariff in any market structure, i.e., $\frac{d\hat{X}}{dt} < 0$. Noticing that these inequalities hold both in the short run and long run, the sign of welfare-maximizing value of the tariff rate must satisfy the following relationship:

$$\text{sgn}\left(t - \frac{1}{e_s}\right) = \text{sgn}\frac{d\hat{x}}{dt}. \tag{13}$$

The first part of Proposition 3 restates (13) in words, while the second part follows from combining (13) and Proposition 2.

Proposition 3 *Let t^* denote optimal ad-valorem tariff and e_s^* denote elasticity of supply for the intermediate good evaluated at $t = t^*$. Then,*

(*i*) *Optimal tariff (t^*) is strictly lower than inverse of supply elasticity (e_s^*) if and only if intensive margin is increasing in tariff.*

(*ii*) *Optimal tariff (t^*) is strictly lower than inverse of supply elasticity (e_s^*) for all demand functions in the short run, and all strictly convex demand functions in the long run.*

At the heart of the short-run result is underproduction in oligopoly which requires downward adjustment of tariff from the competitive benchmark. In contrast, long run production exhibits scale economies as reflected in the industry average cost given as $\hat{r}(1+t) + \frac{K}{\hat{x}}$. Whether tariff increases the scale of production or not plays an important role in determining whether optimal tariff exceeds or falls short of the competitive benchmark. The long-run result follows from noting that firm-level imports $\hat{x}$ increase (decrease) with tariff if and only if the demand function is strictly concave (convex).

In the remainder of this section, we first examine how Home welfare varies with tariff and determine optimal tariff in the short-run setting, treating the number of importers as fixed (Section 3.1). Subsequently, we derive optimal tariff for the long-run setting in which the number of importers is endogenously determined via free entry (Section 3.2).

3.1 Short Run

Consider first the effect of tariff on Home welfare W in the short run where the number of importers M is fixed. Here, all of the equilibrium values refer to the ones obtained in Section 2.2. Noting $\frac{d\hat{X}}{dt} = M\frac{d\hat{x}}{dt}$ in that setting and using the definition of e_s, (12) is alternatively expressed as

$$\frac{dW}{dt} = (\hat{P} - \hat{r})\frac{d\hat{X}}{dt} - \hat{X}\frac{d\hat{r}}{dt}. \tag{14}$$

From Proposition 1, it follows that both aggregate output $\hat{X}$ and input price $\hat{r}$ are decreasing in t. The first term, $(\hat{P} - \hat{r})\frac{d\hat{X}}{dt}$, captures a welfare loss due to a tariff-induced output reduction. The final good is valued at price of $\hat{P}$ by Home consumers but it is produced at cost of $\hat{r}$ (net of tariff) from Home's perspective. This profit margin, $\hat{P} - \hat{r}$, multiplied by the amount of output lost by tariff, $\frac{d\hat{X}}{dt}$, is the magnitude of the welfare loss. The second term, $\hat{X}\frac{d\hat{r}}{dt}$, captures a welfare gain due to lower input price. Tariff decreases Home demand for intermediate imports, which in turn decreases input price. This can be thought of as terms-of-trade improvement for Home. Optimal tariff strikes a balance between the two—the welfare loss from the reduction in aggregate output and the welfare gain from the terms-of-trade improvement.

To derive optimal tariff from (14), the first-order condition of profit-maximizing problem (3) gives rise to the markup pricing formula that naturally applies to downstream oligopoly:

$$\hat{P} = \frac{\hat{r}(1+t)}{1 - \frac{1}{Me_d}},$$

where $e_d \equiv -\frac{\hat{P}}{\hat{Q}P'(\hat{Q})} (> 0)$ is elasticity of inverse demand for the final good at $P = \hat{P}$. This formula states that Home firms set the equilibrium final good price $\hat{P}$ that equals the markup $\frac{1}{1-\frac{1}{Me_d}}$ over marginal cost $\hat{r}(1+t)$.

Finally, substituting the expressions for $\frac{d\hat{r}}{dt}$ and $\hat{P}$ into (14) and rearranging,

$$\frac{dW}{dt} = \left(\left(\frac{t + \frac{1}{Me_d}}{1 - \frac{1}{Me_d}} \right) e_s - 1 \right) \hat{X} h'(\hat{X}) \frac{d\hat{X}}{dt}.$$

Optimal tariff is given by the value of tariff that solves this equality.

Proposition 4 *Optimal ad-valorem tariff is implicitly given by*

$$t^* = \frac{1}{e_s^*} - \frac{1}{Me_d^*} \left(1 + \frac{1}{e_s^*} \right), \tag{15}$$

where e_d^* and e_s^* denote demand elasticity for the final good and supply elasticity for the intermediate imports respectively, both of which are evaluated at $t = t^*$ and $(X^*, r^*) = (\hat{X}, \hat{r})|_{t=t^*}$. Moreover,

(i) $\lim_{M\to\infty} t^* = \frac{1}{e_s^*} > 0$, $\lim_{e_s\to\infty} t^* = -\frac{1}{Me_d^*} < 0$.

(ii) *Suppose e_s is constant or strictly decreasing in r. There exists a unique M^* such that optimal tariff rate is strictly positive if and only if $M > M^*$.*

(15) shows that when the downstream sector is perfectively competitive ($M = \infty$), we get the familiar result: optimal tariff is strictly positive and its value equals inverse of export supply elasticity for intermediate imports, $t^* = \frac{1}{e_s^*}$. It also shows that when the downstream sector is oligopolistic ($M < \infty$), optimal tariff is strictly lower than the competitive benchmark. As in any trade models, tariff on intermediate imports improves terms-of-trade and increases welfare. In our oligopoly model, however, that improvement comes at the cost of exacerbating underproduction. Tariff reduces output and creates a welfare loss as firms' profit margin, $\hat{P} - \hat{r}(1+t)$, is strictly positive when the number of Home firms is finite. The welfare loss is absent in the competitive benchmark where the welfare loss disappears as the profit margin approaches zero, while keeping the terms-of-trade motive. As a result, in the presence of the oligopolistic downstream sector, a welfare-maximizing government has smaller incentive to impose tariff on intermediate imports.

As optimal tariff is lower than the competitive benchmark, optimal tariff could be negative. To see this, let $e_s \to \infty$. When the supply function becomes perfectly elastic, the terms-of-trade motive for tariff disappears. In that case, the only harmful effect of tariff remains, and import subsidy (i.e., negative tariff) increases Home welfare by raising aggregate output. This setting is similar to a single-stage, Cournot oligopoly with M firms and constant marginal cost, where import subsidy increases welfare by mitigating underproduction in oligopoly. Optimality of import subsidy is not restricted to the case when e_s is large. Even for arbitrarily small e_s, there exist suitable demand functions and market structure such that $t^* < 0$.

Nevertheless, since $\lim_{M\to\infty} t^* > 0$, the possibility of positive optimal tariff remains when M is larger than a threshold value, say M^*. Part (ii) of Proposition 4 says that a sharp characterization exists when either export supply elasticity is constant or decreasing in input price r. This condition ensures that optimal tariff increases with the number of importers M in the downstream sector which in turn implies[7]

$$M > M^* \iff t^* > 0.$$

[7] See Appendix A.1.4 for a formal proof of the statement.

Remarks. We have assumed (i) no Home intermediate input suppliers so that Home specializes in final good production; and (ii) no Foreign consumers so that Home does not export the final good. While these assumptions are made to keep the analysis simple, they are immaterial to our key results. If the first assumption is dropped, profits of Home intermediate input suppliers increase with tariff which increases Home welfare. If the second assumption is dropped, an adverse effect of tariff on consumer surplus is weakened, which also increases Home welfare. Both of these effects work towards increasing optimal tariff without introducing substantive changes in terms of the analysis.

We have also assumed simple one-to-one production technology. In particular, we have considered a single intermediate input, c units of which is used to produce one unit of final good. By focusing on such technology, we have abstracted away from factor price distortion and its related effect on production efficiency. By design, we also ignore feedback effects arising from input-output structure.[8] If the final good were produced using labor and intermediate input, optimal tariff would have been higher as part of the adverse effect could have been moderated by substituting input with labor. Similar effects would arise if there were multiple inputs which were substitutable to some degree.

3.2 Long Run

Consider next the effect of tariff on Home welfare in the long run where the number of importers is endogenous. However, here, all of the equilibrium values refer to the ones obtained in Section 2.3 and we have to take account of the effect on extensive margin. Noting $\frac{d\hat{X}}{dt} = \hat{M}\frac{d\hat{x}}{dt} + \hat{x}\frac{d\hat{M}}{dt}$ in that setting, (12) is alternatively expressed as

$$\frac{dW}{dt} = (\hat{P} - \hat{r})\frac{d\hat{X}}{dt} - \hat{X}\frac{d\hat{r}}{dt} - \frac{d\hat{M}}{dt}K. \tag{16}$$

As in (14), the first two terms in (16) respectively capture the welfare loss from a reduction in aggregate output and the welfare gains from a reduction in the input price, although the magnitude of changes in these terms due to tariff are different from the case where M is fixed. The new term, $-\frac{d\hat{M}}{dt}K$, captures the welfare gain from fixed cost savings due to tariff-induced exit of importers (as the downstream sector entails excess entry), which occurs only with free entry. In the long run, optimal tariff strikes a balance between the three—the welfare loss from the reduction in aggregate output and the welfare gain from the terms-of-trade improvement as well as tariff-induced exit of importers.

To derive optimal tariff from (16), simplify the expression $\frac{d\hat{M}}{dt}$ in (10) as

$$\frac{d\hat{M}}{dt} = \frac{\left(1 + \frac{\hat{\eta}}{2}\right)\hat{X}}{\hat{M}}\frac{d\hat{X}}{dt},$$

where the equality follows from rewriting (3) as $\hat{P} - \hat{r}(1+t) = -P'(\hat{X})\hat{x}$ and the zero-profits condition in (7). Using this equality and noting that $\hat{X}\frac{d\hat{r}}{dt} = \hat{X}h'(\hat{X})\frac{d\hat{X}}{dt}$, we can further express (16) as

$$\frac{dW}{dt} = \left(\left(\frac{t\left(1 - \frac{1}{\hat{M}e_d} - \frac{\hat{\eta}}{2\hat{M}e_d}\right) - \frac{\hat{\eta}}{2\hat{M}e_d}}{1 - \frac{1}{\hat{M}e_d}}\right)e_s - 1\right)\hat{X}h'(\hat{X})\frac{d\hat{X}}{dt}.$$

Setting $\frac{dW}{dt} = 0$ and solving for t gives the expression for optimal tariff in the long run.

[8]See Beshkar and Lashkaripour (2020) for such feedback effects in a neoclassical trade framework.

Proposition 5 *Optimal ad-valorem tariff is implicitly given by*

$$t^* = \frac{1}{e_s^*} + \left(\frac{\frac{\eta^*}{2M^* e_d^*}}{1 - \frac{1}{M^* e_d^*} - \frac{\eta^*}{2M^* e_d^*}}\right)\left(1 + \frac{1}{e_s^*}\right), \tag{17}$$

where e_d^*, e_s^*, and η^* denote the elasticity of demand for the final good, the elasticity of export supply for the intermediate imports, and the elasticity of slope of the inverse demand function respectively, all of which are all evaluated at $t = t^*$ and $(X^*, r^*, M^*) = (\hat{X}, \hat{r}, \hat{M})|_{t=t^*}$. Furthermore,

(*i*) $\lim_{K\to 0} t^* = \frac{1}{e_s^*} > 0$, $\quad \lim_{e_s^*\to\infty} t^* \gtreqless 0 \;\Leftrightarrow\; P''(.) \lesseqgtr 0.$

(*ii*) $t^* \gtreqless \frac{1}{e_s^*} \;\Leftrightarrow\; P''(.) \lesseqgtr 0.$

(*iii*) *For all concave demand functions, optimal tariff is strictly positive. Negative optimal tariff arise only for convex demand functions.*

Part (i) says that when the entry cost is arbitrarily small so that the downstream sector becomes perfectly competitive, optimal tariff t^* approaches $\frac{1}{e_s^*}$—inverse of the export supply elasticity for intermediate imports, as in the short run. When the supply function becomes perfectly elastic, however, optimal tariff is not always negative. When $e_s^* = \infty$ so that the terms-of-trade motive for tariff disappears, tariff affects welfare through two additional channels in free-entry oligopoly. First, as in the short run, tariff worsens welfare by exacerbating underproduction. Second, in contrast to the short run, tariff improves welfare by saving the entry cost due to tariff-induced exit of importers. Evaluating (17) at $e_s^* = \infty$, whether the welfare gain outweighs the welfare loss depends on demand curvature η. For linear demand ($\eta = 0$), the welfare gain and loss are exactly offset and hence free trade is optimal ($t^* = 0$). For concave demand ($\eta > 0$), the welfare gain is greater than the welfare loss and hence positive tariff is optimal ($t^* > 0$), while the opposite is true for convex demand.

To understand the point in more detail, consider an infinitesimally small increase in tariff from free trade. Evaluating (12) at $t = 0$ implies that, starting from free trade, tariff improves Home welfare if and only if

$$\hat{M}(\hat{P} - \hat{r})\frac{d\hat{x}}{dt} > \hat{X}\frac{d\hat{r}}{dt}.$$

While this inequality holds irrespective of market structure, the effect of tariff on intensive margin differs. From Section 2.3, we know that, in endogenous market structure, intensive margin increases with tariff ($\frac{d\hat{x}}{dt} > 0$) only for strictly concave demand, while input price decreases with tariff ($\frac{d\hat{r}}{dt} < 0$) for any demand. Hence, the above inequality suggests that optimal tariff is strictly positive only for strictly concave demand. When the supply function is perfectly elastic ($e_s^* = \infty$), the terms-of-trade motive for tariff disappears ($\frac{d\hat{r}}{dt} = 0$). Then, the sign of optimal tariff is uniquely determined by the sign of $\frac{d\hat{x}}{dt}$ which in turn depends on demand curvature, as stated in part (i).

Part (ii) says that the same logic applies to whether optimal tariff exceeds or fall short of the competitive benchmark. Therefore, for linear demand, $t^* = \frac{1}{e_s^*}$ holds irrespectively of whether there is perfect competition or (free-entry) oligopoly in the downstream sector. However, in general, t^* could be greater or smaller than $\frac{1}{e_s^*}$, depending on demand curvature. Simple inspection of (17) indeed reveals that

$$t^* \gtreqless \frac{1}{e_s^*} \iff \eta^* \gtreqless 0 \iff P''(.) \lesseqgtr 0.$$

Homogeneous product oligopoly exhibits underproduction and excessive entry, i.e., more firms enter and produce too little than socially optimal levels (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). Increase in tariff reduces aggregate output which exacerbates underproduction and decreases welfare; however it reduces the number of Home firms which mitigates excessive entry and increases welfare. Loosely speaking, which effect dominates depends on the effect of tariff on firm-level imports $\hat{x}$. In the presence of entry costs, higher firm-level output implies greater economies of scale and consequently higher welfare. As we have shown in Proposition 2, firm-level imports increase (decrease) with tariff when the demand function is strictly concave (convex). Thus, starting from free trade, a small ad-valorem import tariff (subsidy) improves welfare irrespective of entry costs and market structure if and only if the demand function is strictly concave (convex). When $e_s < \infty$, however, the terms-of-trade motive for tariff resurfaces, giving the government more incentive to impose tariff. Thus, optimal tariff can be positive not only for strictly concave demand functions but also for a large range of parameterizations as indicated in part (iii) of Proposition 5.

4 Heterogeneous Firms

The short-run welfare analysis illustrates how oligopolistic *underproduction* tempers the terms-of-trade motive of tariff while the long-run welfare analysis highlights how tariff can improve welfare by curbing *excessive entry*. A common assumption in these analyses was that firms have identical costs. Here, we relax that assumption and consider cost heterogeneity to focus on a new role of tariff: reallocation.

In addition to *improving terms of trade* and *exacerbating underproduction*—which are present in both short-run and long-run versions of the model—tariff *alters production efficiency* by reallocating output among firms with different costs. As will be clear shortly, demand curvature again plays an important role in this setting: tariff improves production efficiency for all strictly log-concave demand functions by increasing the efficient firms' share of imports while production efficiency worsens for strictly log-convex demand functions.

Change in production efficiency notwithstanding, optimal tariff continues to be lower than the competitive benchmark as long as all firms reduce their imports in response to higher tariff. However, unlike the homogeneous firm setting, some firms might import more in response to increased tariff if costs are sufficiently dispersed. Qualitatively different firm-level response to tariff—some firms import more while others import less—is unique to the heterogeneous firm setting. Whether optimal tariff exceeds or falls short of inverse of supply elasticity depends on share-weighted change in firm-level imports which in turn is influenced by demand curvature and the degree of cost heterogeneity captured by the Herfindahl index.

We show that optimal tariff is lower than the competitive benchmark for linear demand, constant-elasticity demand, and more generally convex demand functions irrespective of cost differences. However, for strictly concave demand functions, the result may not hold when the Herfindahl index is sufficiently large (i.e., costs are sufficiently dispersed). Below we show in details the differential impact of tariff on firms with different costs, and its consequent implications for intensive margin, welfare, and optimal tariff.

To focus on the role of cost heterogeneity in characterizing optimal tariff more sharply, we assume that the number of firms, M, is fixed and abstract from entry-exit considerations. Thus tariff does not induce entry/exit of firms. Firms differ in their labor requirements. We assume a Leontief production function where Home firm i combines one unit of intermediate input with a_i units of labor to produce one unit of final good. Arrange M firms such that $a_i < a_{i+1}$ for $i \in \{1, 2, ..., M-1\}$. To ensure that all M firms are active, we assume that the ad-valorem tariff rate and cost-difference among the firms are not too large. Leontief technology plus unit wage imply that Home firm i's marginal cost is $a_i + r(1+t)$.

Proceeding as in the short-run analysis in Section 2, we find that Home firm i's equilibrium output is

$$q_i = -\frac{P(Q) - a_i - r(1+t)}{P'(Q)}.$$

Not surprisingly, this shows that a more efficient firm with a lower unit labor requirement produces more output. In turn, aggregate output Q uniquely solves

$$MP(Q) + QP'(Q) - M(\bar{a} + r(1+t)) = 0, \tag{18}$$

and $\bar{a} \equiv \frac{\sum_i a_i}{M}$ is average labor unit requirement among M firms. Leontief production technology implies $x_i = q_i$ and consequently $X = \sum_i x_i = \sum_i q_i = Q$. Substituting Q with X in (18) and rearranging yields the inverse demand function for intermediate inputs. Equating demand and supply of intermediate input gives us aggregate intermediate imports $\hat{X}$ and input price $\hat{r}$.

Now we can describe the effect of tariff on equilibrium. From (18), aggregate imports $\hat{X}$ decrease with tariff. Since M is fixed, average imports per firm $\bar{x} \equiv \frac{\sum_i \hat{x}_i}{M}$ decline as well. However, the same does not always hold for each firm's imports $\hat{x}_i$. Using (18) and $q_i = x_i$, we can rewrite Home firm i's imports as $\hat{x}_i = \frac{\hat{X}}{M} + \frac{a_i - \bar{a}}{P'(\hat{X})}$, which upon differentiation and some rearrangement gives

$$\frac{d\hat{x}_i}{dt} = \left(\frac{1}{M} - \frac{\hat{\eta}(a_i - \bar{a})}{\hat{X}P'(\hat{X})}\right)\frac{d\hat{X}}{dt}.$$

Observe that if all firms are identical ($a_i = \bar{a}$), intensive margin always decreases with tariff as in Section 2.2. In contrast, if firms are heterogeneous ($a_i \neq \bar{a}$), the effect on intensive margin critically depends on firm production efficiency a_i as well as demand curvature η. From $a_i - \bar{a} = P'(\hat{X})(\hat{x}_i - \frac{\hat{X}}{M})$ and $\frac{d\hat{X}}{dt} < 0$, it follows that

$$\operatorname{sgn}\frac{d\hat{x}_i}{dt} = \operatorname{sgn}\left(\hat{\eta}\left(\hat{s}_i - \frac{1}{M}\right) - \frac{1}{M}\right). \tag{19}$$

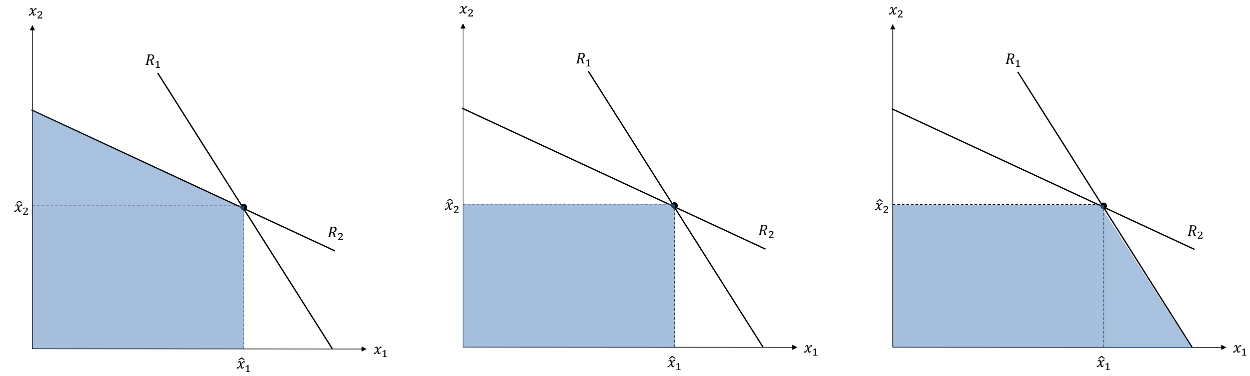
where $\hat{s}_i \equiv \frac{\hat{x}_i}{\hat{X}}$ denotes firm i's import share. (19) shows that all firms cut back their imports ($\frac{d\hat{x}_i}{dt} < 0$) when $\eta \in [-1, 0]$—a condition satisfied by demand functions which are both convex and logconcave. For example, this holds for linear demand ($\eta = 0$) or log-linear demand ($\eta = -1$). More generally, however, all firms do not necessarily cut back their imports in response to increased tariff.

To better appreciate (19), Figure 4 illustrates downstream Cournot duopoly where efficient firm 1 competes with inefficient firm 2 ($a_1 < a_2$). R_i is the reaction function of firm i and the intersection is an initial equilibrium before an increase in tariff. Moreover, noting that the reaction functions shift inwards by tariff, the shared area is the possible region of equilibrium after an increase in tariff. As aggregate imports $\hat{X}(= \hat{x}_1 + \hat{x}_2)$ decrease with tariff, both $\hat{x}_1$ and $\hat{x}_2$ cannot increase or stay the same, but everything else seems possible. However, not everything is possible for every demand function. Figure 3(a) shows that, for constant-elasticity demand, efficient firm 1 always cuts back imports less ($\frac{d\hat{x}_1}{dt} < 0$) while inefficient firm 2 can import more ($\frac{d\hat{x}_2}{dt} > 0$).[9] The opposite is true for strictly concave demand function, as depicted in Figure 3(c). Finally, Figure 3(b) shows that both firms cut back their imports for linear demand.

Proposition 6 elevates this discussion beyond duopoly. It connects possible (and impossible) responses of intensive margin with demand curvature for general demand functions and an arbitrary number of firms.

[9] This effect of trade costs is well known in the heterogeneous firm literature with CES preferences and monopolistic competition, once it is understood that we examine an *increase* in trade costs (rather than a *decrease* in such costs).

Figure 4: Effect of tariff on intensive margin in duopoly



(a) $\eta < -1$ (e.g., constant elas) (b) $\eta \in [-1, 1]$ (e.g., linear) (c) $\eta > 1$ (e.g., strictly concave)

Proposition 6 *All firms reduce imports in response to increased tariff when demand functions are convex but logconcave (i.e.,$\eta \in [0, -1]$). For all other demand functions, a firm i reduce its imports if and only if*

$$\hat{\eta}\left(\hat{s}_i - \frac{1}{M}\right) - \frac{1}{M} > 0.$$

For strictly concave demand functions ($\eta > 0$), there exists $i^* \in [1, M)$ such that a firm i cuts back its imports if and only if it is less efficient than firm i^*. For strictly log-convex demand functions ($\eta < -1$), there exists $i^{**} \in (1, M]$ such that firm i cuts back its imports if and only if it is more efficient than firm i^{**}. In all cases, aggregate imports decline with an increase in tariff.

Clearly, tariff improves production efficiency when efficient firms increase their imports but inefficient firms cut back their imports. However, Proposition 6 suggests that this is not always the case and improvement in production efficiency depends on demand curvature. To understand why demand curvature plays a key role in the presence of cost heterogeneity, it is useful to consider the profit margin of firm i given as $\hat{P} - a_i - \hat{r}(1+t)$. As tariff increases, both $\hat{P}$ and $\hat{r}(1+t)$ increase irrespective of demand curvature but not necessarily by the same amount. Which one increases more depends on elasticity of slope of demand or more precisely on log-curvature. Noting that the profit margin is expressed as $-P'(\hat{X})\hat{x}_i$ and using (19), the effect of tariff on that margin is

$$\text{sgn}\frac{d(\hat{P} - a_i - \hat{r}(1+t))}{dt} = -\text{sgn}(\hat{\eta} + 1).$$

For log-linear demand ($\eta = -1$), $\hat{P}$ and $\hat{r}(1+t)$ increase by the same amount, leaving $\hat{P} - a_i - \hat{r}(1+t)$ unchanged. Since tariff does not change the profit margin, all firms are equally affected by tariff and cut back their imports proportionately. As a result, the ratio of any firm-level variables, such as the import share $\frac{s_i}{s_j}$, remain the same. For strictly log-concave demand functions ($\eta > -1$), however, $\hat{P}$ increases less than $\hat{r}(1+t)$. Since tariff squeezes the profit margin, less efficient firms find it more difficult to maintain their production and inevitably cut back their imports, thereby reallocating the import share from inefficient firms to efficient firms. The opposite is true for strictly log-convex demand functions ($\eta < -1$).

This production efficiency altering the role of tariff affects the characterization of optimal tariff as well. For strictly log-concave demand, tariff reallocates the import share from less efficient firms to more efficient firms. As tariff generates an additional welfare gain associated with reallocation, we can expect that optimal tariff is higher for cost heterogeneity than for identical costs. For strictly log-convex demand, the opposite is true in the sense that tariff mis-reallocates the import share from more efficient firms to less efficient firms, and hence optimal tariff is lower when cost heterogeneity is present. The characterization of optimal tariff, however, must also take into account the predominant concern in oligopoly: underproduction. When all firms have identical costs, tariff exacerbates underproduction and optimal tariff is always lower than the competitive benchmark. When introducing cost heterogeneity, tariff has a new role in improving production efficiency and optimal tariff can be higher lower than the competitive benchmark. Thus it is necessary to check whether the improvement in production efficiency outweighs the exacerbation in underproduction.

Consider then the effect of tariff on welfare. In the current setting, welfare can be expressed as

$$W = \int_0^{\hat{X}} P(y)dy - \hat{r}\hat{X} - \sum_{i=1}^{M} a_i \hat{x}_i - MK + \bar{L}.$$

Upon some rearrangement, the first-order condition of welfare-maximization problem implies

$$\frac{dW}{dt} = \sum_{i=1}^{M} \left(\hat{P} - \hat{r}(1+t) - a_i\right) \frac{d\hat{x}_i}{dt} + \hat{r}\left(t - \frac{1}{e_s}\right) \frac{d\hat{X}}{dt}.$$

The above condition looks very similar to (12) that we have derived in the beginning of Section 3. In particular, when all firms are identical ($a_i = a = 0$), the import level should be the same ($\hat{x}_i = \hat{x}$) and hence we retrieve (12) indicating that the welfare impact of tariff is captured only by the response of intensive margin to tariff. When firms are heterogenous, however, not only does the import level differ but also the response of imports to tariff differs across firms, depending on firm production efficiency and demand curvature, as described above. Since tariff increases firm-level imports for some firms and decrease for others, the effect on intensive margin $\frac{d\hat{x}_i}{dt}$ does not explain the sign of $t - \frac{1}{e_s}$ as in Section 3.

Nonetheless, with slight modification, it is possible to obtain the positive association between optimal tariff and intensive margin. For that purpose, express the profit margin in terms of import share: $\hat{P} - \hat{r}(1+t) - a_i = -P'(\hat{X})\hat{x}_i = -\hat{s}_i P'(\hat{X})\hat{X}$. Substituting this and expressing $-P'(\hat{X})\hat{X}$ as $\frac{\hat{P}}{e_d}$, we can rewrite $\frac{dW}{dt} = 0$ as

$$\frac{\hat{P}}{Me_d}\left(\sum_{i=1}^{M} \hat{s}_i \frac{d\hat{x}_i}{dt}\right) + \hat{r}\left(t - \frac{1}{e_s}\right)\frac{d\hat{X}}{dt} = 0,$$

where $\sum_{i=1}^{M} \hat{s}_i \frac{d\hat{x}_i}{dt}$ is share-weighted intensive margin. Finally, noting that the response of individual intensive margin to tariff $\frac{d\hat{x}_i}{dt}$ depends on demand curvature as in (19) and using $\frac{d\hat{X}}{dt} < 0$, this implies

$$\operatorname{sgn}\left(t - \frac{1}{e_s}\right) = \operatorname{sgn}\left(\sum_{i=1}^{M} \hat{s}_i \frac{d\hat{x}_i}{dt}\right) = \operatorname{sgn}\left(\hat{\eta}\left(H - \frac{1}{M}\right) - \frac{1}{M}\right). \tag{20}$$

where $H \equiv \sum_{i=1}^{n} \hat{s}_i^2$ is Herfindahl index. It is clear that (20) is a counterpart to (13) in Section 3. In view of that similarity, (20) implies that the association is restored by replacing simple average with weighted average. Unbundling (20) further, we get our final result.

Proposition 7 *Optimal tariff is strictly lower than inverse of supply elasticity if and only if*

$$\eta(MH - 1) < 1,$$

where H^* denotes the Herfindahl index evaluated at $t = t^*$. The condition holds for linear demand, constant-elasticity demand and indeed for all strictly convex demand functions irrespective of the cost-differences across firms. However, for strictly concave demand, the conditions holds only if the costs are not too dispersed.

The first thing to note is that the condition in Proposition 7 always holds when firms are identical. In that case, $s_i = \frac{1}{M}$, $H = \frac{1}{M}$ and the above inequality is always satisfied irrespective of demand curvature. Why does the condition always hold for all convex demand functions ($\eta \leq 0$)? Note that the first equality in (20) implies that a necessary condition for optimal tariff to exceed inverse of supply elasticity is that some firm must increase its imports in response to higher tariff. From Proposition 6 we know this cannot occur for linear, log-linear, or any demand functions which satisfy $\eta \in [-1, 0]$. While the possibility $\frac{d\hat{x}_i}{dt} > 0$ exists for $\eta < -1$, the reallocation is, loosely speaking, of the wrong kind, i.e. from efficient firms to inefficient firms which calls for lower rather than higher optimal tariff. Only for strictly lonconcave demand ($\eta > 1$), does tariff reallocate the import share from inefficient firms to efficient firms. As tariff improves production efficiency, the government have incentive to set high tariff and hence $t^* - \frac{1}{e_s} > 0$ can hold provided costs are sufficiently dispersed (i.e., the Herfindahl index is large enough). This is realized in equilibrium since tariff leads to improvement in production efficiency that is greater than exacerbation in underproduction, as raised above.

For a concrete example, consider the class of inverse demand functions given by

$$P = a - \frac{bQ^{1+\eta}}{1+\eta},$$

where a, b, and η are strictly positive constants. As this class of demand is strictly concave ($\eta > 0$), there exists $M > 1 + \frac{1}{\eta}$ such that $t^* > \frac{1}{e_s}$ hold when firms are sufficiently heterogeneous and consequently H is suitably large.[10]

To summarize, tariff improves terms-of-trade (reduces input price) and amplifies underproduction in oligopoly. These effects underpin all versions of the oligopoly model and work towards reducing optimal tariff (t^*) below the inverse of supply elasticity ($\frac{1}{e_s}$) in presence of vertical linkages. When the market structure is endogenous, tariff can improve welfare further by curbing excess entry and saving entry costs. Under cost heterogeneity, tariff can increase welfare by reallocating import shares from less efficient firms to more efficient firms and improving production efficiency. Despite welfare improvement from rationalization of the industry and welfare changes from reallocation of import shares, $t^* < \frac{1}{e_s}$ hold for all strictly convex demand functions. For linear demand, $t^* \leq \frac{1}{e_s}$, where the equality holds under both free entry and heterogeneous costs version of the model. For strictly concave demand functions, optimal tariff is sensitive to the details of the oligopoly environment: $t^* < \frac{1}{e_s}$ when M is fixed, $t^* > \frac{1}{e_s}$ when M is endogenously determined via free entry, and finally t^* could be higher or lower than $\frac{1}{e_s}$ depending on the degree of cost heterogeneity and the number of firms when cost heterogeneity is present. Negative optimal tariff can arise for all demand functions in all versions of the model except when demand function is strictly concave and the market structure is endogenously determined via free entry.

[10] Note that aggregate imports do not depend on how a_i are distributed as long as their sum and consequently their mean are constant. However any increase in dispersion in a_i causes H to increase as firms' import shares get more dispersed.

5 Discussion

So far we have assumed that (i) Home firms produce a homogeneous good; and (ii) import tariff is the only policy instrument. Simplicity notwithstanding, this highlights the importance of *oligopoly* and *free entry* in determining the effect of tariff on market structure and welfare. Our first choice is in line with the literature on imperfect competition in international trade where, typically, oligopoly is paired with a homogeneous good, and monopolistic competition is paired with a differentiated good. Below, in Sections 5.1, we show how our results can be extended to incorporate product differentiation in the downstream sector. Our second choice follows standard practice in the trade policy literature where tariff is typically the key policy instrument. Section 5.2 considers an extension where the Home government sets the number of importers (or equivalently entry taxes) as well as tariff. We show that the presence of such active competition policy lowers levels of optimal tariff. While the issues raised in (i)-(ii) might seem disparate, the analyses in Sections 5.1-5.2 suggest a common theme: optimal tariff is lower compared to the baseline case in Section 3.

5.1 Product Differentiation

It is is straightforward to incorporate differentiated products in our framework. Suppose that each of M Home firms now produces a distinct variety. Let q_i and p_i denote the quantity and price of variety i. Assume that the representative consumer at Home maximizes $U(q_1, q_2, ..., q_M) + y$, where y is the numeraire good and

$$U(q_1, q_2, ..., q_M) = a \sum_{i=1}^{M} q_i - \sum_{i=1}^{M} \frac{q_i^2}{2} - \frac{1}{2} \sum_{j \neq i} q_i q_j.$$

This quadratic utility specification gives rise to a linear demand system:

$$p_i = a - q_i - b \sum_{j \neq i} q_j.$$

The parameter $b \in [0, 1]$ captures the degree of product differentiation. As b increases, varieties become less differentiated. When $b = 1$ we get linear demand for homogeneous goods. As in our baseline case, we assume one-to-one production technology and a perfectly competitive upstream sector.

Proceeding as in Section 2, the short-run demand for the intermediate good is

$$r = \frac{a - \frac{2+(M-1)b}{M} X}{1 + t} = g(X, M, t).$$

Using the zero-profits condition $(p_i - r(1 + t))q_i - K = 0$ where K denotes entry costs, we obtain the long-run demand curve for intermediate imports:

$$r = \frac{a - (2 - b)\sqrt{K} - bX}{1 + t} = G(X, t).$$

The equilibrium value of aggregate imports is obtained by equating $G(\hat{X}, t) = h(\hat{X})$. The number of importers and volume of imports per firm are given by $\hat{M} = \frac{\hat{X}}{\sqrt{K}}$ and $\hat{x} = \frac{\hat{X}}{\hat{M}} = \sqrt{K}$ respectively. Both aggregate volume of imports ($\hat{X}$) and the number of importers ($\hat{M}$) decrease with tariff. Further, firm-level imports ($\hat{x}$) do not vary with tariff when demand is linear, as in the long-run analysis in Section 2.3.

Substituting $q_i = \hat{x} = \sqrt{K}$ in $U(.)$ and simplifying, we can express Home welfare as

$$W = \hat{X}\left(a - \frac{(1+b(\hat{M}-1)}{2}\sqrt{K}\right) - \hat{r}\hat{X} - \hat{M}K + \bar{L}.$$

Differentiating W with respect to t and setting $\frac{dW}{dt} = 0$, optimal tariff is expressed as

$$t^* = \frac{1}{e_s^*} - \frac{(1-b)\sqrt{K}}{2r^*},$$

where $r^* = \hat{r}$ at $t = t^*$. Observe that, as in Section 3, optimal tariff equals inverse of export supply elasticity when products are homogeneous ($b = 1$) or entry costs are vanishingly small ($K = 0$). Comparing this with (17) reveals that optimal tariff is lower in the presence of product differentiation ($b < 1$). Imposing tariff worsens welfare by exacerbating underproduction and increases welfare by improving terms-of-trade and saving entry costs. These welfare effects are present irrespective of whether the final good is homogeneous or differentiated. However, when products are differentiated, fewer firms (resulting from higher tariff) also imply fewer varieties which lowers welfare. This additional source of a welfare loss lowers optimal tariff. In fact, when the export supply elasticity is large, import subsidy could be optimal—a possibility that does not arise for linear demand in a homogeneous product setting.

5.2 Other Policies

Multiple Instruments. We have assumed that a welfare-maximizing Home government has a single policy instrument at its disposal of and that instrument is import tariff. This is in line with tradition in the literature of optimal tariff. We can extend our framework to the case where the government uses per unit consumption subsidy denoted by c_y. In that case, (15) could be expressed as

$$(1+t^*)(1-c_y) = \left(1 - \frac{1}{Me_d^*}\right)\left(1 + \frac{1}{e_s^*}\right).$$

Setting $c_y = \frac{1}{Me_d^*}$ yields the classical result: $t^* = \frac{1}{e_s^*}$. This argument needs to be qualified in the presence of entry/exit consideration. With appropriate per-unit subsidies, only one downstream firm is needed to produce the entire output as the final good is homogeneous. Per-unit instruments alone cannot achieve that. An entry tax is needed. The first-best allocation could be achieved with a entry tax correcting excessive entry, a subsidy correcting underproduction, and an import tariff improving terms of trade. Thus, the absence of other policy instruments play a key role in our environment. See Lashkaripour and Lugovskyy (2023) for a related discussion of the first-best, second-best, and third-best policies in the presence of input-output linkages.

Competition Policy. Instead of dwelling more on per unit taxes and subsidies, we conclude our discussion by looking at a scenario where such instruments are not available. The only instrument at government's disposal is entry tax which influence the market structure. For simplicity, we work with an equivalent competition policy problem: the Home government can directly choose the number of firms (M) to maximize welfare. In reality, a government may indirectly affect market structure by entry taxes/subsidies, merger approvals and ownership restrictions. However, choosing the number of firms directly captures the essence of competition policy in a simple fashion (Horn and Levinsohn, 2001).

To highlight the role of competition policy in our framework, consider first an environment with free trade (i.e., $t = 0$). Differentiating welfare with respect to M, the impact of competition policy on welfare is given by

$$\frac{dW}{dM} = (\hat{P} - \hat{r})\frac{d\hat{X}}{dM} - \hat{X}\frac{d\hat{r}}{dM} - K,$$

where $\frac{d\hat{X}}{dM} > 0$ and $\frac{d\hat{r}}{dM} > 0$ from Proposition 1. Thus an increase in M leads to welfare gains associated with higher output but welfare losses associated with higher input price and more entry costs. Let M^0 denote the welfare-maximizing number of firms, and let M_f denote the free-entry number of firms. Clearly these numbers satisfy the above government problem and the zero-profits condition, respectively. Decomposing $\frac{d\hat{X}}{dM} = \hat{x} + M\frac{\partial \hat{x}}{\partial M}$ and noting $(\hat{P} - \hat{r})\hat{x} - K = 0$ in free-entry equilibrium, the effect of M on welfare in such equilibrium is

$$\left.\frac{dW}{dM}\right|_{M=M_f} = (\hat{P} - \hat{r})\frac{\partial \hat{x}}{\partial M} - \hat{X}\frac{d\hat{r}}{dM} < 0,$$

where the inequality comes from Proposition 1: $\frac{d\hat{x}}{dM} < 0$ due to the business-stealing effect and $\frac{d\hat{r}}{dM} > 0$ as above. This suggests that restricting the number of importers at margin improves welfare in free-entry equilibrium. Intuitively, marginal entrants ignore the business stolen from existing firms, which leads to excessive entry in homogeneous product oligopoly (Mankiw and Whinston, 1986; Suzumura and Kiyono, 1987). In our setting, marginal entrants also ignore the negative impact on terms-of-trade, which exacerbates excess entry further. Thus, for any tariff rate, the welfare maximizing number of firms is smaller than the free-entry number of firms, i.e., $M^0 < M_f$.

Now let us re-introduce tariff. Suppose that a welfare-maximizing Home government chooses not only the number of importers M but also the tariff rate t, so that it can use both trade and competition policies at the same time. How does that affect optimal tariff? Observe that

$$\frac{dW}{dt} = \frac{\partial W}{\partial t} + \frac{\partial W}{\partial M}\frac{dM^0(t)}{dt}.$$

As the number of importers adjusts optimally in equilibrium by a government, solving for optimal tariff boils down to choosing the value of t such that $\frac{\partial W}{\partial t} = 0$ or equivalently (14)—the welfare-maximizing problem in the baseline model. Hence, optimal tariff in this case is expressed as

$$t^* = \frac{1}{e_s^*} - \frac{1}{M^* e_d^*}\left(1 + \frac{1}{e_s^*}\right),$$

where all the expressions are evaluated at $t = t^*, M = M^* = M^0(t^*)$.

Not surprisingly, the expression for optimal tariff in the presence of active competition policy is the same as t^* in (15)—short-run optimal tariff—with one difference: the value of $M(= M^*)$ is endogenously determined by a welfare-maximizing Home government. As in the short run, optimal tariff is always less than the inverse of export supply elasticity, and in fact be negative if entry costs are large (and M^* is small). The possibility of t^* exceeding $\frac{1}{e_s}$ arises with free entry as higher tariff can partially mitigate the welfare loss from excessive entry. With competition policy in place, that welfare enhancing effect of tariff goes away, since the number of importers is optimally chosen. However, the welfare worsening effect of tariff (through exacerbating underproduction) still remains which in turn lowers t^* below $\frac{1}{e_s}$—the optimal tariff under perfect competition (where only the terms-of-trade motive of tariff is present).

6 Conclusion

Recent years have witnessed a significant increase in vertical specialization and a concomitant growth in trade in intermediate goods. While there is a burgeoning literature exploring firms' participation in global supply chains, normative analysis of trade policy in the presence of such production structure have started to receive attention only recently. We contribute to that literature by examining optimal input tariff in environments which capture a significant fraction of world trade in intermediate inputs: (i) final goods and intermediate goods producers interact in markets, (ii) market structure is oligopoly, and (iii) most importantly, there is free entry and exit of oligopolistic firms.

We show that whether optimal tariff exceeds or fall short of competitive benchmark—i.e., inverse of foreign export supply elasticity—depends on how intensive margin responds to tariff. In the absence of free entry, firm-level imports always increases with a reduction in tariff and optimal tariff is strictly lower than inverse of export supply elasticity. In the presence of free entry, however, intensive margin increases with tariff if and only if the demand function is strictly convex. Accordingly, optimal tariff is strictly lower (higher) than inverse of export supply elasticity if and only if the demand function is strictly convex. Further, with cost heterogeneity, optimal tariff is unambiguously lower than the competitive benchmark only for linear and strictly convex demand functions. Under strictly concave demand, optimal tariff exceeds the competitive benchmark when cost heterogeneity is sufficiently high. While oligopoly brings tariff down from the competitive benchmark in any case, entry considerations and heterogeneity puts upward pressure on tariff, since, in a homogeneous product setting like ours, tariff improves welfare by mitigating excess entry—where the number of firms is endogenously determined—and improves production efficiency (e.g., for log-concave demand functions)—where firms exhibit cost heterogeneity.

An important assumption maintained throughout the paper is a non-strategic upstream sector which does not engage actively in trade policy. Theoretically, a natural next step is to consider vertical specialization with two countries (Home and Foreign say) where consumers reside in both countries that set tariff strategically—Home on intermediate goods and Foreign on final goods. As in all trade frameworks, the terms-of-trade motive for tariff would be present here as well. However, complementarity between intermediate goods and final goods might prompt each government to set lower tariff than it would have in the absence of vertical specialization. In future research, we plan to explore these issues and related implications for trade agreements when countries are specialized in different layers of production.

Appendix: Proofs

A.1 Proof of Proposition 1

Differentiating (6) and rearranging, we get

$$\frac{d\hat{X}}{dt} = \frac{g_t}{h'(X) - g_X} < 0, \quad \frac{d\hat{X}}{dM} = \frac{g_M}{h'(X) - g_X} > 0, \tag{A.1}$$

where the inequalities follow from noting that $h'(X) > 0$ and

$$g_X = \frac{(M+1)P'(X) + XP''(X)}{M(1+t)} < 0, \quad g_M = -\frac{XP'(X)}{M^2(1+t)} > 0, \quad g_t = -\frac{g(.)}{1+t} < 0.$$

Applying (A.1) to the input supply function (2), we get

$$\frac{d\hat{r}}{dt} = h'(X)\frac{d\hat{X}}{dt} < 0, \quad \frac{d\hat{r}}{dM} = h'(X)\frac{d\hat{X}}{dM} > 0,$$

and

$$\frac{d\hat{x}}{dt} = \frac{1}{M}\frac{d\hat{X}}{dt} < 0,$$

where the inequalities follow from noting that $\frac{d\hat{X}}{dt} < 0$. Finally, rearranging (4), we get

$$P(\hat{X}) + P'(\hat{X})\hat{x} = \hat{r}(1+t).$$

Differentiating the above equality yields:

$$P'(\hat{X})\frac{d\hat{x}}{dM} = (1+t)\frac{d\hat{r}}{dM} - (P'(\hat{X}) + \hat{x}P''(\hat{X}))\frac{d\hat{X}}{dM}. \tag{A.2}$$

The right-hand side of (A.2) is positive since $\frac{d\hat{r}}{dM} > 0$, $\frac{d\hat{X}}{dM} > 0$ and $P'(\hat{X}) + \hat{x}P''(\hat{X}) < 0$. Thus, $\frac{d\hat{x}}{dM} < 0$ since $P'(\hat{X}) < 0$.

A.2 Proof of Uniqueness of Q

Consider first the short-run equilibrium. Define the left-hand side of (4) as

$$f(Q) \equiv MP(Q) + QP'(Q) - Mr(1+t).$$

Differentiating $f(Q)$ with respect to Q and rearranging we get

$$f'(Q) = (M+1)P'(Q) + QP''(Q) < 0,$$

where the inequality follows from applying (1). The inequality implies that the left-hand side of (4) is strictly decreasing in Q which in turn ensures that q and Q in the short-run equilibrium are unique.

Consider next the long-run equilibrium. Define the left-hand side of (8) as

$$F(Q) \equiv M(Q)P(Q) + QP'(Q) - M(Q)r(1+t).$$

Differentiating $F(Q)$ with respect to Q and rearranging we get

$$F'(Q) = \frac{2M(Q)P'(Q) + QP''(Q)}{2} < 0,$$

where the inequality follows from applying (1). The inequality implies that q and Q in the long-run equilibrium are also unique.

A.3 Proof of Proposition 2

Differentiating the equality of demand and supply in the long run, we get

$$\frac{d\hat{X}}{dt} = \frac{G_t}{h'(X) - G_X} < 0, \tag{A.3}$$

where the inequality follows from noting that $h'(X) > 0$ and

$$G_X = g_X + g_M M'(X) = \frac{2M(X)P'(X) + XP''(X)}{2M(X)(1+t)} < 0, \quad G_t = -\frac{G(.)}{1+t} < 0.$$

Applying (A.3) to the input supply function (2), we get

$$\frac{d\hat{r}}{dt} = h'(X)\frac{d\hat{X}}{dt} < 0.$$

From (10) we know that

$$\frac{d\hat{M}}{dt} = -\frac{P'(\hat{X})\hat{x}(1+\frac{\eta(\hat{X})}{2})}{K}\frac{d\hat{X}}{dt} > 0.$$

From (11) we have

$$\frac{d\hat{x}}{dt} = \frac{1}{2\hat{x}}\frac{KP''(\hat{X})}{[P'(\hat{X})]^2}\frac{d\hat{X}}{dt}.$$

Since $\frac{d\hat{X}}{dt} < 0$, it follows that

$$\frac{d\hat{x}}{dt} < 0 \quad \Longleftrightarrow \quad P''(.) > 0.$$

A.4 Proof of Proposition 4

The expression for t^* in (15) as well as the proof part (i) follows from the text preceding Proposition 4. To prove part (ii) it suffices to show that t^* is strictly increasing in M. Rewrite (14) as

$$\frac{\hat{P}}{\hat{r}} - \left(1 + \frac{1}{e_s}\right) = 0. \tag{A.4}$$

Suppose t remains at t^*. From Proposition 1, we know that as M increases, $\hat{P}$ declines and $\hat{r}$ increases. Thus, $\frac{\hat{P}}{\hat{r}}$ decreases with an increase in M. Furthermore, as $\hat{r}$ increases, e_s decreases. Thus, the left-hand side of (A.4) decreases with an increase in M and falls below zero. An increase in t restores the equality in (A.4), since Proposition 1 implies that effect of an increase on t on $\hat{P}$, $\hat{r}$, and consequently on e_s is exactly the opposite to that of an increase in M. Thus t^* is strictly decreasing in M.

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