Trade with Search Frictions: Identifying New Gains from Trade

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- Firms often search for suppliers to procure specialized inputs:
  - While a few core inputs are made in-house, other non-core inputs are largely purchased from outside suppliers
  - IT revolution makes it easier to search for suppliers not only within borders but also across borders
  - Access to a wide range of outsourced inputs improves production technology of firms
  - $\Rightarrow$  Consider Apple's sourcing strategy

### Motivation



- Search and matching  $\Rightarrow$  Input trade:
  - About two thirds of world trade are accounted by intermediate inputs (Johnson and Noguera, 2012)
  - Traded goods produced at the upstream stage have been rapidly increasing (Antràs et al., 2012)
  - The share of differentiated inputs has more than doubled between 1962–2000 (Antràs and Staiger, 2012)

#### Question and results

- Question:
  - What is the welfare impact of economic integration through trade in the presence of search frictions?
- Two types of economic integration:
  - Goods market integration ⇒ Trade allows firms to ship final products abroad (in classical sense)
  - Matching market integration ⇒ Trade allows firms to source intermediate
    inputs from abroad

#### Question and results



Goods market integration  $\Rightarrow$  Welfare gains are amplified

#### Question and results



Matching market integration  $\Rightarrow$  Welfare losses may occur

- Key assumptions:
  - Firms and suppliers randomly match and bargain over generated surplus (Pissarides, 2000)
  - Firms and suppliers have one-to-one relationships in their search process (Sugita et al., 2021)
  - Matched firms can enjoy a love-of-variety effect from an input expansion (Ethier, 1982; Romer, 1990; Grossman and Helpman, 1991)



#### • Consumer preferences:

$$U = \left(\int_{\omega} y(\omega)^{\frac{\sigma-1}{\sigma}} d\omega\right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1$$

• Demand and expenditure for variety  $\omega$ :

$$egin{aligned} y(\omega) &= A p(\omega)^{-\sigma} \ r(\omega) &= A p(\omega)^{1-\sigma} \end{aligned}$$

where A is the index of industry demand

#### Setup

• Firm technology:

$$y(\omega) = \left( (x^{\mathsf{F}}(\omega))^{\frac{\sigma-1}{\sigma}} + \mathbb{1}(\omega)(x^{\mathsf{S}}(\omega))^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

where both inputs are produced competitively

• Firm marginal cost:

$$\boldsymbol{c}(\omega) = \left( (\boldsymbol{w}\boldsymbol{a}^{\mathsf{F}})^{1-\sigma} + \mathbb{1}(\omega) (\boldsymbol{w}\boldsymbol{a}^{\mathsf{S}})^{1-\sigma} \right)^{\frac{1}{1-\sigma}} = \frac{\boldsymbol{w}\boldsymbol{a}^{\mathsf{F}}}{\varphi(\omega)}$$

where

$$\varphi(\omega) \equiv \left(1 + \mathbb{1}(\omega) \left(\frac{a^{\mathsf{F}}}{a^{\mathsf{S}}}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$$



• Profit-maximization problem:

$$\max_{x^{F}(\omega), x^{S}(\omega)} r(\omega) - wa^{F}x^{F}(\omega) - \mathbb{1}(\omega)wa^{S}x^{S}(\omega)$$

• Optimal pricing, output and revenue:

$$p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{wa^{F}}{\varphi}$$
$$y(\varphi) = A \left(\frac{\sigma - 1}{\sigma} \frac{\varphi}{wa^{F}}\right)^{\sigma}$$
$$r(\varphi) = A \left(\frac{\sigma - 1}{\sigma} \frac{\varphi}{wa^{F}}\right)^{\sigma - 1}$$



• Number of matches:

$$m(u^F, u^S)$$

which satisfies CRS in matching

• Probability of matches:

$$\mu^{F} \equiv m(u^{F}, u^{S})/u^{F} = m(1, \theta)$$
  
$$\mu^{S} \equiv m(u^{F}, u^{S})/u^{S} = m(1/\theta, 1) = \mu^{F}/\theta$$

where  $\theta \equiv u^S/u^F$ 

• Probability of a bad shock:  $\delta$ 



• Search process for firms:



• The law of motion:

$$\dot{N}^{F} = \delta N^{F} - N_{e}^{F}$$
$$\dot{u}^{F} = (\delta + \mu^{F})u^{F} - N_{e}^{F}$$
$$\dot{N}^{F} - \dot{u}^{F} = \delta(N^{F} - u^{F}) - \mu^{F}u^{F}$$



• When only matched firms export, the Bellman equations are given by

$$\gamma V^{F} = \frac{r}{\sigma} + \mu^{F} \left( V^{F}(\varphi) - F_{x} - V^{F} \right) - \delta V^{F} + \dot{V}^{F}$$
$$\gamma V^{F}(\varphi) = \frac{r^{F}(\varphi)}{\sigma} - \delta V^{F}(\varphi) + \dot{V}^{F}(\varphi)$$
$$\gamma V^{S} = \mu^{S} \left( V^{S}(\varphi) - F_{d} - V^{S} \right) - \delta V^{S} + \dot{V}^{S}$$
$$\gamma V^{S}(\varphi) = \frac{r^{S}(\varphi)}{\sigma} - \delta V^{S}(\varphi) + \dot{V}^{S}(\varphi)$$

where  $F_d$  and  $F_x$  are a one-time investment cost

• Assuming that  $\gamma = 0$  and setting  $\dot{V}^F = \dot{V}^F(\varphi) = 0$ :

$$V^{F} = \frac{r}{\delta\sigma} + \left(\frac{\mu^{F}}{\delta + \mu^{F}}\right) \left(\frac{r^{F}(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_{x}\right)$$
$$V^{F}(\varphi) = \frac{r^{F}(\varphi)}{\delta\sigma}$$

where the probability  $\delta$  introduces an effect similar to time discounting

• Similarly, setting  $\dot{N}^F = \dot{u}^F = \dot{N}^F - \dot{u}^F = 0$ :

$$n = \left(\frac{\mu^{\mathsf{F}}}{\delta + \mu^{\mathsf{F}}}\right) \mathsf{N}^{\mathsf{F}}$$

where  $n \equiv N^F - u^F$ 

• Bargaining within matched agents:

$$\max_{\frac{r^{F}(\varphi)}{\sigma}, \frac{r^{S}(\varphi)}{\sigma}} \left(V^{F}(\varphi) - F_{x} - V^{F}\right) \left(V^{S}(\varphi) - F_{d} - V^{S}\right)$$

subject to  $r^{F}(arphi)/\sigma + r^{S}(arphi)/\sigma = r(arphi)/\sigma$ 

• Optimal sharing rule:

$$\frac{r^{F}(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_{x} = \beta \left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_{d} - F_{x}\right)$$
$$\frac{r^{S}(\varphi)}{\delta\sigma} - F_{d} = (1 - \beta) \left(\frac{r(\varphi)}{\delta\sigma} - \frac{r}{\delta\sigma} - F_{d} - F_{x}\right)$$
$$= (\delta + u^{F})/(2\delta + u^{F} + u^{S})$$

where  $\beta \equiv (\delta + \mu^F)/(2\delta + \mu^F + \mu^S)$ 

#### • FE conditions:

$$V_e^F \equiv V^F - F_e^F = 0$$
$$V_e^S \equiv V^S - F_e^S = 0$$

• From the steady-state relationships, this can be written as

$$\frac{r}{\sigma} + \frac{n}{N^F} \beta \left( \frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d - f_x \right) - f_e^F = 0$$
$$\frac{n}{N^S} (1 - \beta) \left( \frac{r(\varphi)}{\sigma} - \frac{r}{\sigma} - f_d - f_x \right) - f_e^S = 0$$

where  $f_d \equiv \delta F_d$ ,  $f_x \equiv \delta F_x$ ,  $f_e^F \equiv \delta F_e^F$  and  $f_e^S \equiv \delta F_e^S$ 



$$\theta = u^S / u^F = (N^S - n) / (N^F - n)$$

- *FF* curve  $\theta \uparrow \Rightarrow \mu^F \uparrow \Rightarrow r/\sigma \downarrow$
- SS curve  $\theta \uparrow \Rightarrow \mu^{S} \downarrow \Rightarrow r/\sigma \uparrow$
- $\theta$  and  $r/\sigma$  are consistent with free entry in X-integration equilibrium



• Impact of X-integration

 $heta > heta_{a}$   $r/\sigma < r_{a}/\sigma$ 

 Matched firms get a larger rent by reductions in trade costs (*τ<sub>x</sub>*, *f<sub>x</sub>* ↓)

$$rac{r(arphi)}{\sigma} - rac{r}{\sigma} - f_d - f_x \quad \uparrow$$

which induces new entry of agents

- Gains from trade (GFT) in X-integration:
  - r/σ < r<sub>a</sub>/σ ⇒ Resources are reallocated from (less efficient) unmatched firms to (more efficient) matched firms
  - **2**  $\theta > \theta_a \implies$  Firms have the higher probability to meet suppliers  $(n/N^F > n_a/N_a^F)$ , enhancing overall production efficiency of the industry

#### • GFT are expressed as

$$\frac{W}{W_a} = \left[ \left( \frac{N_a^F + (\varphi^{\sigma-1} - 1)n_a}{N^F + (\varphi^{\sigma-1} - 1)n} \right) \lambda \right]^{-\frac{1}{\sigma-1}}$$

where  $\lambda$  is the expenditure share on domestic goods

- In Krugman (1980) where  $n = n_a = 0$  and  $N^F = N_a^F$ , this ratio is simply given as  $W/W_a = \lambda^{-1/(\sigma-1)}$  (Arkolakis et al., 2012)
- In our model where  $n/N^F > n_a/N_a^F$ , the values in the brackets (endogenous firm matches) matter for welfare
- 0 Numerical solutions  $\Longrightarrow$  GFT are 0.9% without search but 2.4% with search





- Impact of M-integration  $\theta < \theta_a$  $r/\sigma > r_a/\sigma$
- M-integration has three types of firms
  - Least efficient unmatched firms
  - Moderately efficient firms matched with Foreign suppliers
  - Most efficient firms matched with Home suppliers

- Main findings:
  - Search frictions in workhorse trade models may lead to contrasting welfare effects from economic integration
  - Goods market integration  $\Rightarrow$  Welfare gains are amplified
  - Matching market integration  $\Rightarrow$  Welfare losses may occur