Country Size, Technology, and Ricardian Comparative Advantage*

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Abstract

We develop a Ricardian model with heterogeneous firms in which country size and technology play a crucial role in the firm-level variables. We show that a country with larger size and better technology exhibits higher productivity and lower price-cost margins even under assumptions of C.E.S. preferences and monopolistic competition. Welfare is higher in this country, not only due to increased product variety but also due to increased competition in a domestic market. We also show that country size and technology impact critically on the intensive margin as well as the extensive margin in the gravity equation.

Keywords: Ricardian comparative advantage, country size, technology, heterogeneous firms
JEL Classification Numbers: F12, F14

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1 Introduction

Old trade theory based on comparative advantage has regained empirical relevance. In contrast to the latter half of the twentieth century where the bulk of world trade was dominated between similar industrial countries, recent years have witnessed rapidly rising trade between developed and less developed countries with lower wages, especially China. In this dissimilar-dissimilar trade, the two different types of countries not only exchange different goods across sectors by engaging in horizontal specialization, but also exchange similar goods within sectors by engaging in vertical differentiation. For instance, Schott (2008) finds empirical evidence that, while China's export bundle increasingly and disproportionately resembles that of the most developed countries in the OECD between 1972 and 2001, Chinese exports are less sophisticated and sell for a substantial discount relative to OECD varieties within narrowly defined products.

This paper develops a general-equilibrium Ricardian model with heterogeneous firms in which country size and technology play a crucial role in the firm-level variables. Following Dornbusch, Fischer, and Samuelson (1977) (henceforth DFS), we assume that productivity levels of two countries vary systematically across a continuum of sectors, where its equilibrium determines the relative wage and trade structure. Moreover, productivity levels of firms producing differentiated varieties under monopolistic competition are drawn idiosyncratically from a fixed distribution a la Melitz (2003). The interplay of these two-dimensional productivity differences helps explain vertical differentiation as well as horizontal specialization of trade between dissimilar countries in a single unified framework, shedding new light on the role of country size and technology in the selection of entry into domestic and export markets in the Ricardian model.

We show that a country with larger size and better technology exhibits higher productivity and lower price-cost margins even under assumptions of C.E.S. preferences and monopolistic competition. Consider, for example, the impact of country size. As in DFS, an increase in country size expands the range of sectors over which a growing country has a comparative advantage by reducing its relative wage. In our Ricardian model with heterogeneous firms, the lower relative wage reduces the price-cost margins (defined as the price *minus* the cost) in a growing country relative to another country, even though the markups (defined as the price *over* the cost) are constant due to C.E.S. preferences. As a result, country size increases the degree of competition in a domestic market and raises the productivity cutoff of domestic production, which is a sufficient statistic for welfare. Welfare is higher in a larger country, not only due to increased product variety but also due to increased competition in a domestic market. This stands in sharp contrast to the standard heterogeneous-firm model with C.E.S. preferences (e.g., Melitz, 2003) in which all the firm-level variables are independent of country size. Thus, our Ricardian model can overcome the drawbacks of pro-competitive effects of trade that typically arise in C.E.S. preferences and monopolistic competition, while preserving the usefulness of the workforce model in the new trade theory literature. Technology also has a similar impact on the price-cost margins and welfare in our Ricardian model. We show that, although a country with more advanced technology entails the higher relative wage, technology nonetheless reduces the price-cost margins in this country relative to another country and raises welfare there.

Country size has an impact on the firm-level variables in our Ricardian model through which an increase in country size endogenously reduces the relative wage, which in turn lowers the marginal cost of firms and raises aggregate productivity. The channel is not operative if the relative wage is exogenously fixed, in which case not only the markups but also the price-cost margins are constant. Our result has a similar flavor to those in the existing models that an increase in country size as a result of free trade agreements allows firms to engage in technology upgrading, which in turn lowers their marginal cost and raises efficiency (Bustos, 2011; Lileeva and Trefler, 2009). The current paper takes technology as exogenously fixed and uncorrelated with country size, but instead assumes that technology follows the law of Ricardian comparative advantage where country size has an impact on the relative wage. The influence of country size on firm selection is also similar with that in a model with quasi-linear-quadratic preferences (Melitz and Ottaviano, 2008). In that model, competition by country size operates through increased goods market competition that puts downward pressure on residual demands. In the current paper, in contrast, competition by country size operates through increased factor market competition that reduces the relative wage.

The equilibrium property of our model helps understand the role of country size and technology in the gravity equation. A recent body of empirical evidence using the firm-level dataset has highlighted the importance of decomposing aggregate trade flows into the extensive margin and the intensive margin, where the former refers to the number of exporting firms and the latter refers to the average export sales per firm (Bernard et al., 2007a; Helpman et al., 2008). If the gravity equation employs C.E.S. preferences and monopolistic competition, the model predicts that country size and technology affect only the number of export variety (extensive margin), leaving the average export sales (intensive margin) independent of these integrants. However, empirical evidence suggests that country size and technology have critical impacts not only on the extensive margin but also on the intensive margin in estimating trade flows. For example, Bernard et al. (2007a) show that GDP (a proxy of country size) impacts positively on the extensive margin, whereas it impacts negatively on the intensive margin in aggregate U.S. exports in 2000. Our model offers a possible explanation for this empirical pattern by allowing country size to affect the firm-level variables.

The model outcome in our paper arises only when allowing for country asymmetry in technology and thereby the relative wage is endogenously determined in equilibrium. In this respect, our paper is particularly close to Bernard et al. (2007b) who allow for country asymmetry in factor proportions and thereby factor price equalization (FPE) does not necessarily hold. They find in the environment that trade-induced resource reallocations are more significant in comparative advantage sectors than in comparative disadvantage sectors, creating a new welfare gain from trade. Although this finding is similar to that in our Ricardian model, one of the most important differences is that they have to resort to numerical simulations for the outside FPE region. (At least they do not examine the impact of country endowments on the firm-level variables in their analytical sections.) In contrast, we are able to analytically examine the impact of country size and technology on the firm-level variables even in the model with C.E.S. preferences, which proves to be useful for analyzing its consequence on welfare and the margins of aggregate trade flows in the gravity equation as noted above. This feature of the Ricardian model is not to imply however that the Heckscher-Ohlin model is less important.

Indeed it is well-known that the two-factor Heckscher-Ohlin model has the advantage of being able to provide a rich framework for analyzing distributional consequences from trade, a feature that is missing in the one-factor Ricardian model. Our emphasis is instead that the Ricardian model can provide a different lens through which to understand the real world especially when large countries with different technology engage in international trade by exploiting wage differentials. As observed by Krugman (2008), we believe this is one of the most striking aspects of recent trade flows.

Another novelty of this paper is in examining log-supermodularity studied by Costinot (2009) to explore implications of the Ricardian model with monopolistic competition and heterogeneous firms (Costinot (2009) only considers the Ricardian model with perfect competition). We show that if labor productivity is log-supermodular, not only is aggregate output but also other endogenous aggregate variables – including aggregate sector labor supply and the number of firms – are log-supermodular. As a result, international trade allows laborers to be allocated relatively more to sectors in which each country is relatively more productive, leading to the greater number of firms that operate in these sectors. This finding represents a sharp departure from that in perfect competition in which international trade simply allows all laborers to be allocated to comparative advantage sectors and the number of firms is indeterminate.

A number of papers have employed the DFS model to capture bilateral trade volumes between dissimilar countries. Eaton and Kortum (2002), while keeping perfect competition, extend the DFS model by allowing for an arbitrary number of countries to quantify the effect of country characteristics and geographic barriers on bilateral trade flows. In contrast, while keeping a two-country model, we extend the DFS model by allowing for monopolistic competition and heterogeneous firms to examine the effect of country characteristics and geographic barriers on the selection into export markets. In terms of methodology and objective, this paper is close to Okubo (2009), Fan et al. (2013) and Huang et al. (2017) in that the DFS model is integrated into a multi-sector version of the Melitz model. While these papers restrict their analysis to a Pareto distribution to obtain closed-form solutions, we develop a more general model without imposing specific parameterizations to a fixed distribution, and show with the technique from Costinot (2009) that most results of their models hold in such a setting. More importantly, we show that the Ricardian model can uncover a new insight into the impact of country size and technology on the firm-level variables through an endogenous change in the relative wage, which has not been analytically explored in the existing work.[1]

The influence of the relative wage on firm selection under the assumptions of C.E.S. preferences and monopolistic competition is similar to that in Demidova and Rodríguez-Clare (2013). They show that endogenous wage considerations in the standard heterogeneous-firm model alter drastically the impact of asymmetric trade liberalization on welfare for a liberalizing country in a small economy. Our approach differs from theirs because we study Ricardian comparative advantage and the relative wage in a large economy. Although we focus primarily on the impact of country size and technology, our model is tractable enough to study the impact of asymmetric trade liberalization on the firm-level variables and welfare, yielding a similar result with theirs.

[1]Eaton et al. (2011) extend the Melitz model to examine the role of market size on a hierarchical firm entry pattern by allowing for a market penetration cost, not by allowing market size to affect the firm-level variables as in this paper.

2 Setup

Consider a world composing of two large countries indexed by $i, j = 1, 2$. Throughout this paper, country subscripts are attached to all relevant variables.

2.1 Demand

Country i is populated by the mass of identical consumers $\overline{L}_i$ who devote their income into goods produced in a continuum of sectors over a unit interval $[0, 1]$. The preferences of a representative consumer are Cobb-Douglas across sectors and Dixit-Stiglitz within sectors:

$$U_i = \int_0^1 b_i(z) \ln Q_i(z) dz,$$

where $b_i(z)$ denotes a *constant* share of expenditure spent on sector z, which is *identical* between the two countries, and

$$Q_i(z) = \left[\sum_{h=i,j} \int_{v \in V_i(z)} q_{hi}(v, z)^{\frac{\sigma-1}{\sigma}} dv \right]^{\frac{\sigma}{\sigma-1}},$$

is the set of varieties consumed as an aggregate good in sector z. $V_i(z)$ is the set of available goods in the sector, and $\sigma(> 1)$ is a constant elasticity of substitution between varieties, which is the same across sectors. Given this aggregate good $Q_i(z)$, its dual aggregate price is given by

$$P_i(z) = \left[\sum_{h=i,j} \int_{v \in V_i(z)} p_{hi}(v, z)^{1-\sigma} dv \right]^{\frac{1}{1-\sigma}}.$$

Let $R_i(z) = P_i(z)Q_i(z)$ and $Y_i = w_i\overline{L}_i$ denote aggregate expenditure in sector z and aggregate labor income in the economy, where w_i is a wage rate. Then, the expenditure share $b_i(z)$ is defined as

$$b_i(z) = \frac{P_i(z)Q_i(z)}{Y_i} = \frac{R_i(z)}{w_i\overline{L}_i}, \qquad \int_0^1 b_i(z)dz = 1, \qquad b_i(z) = b_j(z).$$

Thus, the sum of aggregate sector expenditure equals aggregate labor income ($\int_0^1 R_i(z)dz = w_i\overline{L}_i$). As is well-known, the preferences yield the following demand functions for variety v:

$$q_{ji}(v, z) = R_i(z)P_i(z)^{\sigma-1}p_{ji}(v, z)^{-\sigma},$$

where $R_i(z) = b_i(z)w_i\overline{L}_i$ from the definition of $b_i(z)$. In the analysis below, we focus on a particular variety and drop the variety script v for notational simplicity.

It is important to note that demand structure is almost the same as the DFS model with perfect competition, except that all goods are differentiated in this model with monopolistic competition. In particular, the upper-tier Cobb-Douglas preferences imply that differentiated goods in sector z are associated with constant expenditure shares $b_i(z), b_j(z)$, which are exogenous preference parameters.

Further the sub-utility function is C.E.S. in all sectors and a freely traded outside good is excluded, implying that wage rates w_i, w_j cannot be normalized between the two countries. As stressed in the Introduction, this assumption is made to examine the role of *endogenous* factoral terms of trade in the DFS model with monopolistic competition and heterogeneous firms.

2.2 Production

There is a continuum of firms that produce a different variety in each sector. Labor is the only factor of production and firms in country i face a perfectly elastic supply of labor at country size $\overline{L}_i$. Since labor is completely mobile across sectors but immobile across countries, the wage rate w_i is the same across sectors but is different across countries (see Appendix C for empirical relevance on this).

To enter a sector in country i, potential entrants bear a fixed entry cost f_i^e, measured in country i's labor units. Upon paying this entry cost, these entrants draw their productivity level φ from a fixed distribution $G(\varphi)$, and each entrant decides whether to exit or not. If an entrant from country i chooses to serve a market in country j, it pays a variable trade cost $\tau_{ij}(\geq 1)$ and a fixed trade cost $f_{ij}(>0)$, measured in country i's labor units. These costs are the same across sectors and satisfy $\tau_{ii} = 1$ and $\tau_{ij}^{\sigma-1} f_{ij}/f_{ii} > 1$ for $i \neq j$. Labor used by a firm of productivity φ in sector z from country i to country j is a linear cost function of output for domestic production and exporting:

$$\begin{cases} l_{ii}(\varphi, z) = f_{ii} + \frac{q_{ii}(\varphi,z)}{\theta(\varphi,z,\mu)} = f_{ii} + \frac{q_{ii}(\varphi,z)}{\varphi\mu_i(z)} & \text{for domestic production,} \\ l_{ij}(\varphi, z) = f_{ij} + \frac{\tau_{ij} q_{ij}(\varphi,z)}{\theta(\varphi,z,\mu)} = f_{ij} + \frac{\tau_{ij} q_{ij}(\varphi,z)}{\varphi\mu_i(z)} & \text{for exporting,} \end{cases}$$

where $\theta(\varphi, z, \mu)$ is labor productivity.

A few points are in order for this specification. First, labor productivity $\theta(\varphi, z, \mu)$ depends on the three characteristics: (i) firm-specific φ; (ii) sector-specific z; and (iii) country-specific μ. As noted above, each firm has a different productivity level indexed by φ, which is drawn idiosyncratically from a fixed distribution $G(\varphi)$. This distribution is assumed the same across countries and sectors, with support in $[\varphi_{\min}, \infty)$. Moreover, each country also has a different productivity level indexed by $\mu_i(z)$. This productivity denotes country i's ability to produce in sector z, which varies systematically with country characteristics across a continuum of sectors. The cost function indicates that variable costs are lower if φ and $\mu_i(z)$ are greater, while fixed costs are independent of them.

Second, we employ a reduced form of labor productivity $\theta(\varphi, z, \mu) = \varphi\mu_i(z)$ which can be justified by Costinot's (2009) log-supermodular argument.[2] In our model, $\mu_i(z)$ is defined as the inverse of the unit labor requirement indexed by $a_i(z)$: $\mu_i(z) = 1/a_i(z)$. To guarantee the equilibrium, we further restrict the range of $a_i(z)$ such that $\mu_i^{-1}(z)$ exists. Let $\mu(z) \equiv \mu_1(z)/\mu_2(z)(= a_2(z)/a_1(z))$ denote the relative labor productivity (or labor requirement) in country 1. Without loss of generality, we assume that country 1 (country 2) has a relatively bigger cost advantage in high-z (low-z) sectors, which holds under the following assumption:

[2] Costinot (2009) defines Ricardian technological differences as labor productivity satisfying $\theta(\varphi, z, \mu) = h(\varphi)a(z, \mu)$. We adopt a simplified version of his definition in that $h(\varphi) = \varphi$ and $a(z, \mu) = \mu_i(z)$.

Assumption 1 The relative labor productivity $\mu(z) \equiv \mu_1(z)/\mu_2(z)$ is log-supermodular. For $z < z'$,

$$\frac{\mu_1(z)}{\mu_2(z)} \leq \frac{\mu_1(z')}{\mu_2(z')}.$$

Assumption 1 means that the relative labor productivity $\mu_i(z)/\mu_j(z)$ is increasing in the strength of country i's comparative advantage. By assumption of a continuum of sectors, Assumption 1 also means that $\mu(z)$ is increasing in z.

Following the literature, we say that country i has a comparative advantage in producing goods in sector z if country i's unit labor costs are less than or equal to country j's unit labor costs:

$$w_i a_i(z) \leq \tau_{ji} w_j a_j(z) \quad \Longleftrightarrow \quad \frac{w_i}{\tau_{ji} w_j} \leq \frac{\mu_i(z)}{\mu_j(z)}. \tag{1}$$

Let $\omega \equiv w_1/w_2$ denote the relative wage in country 1. Then (1) immediately reveals that country 1 has a comparative advantage in high-z sectors $\bar{z}_1 \leq z \leq 1$, where

$$\bar{z}_1 \equiv \mu^{-1}\left(\frac{\omega}{\tau_{21}}\right). \tag{2}$$

Similarly, country 2 has a comparative advantage in low-z sectors $0 \leq z \leq \bar{z}_2$, where

$$\bar{z}_2 \equiv \mu^{-1}(\tau_{12}\omega). \tag{3}$$

Note that, as long as $\tau_{ji} \geq 1$, these cutoff sectors satisfy $\bar{z}_1 \leq \bar{z}_2$.

Having defined Ricardian comparative advantage, we next turn to firm behavior. Consider sector z in country i where domestic firms in i and foreign firms from j monopolistically compete and choose its price to maximize the profit. Letting $p_{ii}(\varphi, z)$ and $p_{ji}(\varphi, z)$ denote the prices set by domestic firms in i and foreign firms from j, profit maximization yields the following pricing rules:

$$p_{ii}(\varphi, z) = \frac{\sigma}{\sigma - 1}\frac{w_i}{\varphi\mu_i(z)}, \qquad p_{ji}(\varphi, z) = \frac{\sigma}{\sigma - 1}\frac{\tau_{ji} w_j}{\varphi\mu_j(z)}.$$

With these pricing rules, the revenues of domestic firms and foreign firms are respectively given by

$$r_{ii}(\varphi, z) = \sigma B_i(z)\left(\frac{\mu_i(z)}{w_i}\right)^{\sigma-1}\varphi^{\sigma-1}, \qquad r_{ji}(\varphi, z) = \sigma B_i(z)\left(\frac{\mu_j(z)}{\tau_{ji} w_j}\right)^{\sigma-1}\varphi^{\sigma-1},$$

where

$$B_i(z) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma} R_i(z) P_i(z)^{\sigma-1}$$

is the index of aggregate market demand. In the revenues, aggregate market demand $B_i(z)$ is same since both domestic firms and foreign firms sell their goods to consumers in country i. In contrast, country-specific productivity levels $\mu_i(z), \mu_j(z)$ and wage rates w_i, w_j are different since foreign firms make use of foreign technology and labor in j. Note that $p_{ii}(\varphi, z) \leq p_{ji}(\varphi, z)$ and $r_{ii}(\varphi, z) \geq r_{ji}(\varphi, z)$

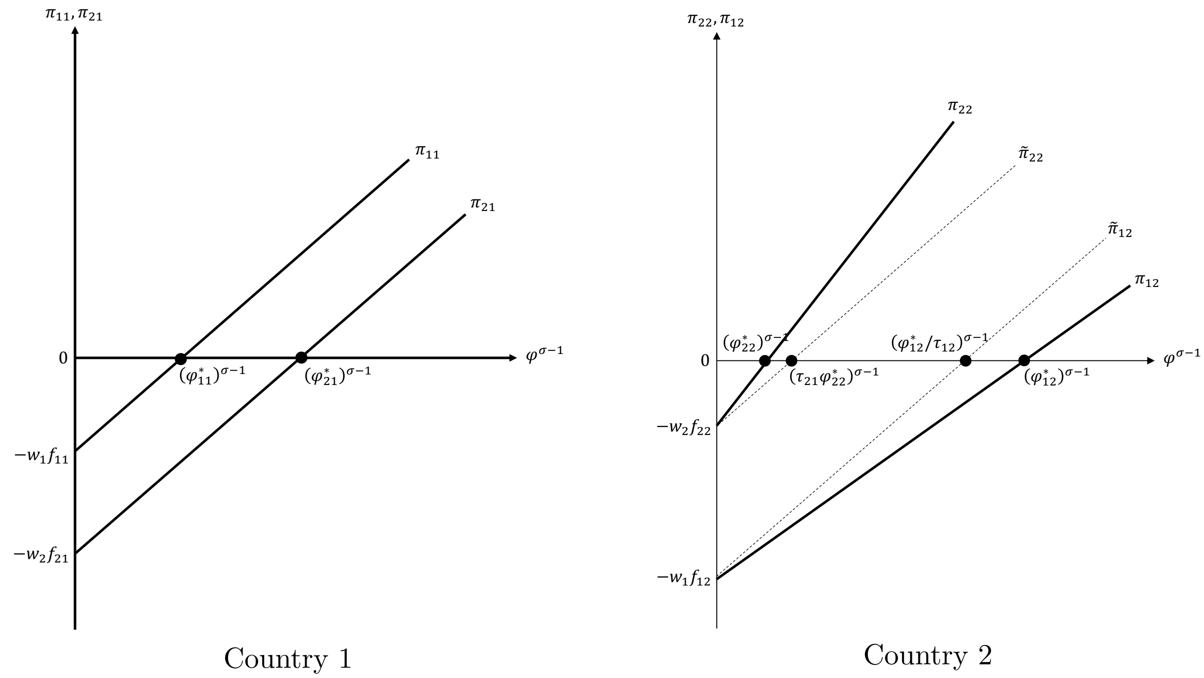


FIGURE 1 – Profits from domestic and export markets in sector $\bar{z}_1$

if and only if (1) holds (for given φ). Thus firms in comparative advantage sectors set the lower price and earn the higher revenue than firms in comparative disadvantage sectors. From these revenues, the operating profits of domestic firms and foreign firms are respectively given by

$$\pi_{ii}(\varphi, z) = \frac{r_{ii}(\varphi, z)}{\sigma} - w_i f_{ii} = B_i(z) \left(\frac{\mu_i(z)}{w_i} \right)^{\sigma-1} \varphi^{\sigma-1} - w_i f_{ii},$$
$$\pi_{ji}(\varphi, z) = \frac{r_{ji}(\varphi, z)}{\sigma} - w_j f_{ji} = B_i(z) \left(\frac{\mu_j(z)}{\tau_{ji} w_j} \right)^{\sigma-1} \varphi^{\sigma-1} - w_j f_{ji}.$$

These profits can be drawn in $(\varphi^{\sigma-1}, \pi_{ji})$ space, with slope $B_i(z) \left(\frac{\mu_j(z)}{\tau_{ji} w_j} \right)^{\sigma-1}$ and intercept $-w_j f_{ji}$. Since the profits depend on z, these must vary across sectors. Figure 1 depicts $\pi_{ii}(\varphi, z)$ and $\pi_{ji}(\varphi, z)$ for country 1's market ($i = 1$) and country 2's market ($i = 2$) in the cutoff sector $\bar{z}_1$. From (2), it follows immediately that $\pi_{11}(\varphi, z)$ and $\pi_{21}(\varphi, z)$ are parallel for country 1's market in that sector. Similarly, denoting by $\tilde{\pi}_{22}(\varphi, z)$ and $\tilde{\pi}_{12}(\varphi, z)$ the operating profits adjusting the variable trade costs with slopes $B_2(z) \left(\frac{\mu_2(z)}{\tau_{21} w_2} \right)^{\sigma-1}$ and $B_2(z) \left(\frac{\mu_1(z)}{w_1} \right)^{\sigma-1}$, these are also parallel for country 2's market in the sector. From (3), the converse is true for the respective markets in another cutoff sector $\bar{z}_2$.

3 Equilibrium

This section examines the interplay among the key endogenous variables of the model and addresses comparative static questions in general equilibrium.

3.1 Equilibrium Conditions

In this subsection, we outline several equilibrium conditions that play a central role in characterizing the endogenous variables in general equilibrium. In the subsequent subsections, we solve this general-equilibrium model with some restrictions on the exogenous variables.

Firstly, a zero profit condition holds for *all* sectors $z \in [0,1]$ of the domestic and export markets. The productivity cutoffs that satisfy $\pi_{ii}(\varphi_{ii}^*, z) = 0$ and $\pi_{ij}(\varphi_{ij}^*, z) = 0$ are respectively given by

$$B_i(z) \left(\frac{\mu_i(z)}{w_i}\right)^{\sigma-1} (\varphi_{ii}^*(z))^{\sigma-1} = w_i f_{ii}, \tag{4}$$

$$B_j(z) \left(\frac{\mu_i(z)}{\tau_{ij} w_i}\right)^{\sigma-1} (\varphi_{ij}^*(z))^{\sigma-1} = w_i f_{ij}. \tag{5}$$

Since (4) and (5) respectively apply to domestic firms in i and exporting firms from i to j, aggregate market demands $B_i(z), B_j(z)$ are different between (4) and (5), but the country-specific productivity level $\mu_i(z)$ and wage rate w_i are the same.

Secondly, a free entry condition holds for *all* sectors:

$$\int_{\varphi_{ii}^*(z)}^{\infty} \pi_{ii}(\varphi, z) dG(\varphi) + \int_{\varphi_{ij}^*(z)}^{\infty} \pi_{ij}(\varphi, z) dG(\varphi) = w_i f_i^e, \tag{6}$$

where the first and second terms in the left-hand side respectively denote the expected operating profits in the domestic and export markets by potential entrants. The sum of these expected profits should be equal to the fixed entry cost $w_i f_i^e$. Note that (6) holds so long as there is a positive mass of potential entrants denoted by $M_i^e(z)$. In this paper, we focus on the case where $M_i^e(z) > 0$ in all sectors and international trade leads both countries to incomplete specialization.[3]

Finally, a labor market clearing condition must be taken into account:

$$\int_0^1 M_i^e(z) \int_{\varphi_{ii}^*(z)}^{\infty} l_{ii}(\varphi, z) dG(\varphi) dz + \int_0^1 M_i^e(z) \int_{\varphi_{ij}^*(z)}^{\infty} l_{ij}(\varphi, z) dG(\varphi) dz + \int_0^1 M_i^e(z) f_i^e dz = \overline{L}_i, \tag{7}$$

where the first and second terms in the left-hand side are respectively the expected amounts of labor for domestic production and exporting by potential entrants, and the third is the expected amounts of labor for investment by these entrants. The sum of these expected amounts of labor should be equal to the fixed aggregate labor supply $\overline{L}_i$.

Now, it is possible to endogenize the important variables in general equilibrium. Since there are the eight equations ((4), (5), (6) and (7) that hold in countries 1 and 2), these conditions provide implicit solutions for the following eight unknowns:

$$\varphi_{11}^*(z),\ \varphi_{22}^*(z),\ \varphi_{12}^*(z),\ \varphi_{21}^*(z),\ B_1(z),\ B_2(z),\ w_1,\ w_2,$$

[3] If the expected profits are smaller than the fixed entry cost, $M_i^e(z) = 0$ and county j specializes in this sector. For the sake of parsimony, we rule out this case by imposing some restrictions on the exogenous variables. (The condition of incomplete specialization is given later).

where (7) for $i = 2$ can be omitted by Walras's law, thereby normalizing $w_2 = 1$ as a numeraire of the model. The mass of potential entrants $M_i^e(z)$ is written as a function of these eight unknowns as will be shown later.

As is evident from the dependence of z among the eight unknowns, the productivity cutoffs and the aggregate market demands are allowed to vary across sectors; in contrast, the wage rates are the same across sectors due to perfect inter-sectoral mobility of labor. This means that (4), (5) and (6) for $i = 1, 2$ are the six equations that characterize $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$ in each sector, whereas (7) for $i = 1, 2$ are the two additional equations that characterize $\{w_1, w_2\}$, aggregating the use of labor across all sectors in each country. From this reason, the next subsections first characterize the sectoral equilibrium by focusing on (4), (5) and (6), and then explore the full general equilibrium by integrating (7) into the model.

3.2 Sectoral Equilibrium

This subsection sets forth characterizing the sectoral equilibrium composing of the six unknowns. It is however difficult to solve the model with asymmetric countries in the general setting; in particular, closed-form solutions of these unknowns cannot be obtained without specifying a functional form of a fixed distribution $G(\varphi)$. To avoid this difficulty, the main analysis is devoted to characterizing the *relative* terms of these unknowns, instead of the absolute terms of them.

In what follows, we characterize the sectoral equilibrium in terms of the relative market demand and the relative productivity cutoffs, and demonstrate that these relative variables systematically vary across sectors according to each country's comparative advantage. First, dividing (4) by (5), the relative market demand is given by

$$\frac{B_i(z)}{B_j(z)} = \left(\frac{1}{\tau_{ij}} \frac{\varphi_{ij}^*(z)}{\varphi_{ii}^*(z)}\right)^{\sigma-1} \frac{f_{ii}}{f_{ij}}. \tag{8}$$

In the sectoral equilibrium, not only $B_i(z), B_j(z)$ but $\varphi_{ii}^*(z), \varphi_{ij}^*(z)$ are also endogenously determined, and hence we need to explicitly take account of their interactions. Solving the system of equations (4), (5) and (6) simultaneously leads to the following lemma regarding the relative market demand (see Appendix A for proof).[4]

Lemma 1 *The relative market demand $B(z) \equiv B_1(z)/B_2(z)$ is log-submodular. For $z < z'$,*

$$\frac{B_1(z)}{B_2(z)} \geq \frac{B_1(z')}{B_2(z')}.$$

Lemma 1 means that the relative market demand $B_i(z)/B_j$ is decreasing in the strength of country i's comparative advantage. The intuition stems from the fact that $B_i(z)/B_j(z)$ is proportional to the relative price index $P_i(z)/P_j(z)$. By definition, the stronger is country i's comparative advantage,

[4] All lemmas and propositions hold for a general distribution function until Section 3.6.

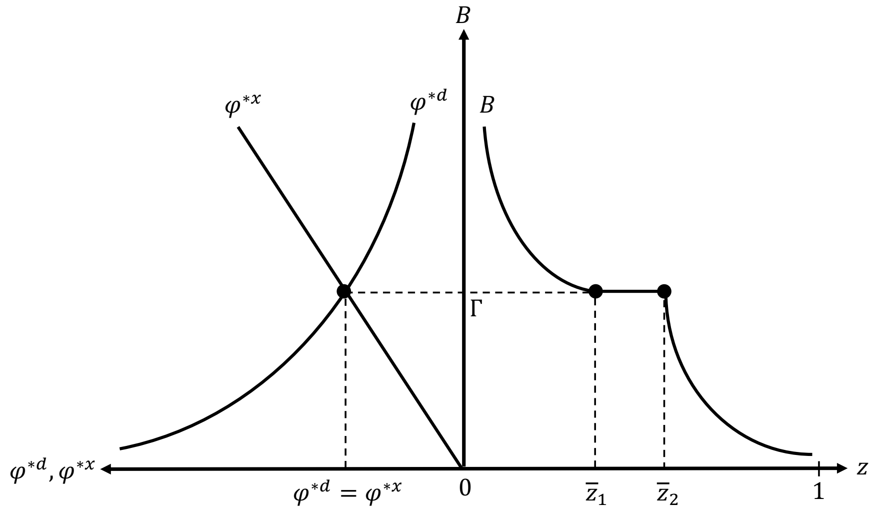


FIGURE 2 – Relative market demand and productivity cutoffs across sectors

the more productive is country i relative to country j, and the lower is $P_i(z)$ relative to $P_j(z)$. As a result, $P_i(z)/P_j(z)$ (and $B_i(z)/B_j(z)$) is decreasing in the strength of i's comparative advantage. Since country 1 (country 2) has a comparative advantage in high-z (low-z) sectors under Assumption 1, Lemma 1 alternatively means that, by assumption of a continuum of sectors, $B(z) \equiv B_1(z)/B_2(z)$ is decreasing in z. The first quadrant of Figure 2 depicts this relationship in (z, B) space.

Next, dividing (4) of i by (4) of j, the relative domestic productivity cutoff is given by

$$\frac{\varphi^*_{ii}(z)}{\varphi^*_{jj}(z)} = \frac{w_i}{w_j}\frac{\mu_j(z)}{\mu_i(z)}\left(\frac{w_i}{w_j}\frac{f_{ii}}{f_{jj}}\frac{B_j(z)}{B_i(z)}\right)^{\frac{1}{\sigma-1}}. \tag{9}$$

Similarly, dividing (5) of i by (5) of j, the relative export productivity cutoff is given by

$$\frac{\varphi^*_{ij}(z)}{\varphi^*_{ji}(z)} = \frac{\tau_{ij}w_i}{\tau_{ji}w_j}\frac{\mu_j(z)}{\mu_i(z)}\left(\frac{w_i}{w_j}\frac{f_{ij}}{f_{ji}}\frac{B_i(z)}{B_j(z)}\right)^{\frac{1}{\sigma-1}}. \tag{10}$$

Since B's are only different endogenous variables between (9) and (10), we have the following lemma.

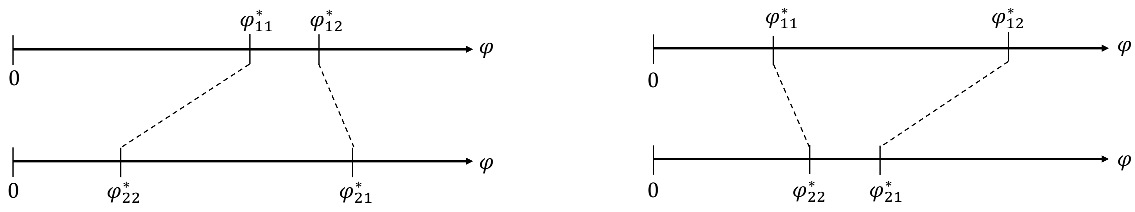
Lemma 2

(i) The relative domestic productivity cutoff $\varphi^{*d}(z) \equiv \varphi^*_{11}(z)/\varphi^*_{22}(z)$ is log-supermodular. For $z < z'$,

$$\frac{\varphi^*_{11}(z)}{\varphi^*_{22}(z)} \leq \frac{\varphi^*_{11}(z')}{\varphi^*_{22}(z')}.$$

(ii) The relative export productivity cutoff $\varphi^{*x}(z) \equiv \varphi^*_{12}(z)/\varphi^*_{21}(z)$ is log-submodular. For $z < z'$,

$$\frac{\varphi^*_{12}(z)}{\varphi^*_{21}(z)} \geq \frac{\varphi^*_{12}(z')}{\varphi^*_{21}(z')}.$$



Country 1 Country 2

FIGURE 3 – Productivity cutoffs in each country's comparative advantage sectors

Lemma 2 means that the relative domestic (export) productivity cutoff is increasing (decreasing) in the strength of i's comparative advantage. In fact, solving the system of equations (4), (5) and (6) reveals that

$$\varphi_{11}^{*'}(z) \geq 0,\ \varphi_{22}^{*'}(z) \leq 0,\ \varphi_{12}^{*'}(z) \leq 0,\ \varphi_{21}^{*'}(z) \geq 0. \tag{11}$$

Thus, the stronger is each country's comparative advantage, the more intense is firm selection in the domestic market ($\varphi_{11}^{*'}(z) \geq 0$, $\varphi_{22}^{*'}(z) \leq 0$), but the less intense is firm selection in the export market ($\varphi_{12}^{*'}(z) \leq 0$, $\varphi_{21}^{*'}(z) \geq 0$), making stronger comparative advantage sectors more open. Note that

$$\varphi^{*d}(z) \gtreqless \varphi^{*x}(z) \quad \Longleftrightarrow \quad B(z) \lesseqgtr \sqrt{\left(\frac{\tau_{21}}{\tau_{12}}\right)^{\sigma-1} \frac{f_{11}f_{21}}{f_{22}f_{12}}} \equiv \Gamma,$$

where $\varphi^{*d}(z) = \varphi^{*x}(z)$ and $B(z) = \Gamma$ for $z = \bar{z}_1, \bar{z}_2$ (see Appendix A). It then follows from Lemma 1 that $B(z)$ is *weakly* decreasing in z with $B(z) = \Gamma$ for $z \in [\bar{z}_1, \bar{z}_2]$. The second quadrant of Figure 2 depicts this relationship in (B, φ^*) space.

Finally, combining the first and second quadrants of Figure 2, we obtain the sectoral equilibrium characterized by the relative market demand and the relative productivity cutoffs:

$$\begin{aligned} 0 \leq z \leq \bar{z}_2 \quad &\Longleftrightarrow \quad B(z) \geq \Gamma \quad \Longleftrightarrow \quad \varphi^{*x}(z) \geq \varphi^{*d}(z), \\ \bar{z}_1 \leq z \leq 1 \quad &\Longleftrightarrow \quad B(z) \leq \Gamma \quad \Longleftrightarrow \quad \varphi^{*x}(z) \leq \varphi^{*d}(z). \end{aligned} \tag{12}$$

Figure 3 depicts the relationship among the productivity cutoffs in the comparative advantage sectors of country 1 ($\bar{z}_1 \leq z \leq 1$) and country 2 ($0 \leq z \leq \bar{z}_2$). From (11), the gap between $\varphi_{ij}^*(z)$ and $\varphi_{ii}^*(z)$ is decreasing in the strength of country i's comparative advantage, and from (12), this gap is relatively narrower than the gap between $\varphi_{ji}^*(z)$ and $\varphi_{jj}^*(z)$ in country i's comparative advantage sectors. These findings can be seen more formally in terms of their relative gap:

$$\frac{\varphi_{ij}^*(z)}{\varphi_{ii}^*(z)} = \tau_{ij} \left(\frac{B_i(z)}{B_j(z)} \frac{f_{ij}}{f_{ii}}\right)^{\frac{1}{\sigma-1}}. \tag{13}$$

From Lemma 1, $\varphi_{ij}^*(z)/\varphi_{ii}^*(z)$ is decreasing in the strength of i's comparative advantage. In addition, from (12), $\varphi_{ij}^*(z)/\varphi_{ii}^*(z)$ is smaller than $\varphi_{ji}^*(z)/\varphi_{jj}^*(z)$ in i's comparative advantage sectors.

(13) shows that the selection into export markets occurs in the comparative *disadvantage* sectors of both countries.[5] In the comparative *advantage* sectors, the selection occurs in both countries if

$$\varphi_{12}^*(z) > \varphi_{11}^*(z),\ \varphi_{21}^*(z) > \varphi_{22}^*(z) \iff \frac{1}{\tau_{12}^{\sigma-1}}\frac{f_{11}}{f_{12}} < B(z) < \tau_{21}^{\sigma-1}\frac{f_{21}}{f_{22}}, \tag{14}$$

whereas this does not hold in country i if

$$\varphi_{ii}^*(z) \geq \varphi_{ij}^*(z) \iff \begin{cases} B(z) \leq \frac{1}{\tau_{12}^{\sigma-1}}\frac{f_{11}}{f_{12}} & \text{for } i=1, \\ B(z) \geq \tau_{21}^{\sigma-1}\frac{f_{21}}{f_{22}} & \text{for } i=2. \end{cases}$$

Clearly, the selection might not occur in the strong comparative *advantage* sectors. If $\varphi_{ii}^*(z) \geq \varphi_{ij}^*(z)$, however, all surviving firms in i could export to j, which is not supported by empirical evidence (see Bernard et al., 2007a). Thus, we hereafter assume that (14) is satisfied across sectors of both countries in the following analysis.

Recall that we have assumed (6) for all sectors and no country fully specializes in any sector, i.e., $M_e^i(z) > 0$ for all z. To derive $M_i^e(z)$, rewrite the price index $P_i(z)$ as

$$(P_i(z))^{1-\sigma} = M_i^e(z)\left(\frac{\sigma}{\sigma-1}\frac{w_i}{\mu_i(z)}\right)^{1-\sigma} V(\varphi_{ii}^*(z)) + M_j^e(z)\left(\frac{\sigma}{\sigma-1}\frac{\tau_{ji}w_j}{\mu_j(z)}\right)^{1-\sigma} V(\varphi_{ji}^*(z)),$$

where $V(\varphi^*) \equiv \int_{\varphi^*}^{\infty} \varphi^{\sigma-1}dG(\varphi)$ is strictly decreasing in φ^*. Solving $P_i(z)$ and $P_j(z)$ for $M_i^e(z)$ and $M_j^e(z)$ and using $B_i(z)$, we obtain the mass of potential entrants:

$$M_i^e(z) = \frac{1}{\sigma}\left(\frac{w_i}{\mu_i(z)}\right)^{1-\sigma} \frac{V(\varphi_{jj}^*(z))\frac{R_i(z)}{B_i(z)} - \tau_{ji}^{1-\sigma}V(\varphi_{ji}^*(z))\frac{R_j(z)}{B_j(z)}}{\Delta(z)},$$

where

$$\Delta(z) \equiv V(\varphi_{ii}^*(z))V(\varphi_{jj}^*(z)) - (\tau_{ij}\tau_{ji})^{(1-\sigma)}V(\varphi_{ij}^*(z))V(\varphi_{ji}^*(z)).$$

Note that $\Delta(z)$ is positive since $\varphi_{ij}^*(z) > \varphi_{ii}^*(z)$ from (14). Then, there is a positive mass of potential entrants in all sectors of both countries if

$$M_1^e(z) > 0,\ M_2^e(z) > 0 \iff \frac{1}{\tau_{12}^{\sigma-1}}\frac{V(\varphi_{12}^*(z))}{V(\varphi_{11}^*(z))}\frac{R_1(z)}{R_2(z)} < B(z) < \tau_{21}^{\sigma-1}\frac{V(\varphi_{22}^*(z))}{V(\varphi_{21}^*(z))}\frac{R_1(z)}{R_2(z)}, \tag{15}$$

whereas this does not hold in country i if

$$M_i^e(z) \leq 0 \iff \begin{cases} B(z) \geq \tau_{21}^{\sigma-1}\frac{V(\varphi_{22}^*(z))}{V(\varphi_{21}^*(z))}\frac{R_1(z)}{R_2(z)} & \text{for } i=1, \\ B(z) \leq \frac{1}{\tau_{12}^{\sigma-1}}\frac{V(\varphi_{12}^*(z))}{V(\varphi_{11}^*(z))}\frac{R_1(z)}{R_2(z)} & \text{for } i=2. \end{cases}$$

[5] Under the condition $\tau_{ij}^{\sigma-1}f_{ij}/f_{ii} > 1$, the comparative disadvantage sectors of country 1, for example, must satisfy

$$0 \leq z < \bar{z}_1 \iff B(z) > \Gamma \implies \varphi_{12}^*(z) > \varphi_{11}^*(z).$$

From Lemma 1 and (11), there might not be a positive mass of entrants in the strong comparative *disadvantage* sectors (see also Huang et al., 2017). For the sake of parsimony, we restrict attention to the situation in which not only is (14) but (15) is also satisfied, so that incomplete specialization occurs in all sectors of both countries. These conditions require country size and technology are not too different between the two countries because $B(z)$ is proportional to $\overline{L}_1/\overline{L}_2$ and $P_1(z)/P_2(z)$.[6]

Proposition 1

(*i*) *The domestic (export) productivity cutoff $\varphi^*_{ii}(z)$ ($\varphi^*_{ij}(z)$) is increasing (decreasing) in the strength of country i's comparative advantage.*

(*ii*) *The relative gap in the productivity cutoff $\varphi^*_{ij}(z)/\varphi^*_{ii}(z)$ is smaller than $\varphi^*_{ji}(z)/\varphi^*_{jj}(z)$ in country i's comparative advantage sectors.*

Proposition 1 shows that aggregate productivity premium of exporting firms relative to domestic firms is smaller, the stronger is each country's comparative advantage in the Ricardian model with monopolistic competition and heterogeneous firms. Note importantly that the findings in Proposition 1 and Figure 3 are very similar to those in Bernard et al. (2007b) who develop the Heckscher-Ohlin model with monopolistic competition and heterogeneous firms. Our contribution is in demonstrating that the relationship between firm selection and comparative advantage rests only on comparative cost advantage, but not on whether the cost advantage stems from factor proportions or technology. Despite this similarity, the difference emerges in the situation in which country endowments impact endogenously on the factor prices across the two countries.

Before proceeding further, it is worth emphasizing that the other aggregate variables in the model exhibit log-supermodularity or log-submodularity. Let $R_{ii}(z)$ and $R_{ij}(z)$ denote aggregate domestic sales and aggregate export sales in sector z from country i to country j:

$$R_{ii}(z) = M^e_i(z) \int_{\varphi^*_{ii}(z)}^{\infty} r_{ii}(\varphi, z) dG(\varphi), \quad R_{ij}(z) = M^e_i(z) \int_{\varphi^*_{ij}(z)}^{\infty} r_{ij}(\varphi, z) dG(\varphi).$$

Similarly, let $L_i(z)$ denote aggregate labor supply in sector z of country i:

$$L_i(z) = M^e_i(z) \int_{\varphi^*_{ii}(z)}^{\infty} l_{ii}(\varphi, z) dG(\varphi) + M^e_i(z) \int_{\varphi^*_{ij}(z)}^{\infty} l_{ij}(\varphi, z) dG(\varphi) + M^e_i(z) f^e_i,$$

where

$$l_{ii}(\varphi, z) = f_{ii} + \frac{\sigma - 1}{\sigma} \frac{r_{ii}(\varphi, z)}{w_i}, \quad l_{ij}(\varphi, z) = f_{ij} + \frac{\sigma - 1}{\sigma} \frac{r_{ij}(\varphi, z)}{w_i}.$$

Noting that these aggregate variables are functions of the endogenous variables in Lemmas 1 and 2, the following lemma is obtained from the characterization of the sectoral equilibrium above.

[6] This comes from noting that $\frac{B_i(z)}{B_j(z)} = \frac{w_i \overline{L}_i}{w_j \overline{L}_j} \left(\frac{P_i(z)}{P_j(z)} \right)^{\sigma - 1}$ where technology indirectly affects the relative price index. While condition (14) is usually imposed in the literature (e.g., Bernard et al., 2007b), condition (15) that rules out the possibility of complete specialization is also often imposed in the literature (e.g., Melitz and Ottaviano, 2008).

Lemma 3

(*i*) *The relative output* $Q(z) \equiv Q_1(z)/Q_2(z)$ *is log-supermodular, whereas the relative price* $P(z) \equiv P_1(z)/P_2(z)$ *is log-submodular. For* $z < z'$*,*

$$\frac{Q_1(z)}{Q_2(z)} \leq \frac{Q_1(z')}{Q_2(z')}, \quad \frac{P_1(z)}{P_2(z)} \geq \frac{P_1(z')}{P_2(z')}.$$

(*ii*) *The relative sales in the domestic market* $R^d(z) \equiv R_{11}(z)/R_{22}(z)$ *and those in the export market* $R^x(z) \equiv R_{12}(z)/R_{21}(z)$ *are log-supermodular. For* $z < z'$*,*

$$\frac{R_{11}(z)}{R_{22}(z)} \leq \frac{R_{11}(z')}{R_{22}(z')}, \quad \frac{R_{12}(z)}{R_{21}(z)} \leq \frac{R_{12}(z')}{R_{21}(z')}.$$

(*iii*) *The relative labor supply* $L(z) \equiv L_1(z)/L_2(z)$ *and the relative mass of potential entrants* $M^e(z) \equiv M_1^e(z)/M_2^e(z)$ *are log-supermodular. For* $z < z'$*,*

$$\frac{L_1(z)}{L_2(z)} \leq \frac{L_1(z')}{L_2(z')}, \quad \frac{M_1^e(z)}{M_2^e(z)} \leq \frac{M_1^e(z')}{M_2^e(z')}.$$

In Lemma 3, the ranking of the relative output and its relative price suggests that each country produces relatively more associated with relatively lower price indices, the stronger is its comparative advantage. The relative sales in the domestic and export markets also belong to the ranking, since each country sells in these markets relatively more, the stronger is its comparative advantage. Finally, the relative labor supply and relative mass of entrants also belong to the ranking, since labor resources are relatively more allocated to sectors in which aggregate output and aggregate sales are greater. Note that since the equality holds for $z \in [\bar{z}_1, \bar{z}_2]$ in Lemmas 1 and 2, the equality also holds for the interval sectors in Lemma 3.

3.3 General Equilibrium

The last subsection characterized the equilibrium vector $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$ for *given* wage rates. Now that the sectoral equilibrium is characterized by these six unknowns, this subsection embeds the sectoral equilibrium into general equilibrium.

To close the model in general equilibrium, we explicitly take account of the labor market clearing condition (7) below. Substituting the amount of labor required by individual firms $l_{ii}(\varphi, z), l_{ij}(\varphi, z)$ into (7) and using (6), equation (7) is simplified as

$$\frac{\int_0^1 R_i(z)dz}{w_i} = \overline{L}_i, \tag{16}$$

where $\int_0^1 R_i(z)dz = \int_0^1 P_i(z)Q_i(z)dz$ is aggregate expenditure in country i. Thus, country i's wage w_i is determined by the equality between aggregate expenditure $\int_0^1 R_i(z)dz$ and aggregate payments to labor $w_i\overline{L}_i$ as in usual general-equilibrium trade models without an outside good.

To derive the relative wage, we first show that (16) is equivalent with the balance-of-payments condition. Since $B_i(z), B_j(z)$ are finite in all sectors under (14), $\varphi^*_{ij}(z), \varphi^*_{ji}(z)$ are finite in all sectors. Moreover, $M^e_i(z), M^e_j(z)$ are positive in all sectors under (15). Then from a fixed distribution with unbounded upper support, it follows that bilateral trade occurs in all sectors:

$$\int_0^1 R_{ij}(z)dz = \int_0^1 R_{ji}(z)dz. \tag{17}$$

Note that aggregate expenditure in i consists of expenditure spent on domestic goods in i and foreign goods from j, $\int_0^1 R_i(z)dz = \int_0^1 (R_{ii}(z) + R_{ji}(z))dz$. On the other hand, aggregate labor income in i consists of revenues earned by domestic firms and exporting firms of i, $w_i\overline{L}_i = \int_0^1 (R_{ii}(z) + R_{ij}(z))\, dz$. As a result, (17) is equivalent with (16) in the sense that both (16) and (17) induce the same equality: $\int_0^1 R_i(z)dz = w_i\overline{L}_i$.

From Lemma 3, aggregate export sales are increasing in the strength of comparative advantage ($\frac{R_{12}(z)}{R_{21}(z)} \leq \frac{R_{12}(z')}{R_{21}(z')}$ for $z < z'$ where equality holds for $z \in [\bar{z}_1, \bar{z}_2]$). Let $\bar{z}_i$ denote the *hypothetical* sector in which net exports are zero in two-way trade. Since net aggregate export sales are the differences between aggregate labor income and aggregate expenditure, (17) is expressed as

$$\int_{\bar{z}_1}^1 (w_1 L_1(z) - R_1(z))\, dz = \int_0^{\bar{z}_2} (w_2 L_2(z) - R_2(z))\, dz,$$

which simply indicates that each country runs trade surplus in the comparative advantage sectors, and trade deficit in the comparative disadvantage sectors. This equation is further rewritten as

$$(\kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1))w_1\overline{L}_1 = (\kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2))w_2\overline{L}_2,$$

where $\kappa_i(\bar{z}_i)$ and $\lambda_i(\bar{z}_i)$ respectively denote the labor share and the expenditure share devoted in i's comparative advantage sectors:

$$\kappa_1(\bar{z}_1) \equiv \int_{\bar{z}_1}^1 \frac{L_1(z)}{\overline{L}_1}dz, \quad \kappa_2(\bar{z}_2) \equiv \int_0^{\bar{z}_2} \frac{L_2(z)}{\overline{L}_2}dz,$$
$$\lambda_1(\bar{z}_1) \equiv \int_{\bar{z}_1}^1 b_1(z)dz, \quad \lambda_2(\bar{z}_2) \equiv \int_0^{\bar{z}_2} b_2(z)dz.$$

Solving the above equation for $\omega \equiv w_1/w_2$ yields

$$\omega = \frac{\kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2)}{\kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1)} \left(\frac{\overline{L}_2}{\overline{L}_1}\right).^7 \tag{18}$$

The other equations that pin down the relative wage are cutoff conditions (2) and (3): $\omega = \tau_{21}\mu(\bar{z}_1)$ and $\omega = \mu(\bar{z}_2)/\tau_{12}$. Then (2), (3) and (18) provide implicit solutions for the following three unknowns:

$$\bar{z}_1,\ \bar{z}_2,\ \omega.$$

[7] From $b_i(z)$, it follows that $\kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1) = \int_{\bar{z}_1}^1 \frac{w_1 L_1(z) - R_1(z)}{w_1\overline{L}_1}dz > 0$ and $\kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2) = \int_0^{\bar{z}_2} \frac{w_2 L_2(z) - R_2(z)}{w_2\overline{L}_2}dz > 0$.

Substituting (2) and (3) into (18) reveals that the right-hand side of (18) is decreasing in ω, which guarantees a unique equilibrium relative wage. The relative wage in (18), together with (2) and (3), determines the pattern of comparative advantage of country 1 and country 2. Given the relative wage, the system of equations (4), (5) and (6) in turn leads to $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$. This completes the characterization of the eight unknowns in general equilibrium.

Proposition 2

(i) *There exist the two cutoff sectors* $\bar{z}_1, \bar{z}_2$ *that pin down comparative advantage of country 1 and country 2 in bilateral trade.*

(ii) *The equilibrium relative wage* ω *is unique.*

While the results in Proposition 2 are similar with the results in DFS, it should be noted that the variable trade costs τ_{12}, τ_{21} do not allow for nontraded goods in the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$ here: each country does trade differentiated goods in $z \in [\bar{z}_1, \bar{z}_2]$, but *net* exports are zero in these sectors. More important differences, however, are comparative static questions for the firm-level variables.

3.4 Comparative Statics

Building on the equilibrium characterization, this subsection addresses comparative static questions with respect to relative country size $\overline{L} \equiv \overline{L}_1/\overline{L}_2$ and relative labor productivity $\mu(z) \equiv \mu_1(z)/\mu_2(z)$ within the ranges of (14) and (15). Regarding $\mu(z)$, we are interested in the effect of uniform changes across sectors.

The comparative static results are facilitated by the recursive structure of the equilibrium: any change in $\overline{L}$ or $\mu(z)$ first has an impact on $\{\bar{z}_1, \bar{z}_2, \omega\}$ from (2), (3) and (18); and the impact of ω on $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$ is then obtained from (4), (5) and (6) for $i = 1, 2$. Let the latter set of the sectoral equilibrium variables express in the relative terms $\{\varphi^{*d}(z), \varphi^{*x}(z), B(z)\}$. The main results are provided in the next proposition.

Proposition 3

(i) *The equilibrium vector* $\{\bar{z}_1, \bar{z}_2, \omega\}$ *characterized by (2), (3) and (18) satisfies*

$$\frac{\partial \bar{z}_1}{\partial \overline{L}} \leq 0, \; \frac{\partial \bar{z}_2}{\partial \overline{L}} \leq 0, \; \frac{\partial \omega}{\partial \overline{L}} \leq 0,$$
$$\frac{\partial \bar{z}_1}{\partial \mu(z)} \leq 0, \; \frac{\partial \bar{z}_2}{\partial \mu(z)} \leq 0, \; \frac{\partial \omega}{\partial \mu(z)} \geq 0.$$

(ii) *The equilibrium vector* $\{\varphi^{*d}(z), \varphi^{*x}(z), B(z)\}$ *characterized by (4), (5) and (6) satisfies*

$$\frac{\partial \varphi^{*d}(z)}{\partial \overline{L}} \geq 0, \; \frac{\partial \varphi^{*x}(z)}{\partial \overline{L}} \leq 0, \; \frac{\partial B(z)}{\partial \overline{L}} \leq 0,$$
$$\frac{\partial \varphi^{*d}(z)}{\partial \mu(z)} \geq 0, \; \frac{\partial \varphi^{*x}(z)}{\partial \mu(z)} \leq 0, \; \frac{\partial B(z)}{\partial \mu(z)} \leq 0.$$

The first part of this proposition is exactly the same as that in DFS. The second part says that an increase in relative country size, for example, makes the relative selection into the domestic (export) market more (less) intense. In fact, solving the system of equations (4), (5) and (6) reveals that

$$\frac{\partial \varphi_{11}^*(z)}{\partial \overline{L}} \geq 0, \ \frac{\partial \varphi_{22}^*(z)}{\partial \overline{L}} \leq 0, \ \frac{\partial \varphi_{12}^*(z)}{\partial \overline{L}} \leq 0, \ \frac{\partial \varphi_{21}^*(z)}{\partial \overline{L}} \geq 0.$$

Thus, if market size is relatively larger in country 1, firm selection is relatively more intense in the country not only for domestic firms ($\frac{\partial \varphi_{11}^*(z)}{\partial \overline{L}} \geq 0$) but for exporting firms from country 2 ($\frac{\partial \varphi_{21}^*(z)}{\partial \overline{L}} \geq 0$). Intuitively, a country with larger size entails the lower relative wage and the lower price-cost margins, which makes competition more intense and raises the productivity cutoffs of domestic and exporting firms operating in that country. While a reduction in the relative wage increases firm revenue, this is smaller than a decline in the price index and hence results in lower firm revenue in a larger country.[8] Consequently, a country with larger size exhibits higher productivity by forcing the least productive firms to exit there.

Our model's prediction for the impact of country size is similar to Melitz and Ottaviano (2008) with quasi-linear-quadratic preferences: a larger country exhibits higher aggregate productivity and lower price-cost margins (see the papers cited therein for evidence). It is important to note, however, that competition by country size operates through different channels. In their paper, increased goods market competition shifts up residual demand price elasticities but factor market competition has no impact due to an outside good that equalizes wages across countries. In the present paper, increased factor market competition reduces the relative wage (and hence the marginal cost) but goods market competition has no impact due to C.E.S. preferences. This is worth emphasizing since it gives rise to different impacts of country size on a trading partner. In contrast to Melitz and Ottaviano (2008) in which country size has no impact on the productivity cutoffs of a trading partner, country size does affect them in the present paper through the relative wage that changes competitiveness across countries. This is the reason why $\frac{\partial \varphi_{22}^*(z)}{\partial \overline{L}} \leq 0, \frac{\partial \varphi_{12}^*(z)}{\partial \overline{L}} \leq 0$.

It is also important to stress that competition by country size has a different impact on the export price that is typically obtained in a quadratic utility model. In that model, the export price is lower when shipped to a larger country with more intense competition which lowers residual demands and puts downward pressure on the export price (Di Comité et al., 2017). In the present model, applying the comparative statics in Proposition 3 to the fob export price $\tilde{p}_{ji}(\varphi, z) = \frac{\sigma}{\sigma-1} \frac{w_j}{\varphi \mu_j(z)}$ implies that, if market size is relatively larger in country 1, the export price from country 2 is relatively higher since intense competition in country 1 increases the relative wage in country 2 in the present model. However, an increase of the export price is dominated by a decline of the price index in the destination country with large market size. As described above, this makes firm revenue lower and firm selection more intense there. Thus the pro-competitive effect also works in the export market as well as the domestic market in the present model, though the mechanism is different from that in the quadratic utility model.

[8] From the comparative statics in Proposition 3, it follows immediately that firm revenue is relatively lower in a country with larger size.

One of key insights from the comparative statics is that, even with C.E.S. preferences, country size does affect firm selection $\varphi_{ii}^*(z), \varphi_{ij}^*(z)$ through an endogenous change in the relative wage in the Ricardian model. (If the relative wage is exogenously fixed, country size impacts only on the mass of potential entrants $M_i^e(z)$ without affecting the firm-level variables.) This finding is in line with recent theoretical work, although the mechanism differs. For example, Bertoletti and Etro (2017) show that national income does affect firm selection through the variable markups when consumers' preferences are represented by additively separable indirect utilities. In our Ricardian model, even though consumers' preferences are represented by C.E.S. and hence the markups are constant, the price-cost margins are no longer constant because per-capita income w_i varies with country size. (If the relative wage is exogenously fixed, not only are the markups but the price-cost margins also are constant.) Our mechanism is close to Demidova and Rodríguez-Clare (2013) for the influence of the relative wage on firm selection in a setting with C.E.S. preferences, but they focus mainly on variable trade costs, rather than country size.

In addition to country size, our Ricardian model is able to show that an increase in relative labor productivity has a similar impact on firm selection:

$$\frac{\partial \varphi_{11}^*(z)}{\partial \mu(z)} \geq 0, \ \frac{\partial \varphi_{22}^*(z)}{\partial \mu(z)} \leq 0, \ \frac{\partial \varphi_{12}^*(z)}{\partial \mu(z)} \leq 0, \ \frac{\partial \varphi_{21}^*(z)}{\partial \mu(z)} \geq 0.$$

While a country with better labor productivity entails the higher relative wage, the relative wage increases proportionally short of an increase in the relative labor productivity. As with country size, this allows the country to entail the lower price-cost margins and gives rise to the pro-competitive effect for domestic and exporting firms operating there.

3.5 Welfare

Let us next consider welfare in the Ricardian model with monopolistic competition and heterogeneous firms. As shown in Appendix A, the real wage is given by

$$\frac{w_i}{P_i(z)} = \frac{\sigma - 1}{\sigma} \left(\frac{b_i(z)\overline{L}_i}{\sigma f_{ii}} \right)^{\frac{1}{\sigma-1}} \mu_i(z)\varphi_{ii}^*(z). \tag{19}$$

In this economy with a continuum of sectors, welfare per worker in country i is defined as

$$W_i = \int_0^1 b_i(z) \ln \left(\frac{w_i}{P_i(z)} \right) dz.$$

Together with (19), this welfare expression means that the productivity cutoff of domestic production $\varphi_{ii}^*(z)$ is a sufficient statistic for welfare (because $b_i(z)$ is constant) in the sense that the impact on welfare is completely determined by the behavior of that productivity cutoff. As shown by Demidova and Rodríguez-Clare (2013), this statement holds true even when wages are an endogenous variable. Applying the comparative static results in Proposition 3 to (19), we have the following proposition within the ranges of (14) and (15).

Proposition 4

(i) A rise in relative country size raises welfare in country 1, but reduces welfare in country 2.

$$\frac{\partial W_1}{\partial \overline{L}} \geq 0, \quad \frac{\partial W_2}{\partial \overline{L}} \leq 0.$$

(ii) A rise in relative labor productivity raises welfare in country 1, but reduces welfare in country 2.

$$\frac{\partial W_1}{\partial \mu(z)} \geq 0, \quad \frac{\partial W_2}{\partial \mu(z)} \leq 0.$$

This result is obtained by noting that the productivity cutoff of domestic production in country 1 (country 2) is increasing (decreasing) in relative country size or relative labor productivity. Note that the welfare implication stands in sharp contrast to the Ricardian model with perfect competition. Regarding the impact of relative country size, for example, a growing country experiences a welfare loss by worsening the terms of trade as in the Ricardian model with perfect competition. At the same time, the lower relative wage reduces the price-cost margins and makes the country's competition more intense. The fact that the productivity cutoff of domestic production rises with country size implies that the welfare loss from the terms of trade is dominated by the welfare gain from increased competition and aggregate productivity in the Ricardian model with monopolistic competition and heterogeneous firms. Similarly, the impact of relative labor productivity also has the different welfare implications since an increase in relative labor productivity leads to a welfare gain for both countries in the Ricardian model with perfect competition.

3.6 Symmetric Costs

To help better understand the equilibrium and its properties, we consider the special case of perfectly symmetric trade and production costs across countries:

$$\tau_{ij} = \tau_{ji} \equiv \tau_x, \quad f_{ij} = f_{ji} \equiv f_x, \quad f_{ii} = f_{jj} \equiv f_d.$$

The only differences across countries are country size and technology. First of all, it should be noted that, even with symmetric costs, wages w_i, w_j are different across countries in the Ricardian model (which will be confirmed later). Given that, the cutoff conditions in (2) and (3) are expressed as

$$\bar{z}_1 = \mu^{-1}\left(\frac{\omega}{\tau_x}\right), \quad \bar{z}_2 = \mu^{-1}(\tau_x \omega),$$

where $\bar{z}_1 \leq \bar{z}_2$, so long as $\tau_x \geq 1$.

The fact that wages are different across countries implies that market demands $B_i(z), B_j(z)$ and productivity cutoffs $\varphi^*_{ii}(z), \varphi^*_{jj}(z), \varphi^*_{ij}(z), \varphi^*_{ji}(z)$ are also different across countries. As in the case of asymmetric costs, these six variables systematically vary with each country's comparative advantage such that $B(z) \equiv \frac{B_1(z)}{B_2(z)}$ and $\varphi^{*x}(z) \equiv \frac{\varphi^*_{12}(z)}{\varphi^*_{21}(z)}$ are decreasing in z and $\varphi^{*d}(z) \equiv \frac{\varphi^*_{11}(z)}{\varphi^*_{22}(z)}$ is increasing in z.

In this special case, however, the trade and production costs do not enter the relative productivity cutoff conditions in (9) and (10), and hence $\varphi^{*d}(z) = \varphi^{*x}(z)$ if and only if $B(z) = 1$. Furthermore, substituting (2) and (3) into (9) and (10) with symmetric costs reveals that $\varphi^{*d}(z) = \varphi^{*x}(z)$ and $B(z) = 1$ for $z = \bar{z}_1, \bar{z}_2$. Thus the sectoral equilibrium depicted by Figure 2 holds in this special case by setting $\Gamma = 1$. The relative gap in the productivity cutoff in (13) is

$$\frac{\varphi^*_{ij}(z)}{\varphi^*_{ii}(z)} = \tau_x \left(\frac{B_i(z)}{B_j(z)} \frac{f_x}{f_d} \right)^{\frac{1}{\sigma-1}},$$

and the characterization of the sectoral equilibrium leads to the relationship between firm selection and comparative advantage in Section 3.2.

Turning to general equilibrium, substituting (2) and (3) into the trade balance condition in (18) with symmetric costs immediately yields

$$\omega = \frac{\kappa_2(\omega\tau_x) - \lambda_2(\omega\tau_x)}{\kappa_1(\omega/\tau_x) - \lambda_1(\omega/\tau_x)} \left(\frac{\overline{L}_2}{\overline{L}_1} \right),$$

which can be solved for the unique equilibrium relative wage. This relative wage is in general not unity and hence wages are different across countries even under the symmetric cost assumption, as noted above. It is then straightforward to show that the analyses of comparative statics and welfare are qualitatively similar with those in Sections 3.4 and 3.5.

3.7 Gravity

We have thus far analyzed the equilibrium characterization and comparative statics. This subsection explores the impact on the extensive and intensive margins, which in turn allows us to derive the gravity equation in our Ricardian model. To obtain closed-form solutions of these two margins, we hereafter assume that firm productivity φ is drawn from a Pareto distribution:

$$G(\varphi) = 1 - \left(\frac{\varphi_{\min}}{\varphi} \right)^k,$$

where $\varphi \geq \varphi_{\min} > 0$ and $k > \sigma - 1$. It is useful to decompose aggregate export sales $R_{ij}(z)$ into

$$\begin{aligned} R_{ij}(z) &= M^e_i(z) \int_{\varphi^*_{ij}(z)}^{\infty} r_{ij}(\varphi, z) dG(\varphi) \\ &= [1 - G(\varphi^*_{ij}(z))] M^e_i(z) \times \frac{1}{[1 - G(\varphi^*_{ij}(z))]} \int_{\varphi^*_{ij}(z)}^{\infty} r_{ij}(\varphi, z) dG(\varphi) \\ &= M_{ij}(z) \times \bar{r}_{ij}(z), \end{aligned}$$

where $M_{ij}(z)$ is the mass of exporting firms (extensive margin), and $\bar{r}_{ij}(z)$ is average sales per firm (intensive margin). Similarly, aggregate domestic sales are decomposed into $R_{ii}(z) = M_{ii}(z) \times \bar{r}_{ii}(z)$. Then, the following lemma records the impact of comparative advantage on the two margins.

Lemma 4

(*i*) *The relative extensive margin of domestic firms* $M^d(z) \equiv M_{11}(z)/M_{22}(z)$ *and that of exporting firms* $M^x(z) \equiv M_{12}(z)/M_{21}(z)$ *are log-supermodular. For* $z < z'$,

$$\frac{M_{11}(z)}{M_{22}(z)} \leq \frac{M_{11}(z')}{M_{22}(z')}, \quad \frac{M_{12}(z)}{M_{21}(z)} \leq \frac{M_{12}(z')}{M_{21}(z')}.$$

(*ii*) *The relative intensive margin of domestic firms* $\bar{r}^d(z) \equiv \bar{r}_{11}(z)/\bar{r}_{22}(z)$ *and that of exporting firms* $\bar{r}^x(z) \equiv \bar{r}_{12}(z)/\bar{r}_{21}(z)$ *are neither log-supermodular nor log-submodular. For* $z < z'$,

$$\frac{\bar{r}_{11}(z)}{\bar{r}_{22}(z)} = \frac{\bar{r}_{11}(z')}{\bar{r}_{22}(z')}, \quad \frac{\bar{r}_{12}(z)}{\bar{r}_{21}(z)} = \frac{\bar{r}_{12}(z')}{\bar{r}_{21}(z')}.$$

Lemma 4 means that the relative mass of domestic firms $M_{ii}(z)/M_{jj}(z)$ and that of exporting firms $M_{ij}(z)/M_{ji}(z)$ are increasing in the strength of i's comparative advantage, whereas the relative average sales of these firms are the same across sectors. To establish this lemma, let us first derive the mass of potential entrants under the Pareto distribution. Applying this specific parameterization to (6) and (7) and rearranging, we have that

$$M_i^e(z) = \frac{\sigma - 1}{k\sigma} \frac{L_i(z)}{f_i^e}. \tag{20}$$

Although $M_i^e(z)$ is a function of the eight unknowns in general (as shown in Lemma 3), it depends only on aggregate labor supply $L_i(z)$ under the Pareto distribution. Using (20), the extensive and intensive margins are expressed as

$$\begin{aligned} M_{ii}(z) &= \left(\frac{\varphi_{\min}}{\varphi_{ii}^*(z)}\right)^k \frac{\sigma - 1}{k\sigma} \frac{L_i(z)}{f_i^e}, \qquad \bar{r}_{ii}(z) = \frac{k\sigma}{k - (\sigma - 1)} w_i f_{ii}, \\ M_{ij}(z) &= \left(\frac{\varphi_{\min}}{\varphi_{ij}^*(z)}\right)^k \frac{\sigma - 1}{k\sigma} \frac{L_i(z)}{f_i^e}, \qquad \bar{r}_{ij}(z) = \frac{k\sigma}{k - (\sigma - 1)} w_i f_{ij}. \end{aligned} \tag{21}$$

Lemma 4 follows immediately from noting Lemma 3 and (21).

The decomposition into the extensive and intensive margins allows us to express aggregate export sales as the gravity equation in the Ricardian model with monopolistic competition and heterogeneous firms. Substituting $\varphi_{ij}^*(z)$ from (5) into $M_{ij}(z)$ given in (21), we obtain the following proposition.

Proposition 5 *Aggregate export sales* $R_{ij}(z)$ *in sector* z *from country* i *to country* j *are given by*

$$R_{ij}(z) = \psi_i L_i(z) B_j(z)^{\frac{k}{\sigma-1}} \left(\frac{\mu_i(z)}{\tau_{ij} w_i}\right)^k (w_i f_{ij})^{1 - \frac{k}{\sigma - 1}}, \tag{22}$$

where $\psi_i \equiv \frac{\sigma-1}{k-(\sigma-1)} \frac{(\varphi_{\min})^k}{f_i^e}$. *An increase in* $R_{ij}(z)$ *due to country* i*'s comparative advantage is mainly accounted for by an increase in the extensive margin.*

The functional form in (22) is similar to the gravity equation in Chaney (2008). In particular, the elasticity of trade flows with respect to variable trade costs depends only on the shape parameter of the distribution k, as variable trade costs affect solely the extensive margin (see Eaton and Kortum (2002) for a similar finding). In contrast to these models, (22) also allows us to examine the impact of comparative advantage on the two margins. To see this, let us express (22) in the relative term:

$$\frac{R_{ij}(z)}{R_{ji}(z)} = \underbrace{\frac{f_j^e}{f_i^e}\frac{L_i(z)}{L_j(z)}\left(\frac{B_j(z)}{B_i(z)}\right)^{\frac{k}{\sigma-1}}\left(\frac{\tau_{ji}w_j}{\tau_{ij}w_i}\frac{\mu_i(z)}{\mu_j(z)}\right)^k}_{\frac{M_{ij}(z)}{M_{ji}(z)}}\underbrace{\left(\frac{w_i}{w_j}\frac{f_{ij}}{f_{ji}}\right)^{1-\frac{k}{\sigma-1}}}_{\frac{\bar{r}_{ij}(z)}{\bar{r}_{ji}(z)}}.$$

From Assumption 1, Lemmas 1 and 3, country i's comparative advantage increases aggregate export sales $R_{ij}(z)$ (relative to $R_{ji}(z)$) by increasing labor allocation in exporting country i ($L_i(z)$), market demand in importing country j ($B_j(z)$), and labor productivity in exporting country i ($\mu_i(z)$), which all contribute to an increase in the extensive margin of trade.

Having described the impact of comparative advantage on the extensive and intensive margins, let us turn to examining the impact of country size and technology on these two margins. From (21), the relative extensive margins, $M^d(z) \equiv M_{11}(z)/M_{22}(z), M^x(z) \equiv M_{12}(z)/M_{21}(z)$, are given by

$$M^d(z) = \frac{L(z)}{(\varphi^{*d}(z))^k}\frac{f_2^e}{f_1^e}, \quad M^x(z) = \frac{L(z)}{(\varphi^{*x}(z))^k}\frac{f_2^e}{f_1^e}. \tag{23}$$

Similarly, the relative intensive margins, $\bar{r}^d(z) \equiv \bar{r}_{11}(z)/\bar{r}_{22}(z), \bar{r}^x(z) \equiv \bar{r}_{12}(z)/\bar{r}_{21}(z)$, are given by

$$\bar{r}^d(z) = \omega\frac{f_{11}}{f_{22}}, \quad \bar{r}^x(z) = \omega\frac{f_{12}}{f_{21}}. \tag{24}$$

Applying the comparative statics in Proposition 3 to (23) and (24) leads to the next proposition.

Proposition 6 *The relative extensive and intensive margins in (23) and (24) satisfy*

$$\begin{aligned}
&\frac{\overline{L}}{M^d(z)}\frac{\partial M^d(z)}{\partial \overline{L}} \le \frac{\overline{L}}{M^x(z)}\frac{\partial M^x(z)}{\partial \overline{L}}, \quad \frac{\partial \bar{r}^d(z)}{\partial \overline{L}} \le 0, \quad \frac{\partial \bar{r}^x(z)}{\partial \overline{L}} \le 0,\\
&\frac{\mu(z)}{M^d(z)}\frac{\partial M^d(z)}{\partial \mu(z)} \le \frac{\mu(z)}{M^x(z)}\frac{\partial M^x(z)}{\partial \mu(z)}, \quad \frac{\partial \bar{r}^d(z)}{\partial \mu(z)} \ge 0, \quad \frac{\partial \bar{r}^x(z)}{\partial \mu(z)} \ge 0.
\end{aligned}$$

This proposition means that country size, for example, impacts positively on the extensive margin, whereas it impacts negatively on the intensive margin in the gravity equation, which accords well with recent empirical evidence (e.g., Bernard et al., 2007a). The comparative static results for the intensive margin are obtained immediately from Proposition 3 and (24). As for the extensive margin, (20) shows that the mass of entrants is proportional to aggregate sector labor supply and there is no home market effect for entry:

$$\frac{M_1^e(z)}{M_2^e(z)} = \frac{L_1(z)}{L_2(z)}\frac{f_2^e}{f_1^e}. \tag{25}$$

In contrast, (23) shows that the masses of domestic firms and exporting firms are not proportional to entry since the productivity cutoffs vary with country size in the Ricardian model with monopolistic competition and heterogeneous firms. Noting that $M^e(z) \equiv M_1^e(z)/M_2^e(z)$ and $L(z) \equiv L_1(z)/L_2(z)$, $M^e(z) = L(z)/(f_1^e/f_2^e)$ from (25) and $M^d(z) = M^e(z)/(\varphi^{*d}(z))^k, M^x(z) = M^e(z)/(\varphi^{*x}(z))^k$ from (23). Further since $\varphi^{*d}(z)$ ($\varphi^{*x}(z)$) is increasing (decreasing) in $\overline{L}$ from Proposition 3, the relative extensive margins in (23) must satisfy

$$\frac{\overline{L}}{M^d(z)}\frac{\partial M^d(z)}{\partial \overline{L}} \leq \frac{\overline{L}}{M^e(z)}\frac{\partial M^e(z)}{\partial \overline{L}} \leq \frac{\overline{L}}{M^x(z)}\frac{\partial M^x(z)}{\partial \overline{L}}.$$

Thus, the mass of exporting firms (domestic firms) increases more (less) than proportionally to entry. This reasoning also explains why relative labor productivity raises both the extensive and intensive margins through the impact on the productivity cutoffs and relative wage. The comparative statics suggest that any change in country size or technology should have an impact not only on the structure of comparative advantage (*inter*-sectoral adjustment) characterized by (2), (3) and (18), but also on the extensive and intensive margins (*intra*-sectoral adjustment) characterized by (4), (5) and (6) in the Ricardian model with monopolistic competition and heterogeneous firms.

We conclude this subsection by examining the the impact on the fraction of firms that export. From (4), (5) and (21), this fraction is given by

$$\frac{M_{ij}(z)}{M_{ii}(z)} = \begin{cases} \left(\frac{1}{B(z)}\frac{1}{\tau_{12}^{\sigma-1}}\frac{f_{11}}{f_{12}}\right)^{\frac{k}{\sigma-1}} & \text{for} \quad i = 1, \\ \left(B(z)\frac{1}{\tau_{21}^{\sigma-1}}\frac{f_{22}}{f_{21}}\right)^{\frac{k}{\sigma-1}} & \text{for} \quad i = 2, \end{cases} \tag{26}$$

which is between zero and unity under (14). Since $B(z)$ is decreasing in z (Lemma 1), (26) shows that $M_{ij}(z)/M_{ii}(z)$ is increasing in the strength of i's comparative advantage and thus log-supermodular ($\frac{M_{12}(z)/M_{11}(z)}{M_{21}(z)/M_{22}(z)} \leq \frac{M_{12}(z')/M_{11}(z')}{M_{21}(z')/M_{22}(z')}$ for $z < z'$). It also follows from (12) that $M_{ij}(z)/M_{ii}(z)$ is greater than $M_{ji}(z)/M_{jj}(z)$ in i's comparative advantage sectors. In addition, from Proposition 3, $M_{ij}(z)/M_{ii}(z)$ is increasing (decreasing) in relative country size $\overline{L}$ for country 1 (country 2). The mechanism of the last result stems from the above comparative statics: an increase in relative country size makes firm selection into the domestic (export) market more (less) intense, which increases the mass of domestic firms (exporting firms) less (more) than proportionally to entry, and consequently raises the fraction of firms that export. It is important to stress that the analysis applies only for large countries where any exogenous shocks in a country have an influence on another country. Though rigorous empirical work examining this channel is yet to come, if we treat the U.S. and China as representatives of such large countries, our theoretical prediction is consistent with the existing evidence: 18% of U.S. firms export in 2002 (Bernard et al., 2007a), while 25% of Chinese firms export in either 1999 or 2007 (Huang et al., 2017).[9] Clearly, $M_{ij}(z)/M_{ii}(z)$ is increasing (decreasing) in relative labor productivity $\mu(z)$ for country 1 (country 2).

[9] As in Bernard et al. (2007a), Huang et al. (2017) focus on manufacturing firms, using the Chinese Annual Industrial Survey that covers all State Owned Enterprises (SOEs) and non-SOEs with annual sales higher than 5 million yuan.

4 Discussions

This section first discusses the impact of variable trade costs, and then relates the DFS model with perfect competition to the DFS model with monopolistic competition and heterogeneous firms.

4.1 Variable Trade Costs

Let us first consider a symmetric reduction in variable trade costs. Simple inspection of (2), (3) and (18) reveals that this reduction always narrows the the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$, but it can shift the relative wage ω in either direction (see the equilibrium relative wage in Section 3.6). Thus it also can shift the equilibrium vector $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$ in either direction too. We can however consider the impact of zero gravity ($\tau_{ij} = \tau_{ji} = 1$) on the equilibrium outcomes. In this setting, country i's unit labor costs are less than or equal to country j's unit labor costs if

$$w_i a_i(z) \leq w_j a_j(z) \quad \Longleftrightarrow \quad \frac{w_i}{w_j} \leq \frac{\mu_i(z)}{\mu_j(z)}.$$

This means the existence of a unique cutoff sector such that country 1 (country 2) has a comparative advantage in high-z (low-z) sectors $\bar{z} \leq z \leq 1$ ($0 \leq z \leq \bar{z}$), where

$$\bar{z} \equiv \mu^{-1}(\omega). \tag{27}$$

In the zero gravity world, we can easily show that (18) is given by

$$\omega = \frac{\kappa_2(\bar{z}) - \lambda_2(\bar{z})}{\kappa_1(\bar{z}) - \lambda_1(\bar{z})} \left(\frac{\overline{L}_2}{\overline{L}_1} \right). \tag{28}$$

As with the main analysis, (27) and (28) provide implicit solutions for $\{\bar{z}, \omega\}$; and (4), (5) and (6) with $\tau_{ij} = \tau_{ji} = 1$ then provide implicit solutions for $\{\varphi_{11}^*(z), \varphi_{22}^*(z), \varphi_{12}^*(z), \varphi_{21}^*(z), B_1(z), B_2(z)\}$. While the equilibrium characterization and comparative statics are similar as before, a key difference arises in the presence of zero gravity: $B(z)$ and $\varphi^{*x}(z)$ are *strictly* decreasing in z, and $\varphi^{*d}(z)$ is *strictly* increasing in z, because the interval sectors disappear. As a result, all the endogenous variables in Lemmas 1–4 are *strictly* log-supermodular or log-submodular (see Appendix B for details).

Although the impact of a symmetric reduction in variable trade costs is ambiguous in this model, the impact of an asymmetric reduction is unambiguous. Noting that τ_{ij} is variable trade costs from country i to country j, let $\tau \equiv \tau_{21}/\tau_{12}$ denote relative variable trade costs in country 1. Obviously, the trade costs are lower when country 1 unilaterally reduces its variable trade costs of importing from country 2. From (2), (3) and (18), a reduction in τ narrows the the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$ as above, but it reduces the relative wage in this case. Given this impact on the relative wage, solving the system of equations (4), (5) and (6) reveals that a reduction in τ has the following impact on the productivity cutoffs within some range of parameterizations:

$$\frac{\partial \varphi_{11}^*(z)}{\partial \tau} \leq 0, \ \frac{\partial \varphi_{22}^*(z)}{\partial \tau} \leq 0, \ \frac{\partial \varphi_{12}^*(z)}{\partial \tau} \geq 0, \ \frac{\partial \varphi_{21}^*(z)}{\partial \tau} \geq 0. \tag{29}$$

Since the productivity cutoff of domestic production is a sufficient statistic for welfare in this model, (29) implies that the liberalizing country gains from a reduction in τ. Intuitively, while liberalization in country 1 leads to a decline in its relative wage, this is smaller than the decline in the price index and hence raises welfare there. This impact on the liberalizing country, which depends crucially on the endogenous relative wage, is the same as Demidova and Rodríguez-Clare (2013) but it is opposite to Demidova (2008) and Melitz and Ottaviano (2008) due to the presence of an outside good. In fact, if wages are exogenously fixed in our model, the liberalizing country loses from such a reduction.[10]

4.2 Relationship to DFS

It is important to stress that, in the DFS model with monopolistic competition and heterogeneous firms, the finding of the DFS model with perfect competition arises as a special case in which product differentiation and firm heterogeneity are absent. If the current model assumes perfect competition, international trade will lead to complete specialization for traded goods in each country, allowing all laborers to be allocated to the comparative advantage sectors:

$$\int_{\bar{z}_1}^{1} \frac{L_1(z)}{\overline{L}_1} dz = \int_{0}^{\bar{z}_2} \frac{L_2(z)}{\overline{L}_2} dz = 1,$$

or $\kappa_1(\bar{z}_1) = \kappa_2(\bar{z}_2) = 1$. Substituting this equality into (18), we obtain

$$\omega = \frac{1 - \lambda_2(\bar{z}_2)}{1 - \lambda_1(\bar{z}_1)} \left(\frac{\overline{L}_2}{\overline{L}_1} \right). \tag{30}$$

The equilibrium characterization determined by (2), (3) and (30) is exactly the same as that of DFS, while making (4), (5) and (6) irrelevant for the analysis of perfect competition.

The above labor reallocation does not occur in the current model since international trade leads to incomplete specialization, allowing laborers to be allocated relatively more to the sectors where each country's comparative advantage is relatively stronger. This implies that the DFS model with perfect competition can be understood as a special case of the DFS model with monopolistic competition and heterogeneous firms in that the equilibrium characterization and comparative statics give rise to exactly the same outcomes as those of the DFS model with perfect competition if we ignore product differentiation and firm heterogeneity. In that sense the DFS model with monopolistic competition and heterogeneous firms can generate richer predictions through the intra-sectoral adjustment in the firm-level variables that are absent in the DFS model with perfect competition.

Finally, we mention a welfare comparison between the two models. While the real wage in the DFS model with perfect competition is given by $w_i/P_i(z) = \mu_i(z)$, we cannot say for sure whether this real wage is necessarily greater or smaller than that in the DFS model with monopolistic competition and heterogeneous firms, which is given by (19). This makes it difficult to compare welfare between the two models.

[10] Since asymmetric trade liberalization reduces the relative wage, it does not always improve welfare of the liberalizing country in the Ricardian model with perfect competition.

5 Conclusions

This paper presented a general-equilibrium Ricardian model with heterogeneous firms to explore the impact of country size and technology on the firm-level variables. We demonstrated that a country with larger size and better technology exhibits higher productivity and lower price-cost margins even under the assumptions of C.E.S. preferences and monopolistic competition by changing the relative wage. Welfare is higher in this country, not only due to increased product variety but also due to increased competition in a domestic market. We also showed that the equilibrium property of our model helps understand the role of country size and technology in the gravity equation. In particular, our model predicts that country size impacts positively on the extensive margin, whereas it impacts negatively on the intensive margin, which accords well with recent empirical evidence using the firm-level dataset. Our model offers a possible explanation for this empirical pattern by allowing country size to affect the firm-level variables, while preserving the usefulness of the workforce model in the new trade theory literature.

The model outcome in our paper depends entirely on supply side characteristics such as wages and labor productivity, and demand side characteristics such as quality and taste do not play a key role. A growing body of empirical evidence suggests however that firm heterogeneity alone cannot account for revenue variation across export markets (Eaton et al., 2011), and recent work theoretically and empirically examines the role of consumer heterogeneity in that variation (Di Comité et al., 2017). To take account of the demand side in the current model, we follow Schott (2008) in interpreting export prices as a signal of vertical differentiation in quality. Under this interpretation, our model allows for differences in quality across export markets in the sense that goods that require relatively high wages are of high quality. From the comparative statics, it then follows that quality of exported goods is higher when selling in a destination country with larger size because wages (and hence quality) are relatively higher in an origin country. Further, we can explain revenue variation for the same variety across export markets: export revenue is lower when selling in a destination country with larger size because higher export prices are dominated by a lower price index there. The Ricardian model thus yields empirically testable predictions that are complement to the existing models.

To make the analysis simple, we have restricted our attention to an open economy and abstracted from comparing welfare in autarky and costly trade, but it is straightforward to extend our setup to explore the impact of trade on inter-/intra-sectoral resource allocations and welfare gains from trade. From the impact of asymmetric trade liberalization on the firm-level variables, we expect that trade liberalization would allocate labor resources relatively more to more productive firms within sectors, whereas these trade-induced reallocations would be more significant in comparative advantage sectors than comparative disadvantage sectors, thereby creating additional welfare gains from trade. The rationale in our analysis suggests that this welfare consequence of trade should be similar between the Ricardian model and the Heckscher-Ohlin model. Since the movement from autarky to costly trade would not lead to the same factor prices between two countries in the Ricardian model, however, this difference in the factor prices would lead to different implications for the role of country endowments in the firm-level variables.

Appendix A: Proofs

A.1 Proofs of Lemmas 1 and 2

We first prove Lemmas 1 and 2. Taking the log and differentiating (4) and (5) with respect to z,

$$\frac{B_1'(z)}{B_1(z)} - (\sigma-1)\frac{\mu_1'(z)}{\mu_1(z)} + (\sigma-1)\frac{\varphi_{11}^{*'}(z)}{\varphi_{11}^*(z)} = 0, \tag{A.1}$$

$$\frac{B_2'(z)}{B_2(z)} - (\sigma-1)\frac{\mu_2'(z)}{\mu_2(z)} + (\sigma-1)\frac{\varphi_{22}^{*'}(z)}{\varphi_{22}^*(z)} = 0, \tag{A.2}$$

$$\frac{B_2'(z)}{B_2(z)} - (\sigma-1)\frac{\mu_1'(z)}{\mu_1(z)} + (\sigma-1)\frac{\varphi_{12}^{*'}(z)}{\varphi_{12}^*(z)} = 0, \tag{A.3}$$

$$\frac{B_1'(z)}{B_1(z)} - (\sigma-1)\frac{\mu_2'(z)}{\mu_2(z)} + (\sigma-1)\frac{\varphi_{21}^{*'}(z)}{\varphi_{21}^*(z)} = 0. \tag{A.4}$$

Further, using (4) and (5), rewrite (6) as

$$f_{ii}J(\varphi_{ii}^*(z)) + f_{ij}J(\varphi_{ij}^*(z)) = f_i^e,$$

where $J(\varphi^*) = \int_{\varphi^*}^{\infty}[(\varphi/\varphi^*)^{\sigma-1} - 1]dG(\varphi)$ is strictly decreasing in φ^*, with $\lim_{\varphi^*\to 0} J(\varphi^*) = \infty$ and $\lim_{\varphi^*\to\infty} J(\varphi^*) = 0$. Differentiating this equality with respect to z and rearranging,

$$\varphi_{12}^{*'}(z) = -C_1(z)\varphi_{11}^{*'}(z), \tag{A.5}$$

$$\varphi_{21}^{*'}(z) = -C_2(z)\varphi_{22}^{*'}(z), \tag{A.6}$$

where $C_i(z) \equiv \frac{f_{ii}J'(\varphi_{ii}^*(z))}{f_{ij}J'(\varphi_{ij}^*(z))} > 0$. Note that (A.1) – (A.6) are six equations which have six unknowns $(\varphi_{11}^{*'}(z), \varphi_{22}^{*'}(z), \varphi_{12}^{*'}(z), \varphi_{21}^{*'}(z), B_1'(z), B_2'(z))$. Substituting (A.5) and (A.6) respectively into (A.3) and (A.4), and subtracting (A.2) and (A.1) respectively from these yields

$$\frac{C_1(z)\varphi_{11}^{*'}(z)}{\varphi_{12}^*(z)} + \frac{\varphi_{22}^{*'}(z)}{\varphi_{22}^*(z)} = \frac{\mu'(z)}{\mu(z)}, \quad \frac{C_2(z)\varphi_{22}^{*'}(z)}{\varphi_{21}^*(z)} + \frac{\varphi_{11}^{*'}(z)}{\varphi_{11}^*(z)} = -\frac{\mu'(z)}{\mu(z)},$$

where $\mu'(z) \geq 0$. These are two equations with two unknowns $(\varphi_{11}^{*'}(z), \varphi_{22}^{*'}(z))$, which are solved for

$$\varphi_{11}^{*'}(z) = \frac{\frac{\mu'(z)}{\mu(z)}\left(\frac{1}{\varphi_{22}^*(z)} + \frac{C_2(z)}{\varphi_{21}^*(z)}\right)}{\Xi(z)}, \quad \varphi_{22}^{*'}(z) = -\frac{\frac{\mu'(z)}{\mu(z)}\left(\frac{1}{\varphi_{11}^*(z)} + \frac{C_1(z)}{\varphi_{12}^*(z)}\right)}{\Xi(z)},$$

where

$$\Xi(z) \equiv \frac{1}{\varphi_{11}^*(z)\varphi_{22}^*(z)}\left(\frac{\varphi_{11}^*(z)\varphi_{22}^*(z)}{\varphi_{12}^*(z)\varphi_{21}^*(z)}C_1(z)C_2(z) - 1\right).$$

From (4), (5) and $C_i(z)$ defined above, $\Xi(z)$ is positive if

$$\frac{J'(\varphi_{11}^*(z))J'(\varphi_{22}^*(z))}{J'(\varphi_{12}^*(z))J'(\varphi_{21}^*(z))} > \tau_{12}\tau_{21}\left(\frac{f_{12}f_{21}}{f_{11}f_{22}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{A.7}$$

which holds for a general distribution function. To show this, differentiating $J(\varphi^*)$ with respect to φ^*, we have

$$J'(\varphi^*) = -\left(\frac{\sigma-1}{\varphi^*}\right)[J(\varphi^*) + 1 - G(\varphi^*)],$$

where $J(\varphi^*) + 1 - G(\varphi^*) = (\varphi^*)^{1-\sigma}V(\varphi^*)$ from the definitions of $J(\varphi^*)$ and $V(\varphi^*)$. Then the above equality is given by

$$J'(\varphi^*) = -(\sigma-1)(\varphi^*)^{-\sigma}V(\varphi^*).$$

Substituting this into (A.7) and (13), (A.7) is rewritten as

$$(\tau_{12}\tau_{21})^{\sigma-1}\left(\frac{V(\varphi^*_{11}(z))V(\varphi^*_{22}(z))}{V(\varphi^*_{12}(z))V(\varphi^*_{21}(z))}\right) > 1,$$

which holds true under (14) because $V(\varphi^*)$ is strictly decreasing in φ^*. Thus $\varphi^{*'}_{11}(z) \geq 0, \varphi^{*'}_{22}(z) \leq 0$, and from (A.5) and (A.6), $\varphi^{*'}_{12}(z) \leq 0, \varphi^{*'}_{21}(z) \geq 0$, and hence $\varphi^{*d'}(z) \geq 0,\ \varphi^{*x'}(z) \leq 0$. From (8), these imply that $B'(z) \leq 0$.

We next show that $\varphi^{*d}(z) = \varphi^{*x}(z)$ and $B(z) = \Gamma$ for $z = \bar{z}_1, \bar{z}_2$. First, from (9) and (10), when $\varphi^{*d}(z) = \varphi^{*x}(z)$, we have $B(z) = \Gamma$ for any z, and hence $B(z)$ is constant and the same across z's, as long as $\varphi^{*d}(z) = \varphi^{*x}(z)$. Next, substituting (2) into (9) and (10), we have for $z = \bar{z}_1$,

$$\varphi^{*d}(\bar{z}_1) = \tau_{21}\left(\omega\frac{f_{11}}{f_{22}}\frac{1}{B(\bar{z}_1)}\right)^{\frac{1}{\sigma-1}}, \quad \varphi^{*x}(\bar{z}_1) = \tau_{12}\left(\omega\frac{f_{12}}{f_{21}}B(\bar{z}_1)\right)^{\frac{1}{\sigma-1}},$$

which suggest that when $\varphi^{*d}(\bar{z}_1) = \varphi^{*x}(\bar{z}_1)$, we have $B(\bar{z}_1) = \Gamma$ for $z = \bar{z}_1$. Similarly, substituting (3) into (9) and (10), when $\varphi^{*d}(\bar{z}_2) = \varphi^{*x}(\bar{z}_2)$, we have $B(\bar{z}_2) = \Gamma$ for $z = \bar{z}_2$. This establishes the desired result.

A.2 Proof of Lemma 3

We first show that $Q(z) \equiv \frac{Q_1(z)}{Q_2(z)}$ is increasing in z whereas $P(z) \equiv \frac{P_1(z)}{P_2(z)}$ is decreasing in z. Using $R_i(z) = b_i(z)w_i\overline{L}_i$ and $b_i(z) = b_j(z)$, we have

$$R(z) \equiv \frac{R_1(z)}{R_2(z)} = \omega\overline{L}. \tag{A.8}$$

Using this, it follows from $B_i(z) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^\sigma}R_i(z)(P_i(z))^{\sigma-1}$ that

$$B(z) \equiv \frac{B_1(z)}{B_2(z)} = \omega\overline{L}(P(z))^{\sigma-1}.$$

Taking the log and differentiating $B(z)$ with respect to z, $\frac{B'(z)}{B(z)} = (\sigma-1)\frac{P'(z)}{P(z)}$. Since $B'(z) \leq 0$ and $\sigma > 1$, we have $P'(z) \leq 0$. Moreover, using $R_i(z) = P_i(z)Q_i(z)$, (A.8) is alternatively expressed as

$$R(z) = P(z)Q(z).$$

Noting that the right-hand side of (A.8) is independent of z, taking the log and differentiating this with respect to z yields $\frac{R'(z)}{R(z)} = \frac{P'(z)}{P(z)} + \frac{Q'(z)}{Q(z)} = 0$. Since $P'(z) \leq 0$, we have $Q'(z) \geq 0$. This proves that $\frac{Q_1(z)}{Q_2(z)} \leq \frac{Q_1(z')}{Q_2(z')}$ and $\frac{P_1(z)}{P_2(z)} \geq \frac{P_1(z')}{P_2(z')}$ for $z < z'$.

We next show that $R^d(z) \equiv \frac{R_{11}(z)}{R_{22}(z)}$ and $R^x(z) \equiv \frac{R_{12}(z)}{R_{21}(z)}$ are increasing in z, where $R_{ii}(z)$ and $R_{ij}(z)$ are expressed as

$$R_{ii}(z) = M_i^e(z) \int_{\varphi_{ii}^*(z)}^{\infty} r_{ii}(\varphi, z) dG(\varphi), \quad R_{ij}(z) = M_i^e(z) \int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi, z) dG(\varphi).$$

Using the definition of $V(\varphi^*)$, rewrite these aggregate revenues as

$$\begin{aligned} R_{ii}(z) &= M_i^e(z) \sigma B_i(z) \left(\frac{\mu_i(z)}{w_i} \right)^{\sigma-1} V(\varphi_{ii}^*(z)), \\ R_{ij}(z) &= M_i^e(z) \sigma B_j(z) \left(\frac{\mu_i(z)}{\tau_{ij} w_i} \right)^{\sigma-1} V(\varphi_{ij}^*(z)), \end{aligned}$$

and hence its ratio is given by

$$\frac{R_{ii}(z)}{R_{ij}(z)} = \tau_{ij}^{\sigma-1} \frac{B_i(z)}{B_j(z)} \frac{V(\varphi_{ii}^*(z))}{V(\varphi_{ij}^*(z))}.$$

Taking the log and differentiating $\frac{R_{ii}(z)}{R_{ij}(z)}$ with respect to z, we have

$$\frac{R_{11}'(z)}{R_{11}(z)} \leq \frac{R_{12}'(z)}{R_{12}(z)}, \quad \frac{R_{21}'(z)}{R_{21}(z)} \leq \frac{R_{22}'(z)}{R_{22}(z)}, \tag{A.9}$$

where the inequalities come from the results in Lemmas 1 and 2. Further, noting that $b_i(z) = b_i(z')$ for $z \neq z'$ and $R_i(z) = R_{ii}(z) + R_{ji}(z) = b_i(z) w_i \overline{L}_i$, we have $R_{ii}'(z) = -R_{ji}'(z)$. Substituting this into (A.9) and rearranging,

$$\begin{aligned} \frac{R_{21}(z)}{R_{22}(z)} R_{12}'(z) \leq R_{11}'(z) \leq \frac{R_{11}(z)}{R_{12}(z)} R_{12}'(z), \\ \frac{R_{22}(z)}{R_{21}(z)} R_{21}'(z) \leq R_{22}'(z) \leq \frac{R_{12}(z)}{R_{11}(z)} R_{21}'(z). \end{aligned} \tag{A.10}$$

Note that $\frac{R_{21}(z)}{R_{22}(z)} < \frac{R_{11}(z)}{R_{12}(z)}$ if and only if $\frac{V(\varphi_{21}^*(z))}{V(\varphi_{22}^*(z))} < \frac{V(\varphi_{11}^*(z))}{V(\varphi_{12}^*(z))}$, which holds true because $\varphi_{ij}^*(z) > \varphi_{ii}^*(z)$ under (14). Then, (A.10) implies that $R_{11}'(z) \geq 0$, $R_{22}'(z) \leq 0$, $R_{12}'(z) \geq 0$, $R_{21}'(z) \leq 0$, which in turn implies that $R^{d'}(z) \geq 0$ and $R^{x'}(z) \geq 0$. This proves that $\frac{R_{11}(z)}{R_{22}(z)} \leq \frac{R_{11}(z')}{R_{22}(z')}$ and $\frac{R_{12}(z)}{R_{21}(z)} \leq \frac{R_{12}(z')}{R_{21}(z')}$ for $z < z'$.

Finally, we show that $L(z) \equiv \frac{L_1(z)}{L_2(z)}$ and $M^e(z) \equiv \frac{M_1^e(z)}{M_2^e(z)}$ are increasing in z. Regarding $L(z)$, we will show in Appendix A.3 that $L_i(z)$ is written as

$$L_i(z) = \frac{R_{ii}(z) + R_{ij}(z)}{w_i}. \tag{A.11}$$

From (A.11), its ratio is given by

$$L(z) \equiv \frac{L_1(z)}{L_2(z)} = \frac{1}{\omega}\frac{R_{11}(z)+R_{12}(z)}{R_{22}(z)+R_{21}(z)}.$$

Since $R'_{11}(z) \geq 0$, $R'_{22}(z) \leq 0$, $R'_{12}(z) \geq 0$, $R'_{21}(z) \leq 0$, (A.11) implies that $L'_2(z) \leq 0 \leq L'_1(z)$, and hence $L'(z) \geq 0$. As for $M_i^e(z)$, from the expression of $M_i^e(z)$ in the main text, it follows that its ratio is given by

$$M^e(z) \equiv \frac{M_1^e(z)}{M_2^e(z)} = \left(\frac{\mu(z)}{\omega}\right)^{\sigma-1} \frac{V(\varphi_{22}^*(z))(P(z))^{1-\sigma} - \tau_{21}^{1-\sigma}V(\varphi_{21}^*(z))}{V(\varphi_{11}^*(z)) - \tau_{12}^{1-\sigma}V(\varphi_{12}^*)(P(z))^{1-\sigma}}.$$

From $\mu'(z) \geq 0, \varphi_{11}^{*'}(z) \geq 0, \varphi_{22}^{*'}(z) \leq 0, \varphi_{12}^{*'}(z) \leq 0, \varphi_{21}^{*'}(z) \geq 0, P'(z) \leq 0$, it follows that $M^{e'}(z) \geq 0$. This proves that $\frac{L_1(z)}{L_2(z)} \leq \frac{L_1(z')}{L_2(z')}$ and $\frac{M_1^e(z)}{M_2^e(z)} \leq \frac{M_1^e(z')}{M_2^e(z')}$ for $z < z'$.

A.3 Proofs of Proposition 2

A.3.1 Proof of Equation (16)

We show that equation (7) is written as equation (16). Aggregate labor supply in sector z of country i is given by

$$L_i(z) = M_i^e(z)\int_{\varphi_{ii}^*(z)}^{\infty} l_{ii}(\varphi,z)dG(\varphi) + M_i^e(z)\int_{\varphi_{ij}^*(z)}^{\infty} l_{ij}(\varphi,z)dG(\varphi) + M_i^e(z)f_i^e. \tag{A.12}$$

Using the amount of labor required by individual firms $l_{ii}(\varphi,z)$ and $l_{ij}(\varphi,z)$, the first two terms in the right-hand side of (A.12) are

$$\begin{aligned}
&M_i^e(z)\int_{\varphi_{ii}^*(z)}^{\infty} l_{ii}(\varphi,z)dG(\varphi) + M_i^e(z)\int_{\varphi_{ij}^*(z)}^{\infty} l_{ij}(\varphi,z)dG(\varphi)\\
&= \frac{M_i^e}{w_i}\left\{[1-G(\varphi_{ii}^*(z))]\,w_i f_{ii} + \frac{\sigma-1}{\sigma}\int_{\varphi_{ii}^*(z)}^{\infty} r_{ii}(\varphi,z)dG(\varphi) + \left[1-G(\varphi_{ij}^*(z))\right]w_i f_{ij} + \frac{\sigma-1}{\sigma}\int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi)\right\}.
\end{aligned}$$

Regarding the last term in the right-hand side of (A.12), on the other hand, let us express the free entry condition in (6) as

$$\begin{aligned}
f_i^e &= \int_{\varphi_{ii}^*(z)}^{\infty} \frac{\pi_{ii}(\varphi,z)}{w_i}dG(\varphi) + \int_{\varphi_{ij}^*(z)}^{\infty} \frac{\pi_{ij}(\varphi,z)}{w_i}dG(\varphi)\\
&= \frac{1}{w_i}\left\{\int_{\varphi_{ii}^*(z)}^{\infty}\frac{r_{ii}(\varphi,z)}{\sigma}dG(\varphi) - [1-G(\varphi_{ii}^*(z))]\,w_i f_{ii} + \int_{\varphi_{ij}^*(z)}^{\infty}\frac{r_{ij}(\varphi,z)}{\sigma}dG(\varphi) - \left[1-G(\varphi_{ij}^*)\right]w_i f_{ij}\right\}.
\end{aligned}$$

Using this expression, the last term in the right-hand side of (A.12) is

$$M_i^e(z)f_i^e = \frac{M_i^e(z)}{w_i}\left\{\frac{1}{\sigma}\int_{\varphi_{ii}^*(z)}^{\infty} r_{ii}(\varphi,z)dG(\varphi) - [1-G(\varphi_{ii}^*(z))]\,w_i f_{ii} + \frac{1}{\sigma}\int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi) - \left[1-G(\varphi_{ij}^*(z))\right]w_i f_{ij}\right\}.$$

Summing up these terms, (A.12) is equivalent with (A.11):

$$\begin{aligned} L_i(z) &= \frac{M_i^e(z)}{w_i}\left\{\int_{\varphi_{ii}^*(z)}^{\infty} r_{ii}(\varphi,z)dG(\varphi) + \int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi)\right\} \\ &= \frac{R_{ii}(z)+R_{ij}(z)}{w_i}. \end{aligned}$$

Integrating the above aggregate labor supply over the interval [0,1] and noting $\int_0^1 L_i(z)dz = \overline{L}_i$,

$$\begin{aligned} \overline{L}_i &= \frac{\int_0^1 R_{ii}(z)dz + \int_0^1 R_{ij}(z)dz}{w_i} \\ &= \frac{\int_0^1 R_{ii}(z)dz + \int_0^1 R_{ji}(z)dz}{w_i} \\ &= \frac{\int_0^1 R_i(z)dz}{w_i}, \end{aligned}$$

where the second equality comes from (17), and the third equality comes from $R_i(z) = R_{ii}(z)+R_{ji}(z)$.

A.3.2 Proof of Equation (18)

We first show that (17) is expressed as

$$\int_{\bar{z}_1}^{1} \left(w_1L_1(z) - R_1(z)\right)dz = \int_0^{\bar{z}_2} \left(w_2L_2(z) - R_2(z)\right)dz. \tag{A.13}$$

Because net exports are zero in the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$, we have $\int_{\bar{z}_1}^{\bar{z}_2} R_{12}(z)dz = \int_{\bar{z}_1}^{\bar{z}_2} R_{21}(z)dz$. Noting this relationship, rewrite (17) as

$$\int_{\bar{z}_1}^{1} \left(R_{12}(z) - R_{21}(z)\right)dz = \int_0^{\bar{z}_2} \left(R_{21}(z) - R_{12}(z)\right)dz.$$

From $w_iL_i(z) = R_{ii}(z)+R_{ij}(z)$ and $R_i(z) = R_{ii}(z)+R_{ji}(z)$, we have $w_iL_i(z)-R_i(z) = R_{ij}(z)-R_{ji}(z)$. Substituting this into the above equality gives us equation (A.13).

We next show that (A.13) is equivalent to (18). By manipulating (A.13),

$$\begin{aligned} &\int_{\bar{z}_1}^{1}\left(\frac{L_1(z)}{\overline{L}_1} - \frac{R_1(z)}{w_1\overline{L}_1}\right)dz = \int_0^{\bar{z}_2}\left(\frac{L_2(z)}{\overline{L}_2}\frac{w_2\overline{L}_2}{w_1\overline{L}_1} - \frac{R_2(z)}{w_2\overline{L}_2}\frac{w_2\overline{L}_2}{w_1\overline{L}_1}\right)dz \\ \iff &\int_{\bar{z}_1}^{1}\left(\frac{L_1(z)}{\overline{L}_1} - b_1(z)\right)dz = \frac{w_2\overline{L}_2}{w_1\overline{L}_1}\int_0^{\bar{z}_2}\left(\frac{L_2(z)}{\overline{L}_2} - b_2(z)\right)dz \\ \iff &(\kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1))w_1\overline{L}_1 = (\kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2))w_2\overline{L}_2, \end{aligned}$$

where the first equality comes from dividing both sides of (A.13) by $w_1\overline{L}_1$, the second equality comes from the definition of $b_i(z)$, and the third equality comes from the definitions of $\kappa_i(\bar{z}_i)$ and $\lambda_i(\bar{z}_i)$. Solving the third equality for $\omega \equiv w_1/w_2$ gives us (18).

Finally, we show that (18) is decreasing in ω. For that purpose, it suffices to prove that (18) is decreasing in $\bar{z}_1, \bar{z}_2$ because $\bar{z}_1$ and $\bar{z}_2$ are increasing in ω (see (2) and (3)). Let

$$\xi_1(\bar{z}_1) \equiv \kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1), \quad \xi_2(\bar{z}_2) \equiv \kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2)$$

respectively denote the denominator and numerator of (18), which are positive for any $\bar{z}_1$ and $\bar{z}_2$. Differentiating these with respect to $\bar{z}_1$ and $\bar{z}_2$ respectively yields

$$\begin{aligned}
\frac{d\xi_1(\bar{z}_1)}{d\bar{z}_1} &= -\frac{L_1(\bar{z}_1)}{\overline{L}_1} + \int_{\bar{z}_1}^{1} \frac{L_1'(\bar{z}_1)}{\overline{L}_1} dz + \frac{R_1(\bar{z}_1)}{w_1\overline{L}_1} = \int_{\bar{z}_1}^{1} \frac{L_1'(\bar{z}_1)}{\overline{L}_1} dz, \\
\frac{d\xi_2(\bar{z}_2)}{d\bar{z}_2} &= \frac{L_2(\bar{z}_2)}{\overline{L}_2} + \int_{0}^{\bar{z}_2} \frac{L_2'(\bar{z}_2)}{\overline{L}_2} dz - \frac{R_2(\bar{z}_2)}{w_2\overline{L}_2} = \int_{0}^{\bar{z}_2} \frac{L_2'(\bar{z}_2)}{\overline{L}_2} dz,
\end{aligned}$$

where the first equality comes from $b_i(z) = \frac{R_i(z)}{w_i\overline{L}_i}$ and the second one comes from $w_i L_i(\bar{z}_i) = R_i(\bar{z}_i)$. Noting that $L_2'(z) \leq 0 \leq L_1'(z)$ (see Appendix A.2) and this property of $L_i(z)$ must hold for $z = \bar{z}_1, \bar{z}_2$, we have $\xi_1'(\bar{z}_1) \geq 0$ and $\xi_2'(\bar{z}_2) \leq 0$. This establishes the desired result.

A.4 Proof of Proposition 3

We first show the comparative statics for $\{\bar{z}_1, \bar{z}_2, \omega\}$ characterized by (2), (3) and (18). Regarding comparative statics with respect to $\overline{L}$, since the right-hand side of (18) is decreasing in ω, a rise in $\overline{L} \equiv \overline{L}_1/\overline{L}_2$ must decrease ω. Further, since the right-hand sides of (2) and (3) are increasing in ω, a rise in $\overline{L}$ must decrease $\bar{z}_1$ and $\bar{z}_2$. Regarding comparative statics with respect to $\mu(z)$, it follows from (2) and (3) that a proportional rise in $\mu(z)$ must increase ω. Further, since the right-hand sides of $\omega = \tau_{21}\mu(\bar{z}_1)$ and $\omega = \mu(\bar{z}_2)/\tau_{12}$ are increasing in $\mu(z)$, a proportional rise in $\mu(z)$ must decrease $\bar{z}_1$ and $\bar{z}_2$. This proves that $\frac{\partial \bar{z}_1}{\partial \overline{L}} \leq 0, \frac{\partial \bar{z}_2}{\partial \overline{L}} \leq 0, \frac{\partial \omega}{\partial \overline{L}} \leq 0, \frac{\partial \bar{z}_1}{\partial \mu(z)} \leq 0, \frac{\partial \bar{z}_2}{\partial \mu(z)} \leq 0, \frac{\partial \omega}{\partial \mu(z)} \geq 0$.

We next show the comparative statics for $\{\varphi^{*d}(z), \varphi^{*x}(z), B(z)\}$ characterized by (4), (5) and (6). Regarding comparative statics with respect to $\overline{L}$, consider a rise in $\overline{L}_1$ (while keeping $\overline{L}_2$ constant) and normalize $w_2 = 1$ as a numeraire of the model. Differentiating (4), (5) and (6) with respect to $\overline{L}_1$ gives us the following six equations:

$$\frac{\dot{B}_1(z)}{B_1(z)} - (\sigma - 1)\frac{\dot{w}_1}{w_1} + (\sigma - 1)\frac{\dot{\varphi}_{11}^*(z)}{\varphi_{11}^*(z)} = \frac{\dot{w}_1}{w_1}, \tag{A.14}$$

$$\frac{\dot{B}_2(z)}{B_2(z)} + (\sigma - 1)\frac{\dot{\varphi}_{22}^*(z)}{\varphi_{22}^*(z)} = 0, \tag{A.15}$$

$$\frac{\dot{B}_2(z)}{B_2(z)} - (\sigma - 1)\frac{\dot{w}_1}{w_1} + (\sigma - 1)\frac{\dot{\varphi}_{12}^*(z)}{\varphi_{12}^*(z)} = \frac{\dot{w}_1}{w_1}, \tag{A.16}$$

$$\frac{\dot{B}_1(z)}{B_1(z)} + (\sigma - 1)\frac{\dot{\varphi}_{21}^*(z)}{\varphi_{21}^*(z)} = 0, \tag{A.17}$$

$$\dot{\varphi}_{12}^*(z) = -C_1(z)\dot{\varphi}_{11}^*(z), \tag{A.18}$$

$$\dot{\varphi}_{21}^*(z) = -C_2(z)\dot{\varphi}_{22}^*(z), \tag{A.19}$$

where a dot is used to represent the derivative with respect to $\overline{L}_1$ (e.g., $\dot{B}_1(z) \equiv \frac{\partial B_1(z)}{\partial \overline{L}_1}$). Note that (A.14) – (A.19) are six equations with six unknowns $(\dot{\varphi}^*_{11}(z), \dot{\varphi}^*_{22}(z), \dot{\varphi}^*_{12}(z), \dot{\varphi}^*_{21}(z), \dot{B}_1(z), \dot{B}_2(z))$. Following the same steps in Appendix A.1, we can solve for

$$\dot{\varphi}^*_{11}(z) = -\frac{\frac{\sigma}{\sigma-1}\frac{\dot{w}_1}{w_1}\left(\frac{1}{\varphi^*_{22}(z)} + \frac{C_2(z)}{\varphi^*_{21}(z)}\right)}{\Xi(z)}, \quad \dot{\varphi}^*_{22}(z) = \frac{\frac{\sigma}{\sigma-1}\frac{\dot{w}_1}{w_1}\left(\frac{1}{\varphi^*_{11}(z)} + \frac{C_1(z)}{\varphi^*_{12}(z)}\right)}{\Xi(z)}.$$

From $\dot{w}_1 \leq 0$, we have $\dot{\varphi}^*_{11}(z) \geq 0, \dot{\varphi}^*_{22}(z) \leq 0$; and from (A.18) and (A.19), $\dot{\varphi}^*_{12}(z) \leq 0, \dot{\varphi}^*_{21}(z) \geq 0$. Further, from (8), we have $\dot{B}(z) \leq 0$. This proves that $\frac{\partial \varphi^*_{11}(z)}{\partial \overline{L}} \geq 0$, $\frac{\partial \varphi^*_{22}(z)}{\partial \overline{L}} \leq 0$, $\frac{\partial \varphi^*_{12}(z)}{\partial \overline{L}} \leq 0$, $\frac{\partial \varphi^*_{21}(z)}{\partial \overline{L}} \geq 0$, $\frac{\partial B(z)}{\partial \overline{L}} \leq 0$ and hence $\frac{\partial \varphi^{*d}(z)}{\partial \overline{L}} \geq 0$, $\frac{\partial \varphi^{*x}(z)}{\partial \overline{L}} \leq 0$.

Regarding comparative statics with respect to $\mu(z)$, consider a proportional rise in $\mu_1(z)$ (while keeping $\mu_2(z)$ constant) and normalize $w_2 = 1$. Differentiating (4), (5) and (6) with respect to $\mu_1(z)$ gives us the following six equations:

$$\frac{\ddot{B}_1(z)}{B_1(z)} + (\sigma-1)\frac{1}{\mu_1(z)} - (\sigma-1)\frac{\ddot{w}_1}{w_1} + (\sigma-1)\frac{\ddot{\varphi}^*_{11}(z)}{\varphi^*_{11}(z)} = \frac{\ddot{w}_1}{w_1}, \tag{A.20}$$

$$\frac{\ddot{B}_2(z)}{B_2(z)} + (\sigma-1)\frac{\ddot{\varphi}^*_{22}(z)}{\varphi^*_{22}(z)} = 0, \tag{A.21}$$

$$\frac{\ddot{B}_2(z)}{B_2(z)} + (\sigma-1)\frac{1}{\mu_1(z)} - (\sigma-1)\frac{\ddot{w}_1}{w_1} + (\sigma-1)\frac{\ddot{\varphi}^*_{12}(z)}{\varphi^*_{12}(z)} = \frac{\ddot{w}_1}{w_1}, \tag{A.22}$$

$$\frac{\ddot{B}_1(z)}{B_1(z)} + (\sigma-1)\frac{\ddot{\varphi}^*_{21}(z)}{\varphi^*_{21}(z)} = 0, \tag{A.23}$$

$$\ddot{\varphi}^*_{12}(z) = -C_1(z)\ddot{\varphi}^*_{11}(z), \tag{A.24}$$

$$\ddot{\varphi}^*_{21}(z) = -C_2(z)\ddot{\varphi}^*_{22}(z), \tag{A.25}$$

where a double dot is used to represent the derivative with respect to $\mu_1(z)$ (e.g., $\ddot{B}_1(z) \equiv \frac{\partial B_1(z)}{\partial \mu_1(z)}$). Solving (A.20) – (A.25), we have

$$\ddot{\varphi}^*_{11}(z) = \frac{\left(\frac{1}{\mu_1} - \frac{\sigma}{\sigma-1}\frac{\ddot{w}_1}{w_1}\right)\left(\frac{1}{\varphi^*_{22}(z)} + \frac{C_2(z)}{\varphi^*_{21}(z)}\right)}{\Xi(z)}, \quad \ddot{\varphi}^*_{22}(z) = -\frac{\left(\frac{1}{\mu_1(z)} - \frac{\sigma}{\sigma-1}\frac{\ddot{w}_1}{w_1}\right)\left(\frac{1}{\varphi^*_{11}(z)} + \frac{C_1(z)}{\varphi^*_{12}(z)}\right)}{\Xi(z)}.$$

From $\ddot{w}_1 \geq 0$, we have $\ddot{\varphi}^*_{11}(z) \geq 0, \ddot{\varphi}^*_{22}(z) \leq 0$ if $\frac{1}{\mu_1(z)} - \frac{\sigma}{\sigma-1}\frac{\ddot{w}_1}{w_1} \geq 0$, or equivalently

$$\frac{\mu(z)}{\omega}\frac{\partial \omega}{\partial \mu(z)} \leq \frac{\sigma-1}{\sigma}. \tag{A.26}$$

Note that not only is the right-hand side of (A.26) but the left-hand side of (A.26) is less than one, because a proportional rise in $\mu_1(z)$ (or $\mu(z)$) means that the relative wage increases proportionally short of an increase in relative labor productivity ($\frac{\partial \omega}{\omega} \leq \frac{\partial \mu(z)}{\mu(z)}$). Under (A.26), $\ddot{\varphi}^*_{11}(z) \geq 0, \ddot{\varphi}^*_{22}(z) \leq 0$, and from (A.24), (A.25) and (8), we have $\ddot{\varphi}^*_{12}(z) \leq 0, \ddot{\varphi}^*_{21}(z) \geq 0$, and $\ddot{B}(z) \leq 0$. This proves that $\frac{\partial \varphi^*_{11}(z)}{\partial \mu(z)} \geq 0$, $\frac{\partial \varphi^*_{22}(z)}{\partial \mu(z)} \leq 0$, $\frac{\partial \varphi^*_{12}(z)}{\partial \mu(z)} \leq 0$, $\frac{\partial \varphi^*_{21}(z)}{\partial \mu(z)} \geq 0$, $\frac{\partial B(z)}{\partial \mu(z)} \leq 0$ and hence $\frac{\partial \varphi^{*d}(z)}{\partial \mu(z)} \geq 0$, $\frac{\partial \varphi^{*x}(z)}{\partial \mu(z)} \leq 0$.

A.5 Proof of Proposition 4

We first show the derivation of (19). From $b_i(z) = \frac{R_i(z)}{w_i \overline{L}_i}$, aggregate market demand $B_i(z)$ is given by

$$B_i(z) = \frac{(\sigma-1)^{\sigma-1}}{\sigma^{\sigma}} b_i(z) w_i \overline{L}_i P_i(z)^{\sigma-1}.$$

Substituting this $B_i(z)$ into (4) and rearranging,

$$\left(\frac{\sigma-1}{\sigma}\frac{P_i(z)}{w_i}\mu_i(z)\varphi_{ii}^*(z)\right)^{\sigma-1} = \frac{\sigma f_{ii}}{b_i(z)\overline{L}_i}.$$

Solving this equality for $w_i/P_i(z)$ establishes the result.

We next show the impact of $\overline{L}$ on each country's welfare. To show this, note first that our Cobb-Douglas demand assumption makes the expenditure share $b_i(z)$ *constant* and thus any change in $\overline{L}$ does not affect $b_i(z)$. Then, applying $\frac{\partial \varphi_{11}^*(z)}{\partial \overline{L}} \geq 0$ and $\frac{\partial \varphi_{22}^*(z)}{\partial \overline{L}} \leq 0$ (see Appendix A.4) to (19) and combining this with the welfare expression establishes the result. The similar proof also applies for the impact of $\mu(z)$ on each country's welfare.

A.6 Proofs of Lemma 4

A.6.1 Proof of Equation (20)

We show the derivation of (20). Applying the Pareto distribution to (A.11) yields

$$L_i(z) = M_i^e(z)\left[\left(\frac{\varphi_{\min}}{\varphi_{ii}^*(z)}\right)^k \left(\frac{k\sigma}{k-(\sigma-1)}\right) f_{ii} + \left(\frac{\varphi_{\min}}{\varphi_{ij}^*(z)}\right)^k \left(\frac{k\sigma}{k-(\sigma-1)}\right) f_{ij}\right]. \tag{A.27}$$

Further, applying the Pareto distribution to (6) yields

$$\left(\frac{\varphi_{\min}}{\varphi_{ii}^*(z)}\right)^k \left(\frac{\sigma-1}{k-(\sigma-1)}\right) f_{ii} + \left(\frac{\varphi_{\min}}{\varphi_{ij}^*(z)}\right)^k \left(\frac{\sigma-1}{k-(\sigma-1)}\right) f_{ij} = f_i^e. \tag{A.28}$$

Substituting (A.28) into (A.27) and rearranging gives us equation (20).

A.6.2 Proof of Equation (21)

We show the derivation of (21). Regarding the extensive margins $M_{ii}(z), M_{ij}(z)$, it follows from the decomposition of aggregate domestic and export sales that

$$M_{ii}(z) = [1 - G(\varphi_{ii}^*(z))]M_i^e(z), \quad M_{ij}(z) = [1 - G(\varphi_{ij}^*(z))]M_i^e(z).$$

Applying the Pareto distribution to $G(\varphi)$ and substituting (20) into the above expressions gives us the result. Regarding the intensive margins $\bar{r}_{ii}(z), \bar{r}_{ij}(z)$, on the other hand, let us consider first the intensive margin of exporting $\bar{r}_{ij}(z)$. By definition, this intensive margin is given by

$$\begin{aligned}
\bar{r}_{ij}(z) &= \frac{1}{1-G(\varphi_{ij}^*(z))}\int_{\varphi_{ij}^*(z)}^{\infty} r_{ij}(\varphi,z)dG(\varphi) \\
&= \frac{1}{1-G(\varphi_{ij}^*(z))}B_j(z)\sigma\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{\sigma-1}V(\varphi_{ij}^*(z)) && \text{(using } V(\varphi)\text{)} \\
&= \left(\frac{\varphi_{ij}^*(z)}{\varphi_{\min}}\right)^k B_j(z)\sigma\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{\sigma-1}\frac{k\varphi_{\min}^k}{k-(\sigma-1)}\frac{1}{(\varphi_{ij}^*(z))^{k-(\sigma-1)}} && \text{(using Pareto)} \\
&= \frac{k\sigma}{k-(\sigma-1)}B_j(z)\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{\sigma-1}(\varphi_{ij}^*(z))^{\sigma-1} \\
&= \frac{k\sigma}{k-(\sigma-1)}B_j(z)\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{\sigma-1}\frac{1}{B_j(z)}\left(\frac{\mu_i(z)}{\tau_{ij}w_i}\right)^{1-\sigma}w_i f_{ij} && \text{(using (5))} \\
&= \frac{k\sigma}{k-(\sigma-1)}w_i f_{ij}.
\end{aligned}$$

By following the similar steps, it is easily confirmed that the intensive margin of domestic production $\bar{r}_{ii}(z)$ is given by

$$\bar{r}_{ii}(z) = \frac{k\sigma}{k-(\sigma-1)}w_i f_{ii}.$$

A.7 Proof of Asymmetric Trade Liberalization

We show the derivation of (29). As with the comparative statics with respect to $\overline{L}$ and $\mu(z)$, consider a decline in τ_{21} (while keeping τ_{12} constant) and normalize $w_2 = 1$. Differentiating (4), (5) and (6) with respect to τ_{21} and solving the resulting six equations,

$$\frac{\partial\varphi_{11}^*(z)}{\partial\tau_{21}} = \frac{-\frac{\sigma}{\sigma-1}\frac{\partial w_1/\partial\tau_{21}}{w_1}\left(\frac{1}{\varphi_{22}^*(z)}+\frac{C_2(z)}{\varphi_{21}^*(z)}\right)+\frac{1}{\tau_{21}\varphi_{22}^*(z)}}{\Xi(z)}, \tag{A.29}$$

$$\frac{\partial\varphi_{22}^*(z)}{\partial\tau_{21}} = \frac{\frac{\sigma}{\sigma-1}\frac{\partial w_1/\partial\tau_{21}}{w_1}\left(\frac{1}{\varphi_{11}^*(z)}+\frac{C_1(z)}{\varphi_{12}^*(z)}\right)-\frac{C_1(z)}{\tau_{21}\varphi_{21}^*(z)}}{\Xi(z)}. \tag{A.30}$$

From $\frac{\partial w_1}{\partial\tau_{21}} \geq 0$, we have $\frac{\partial\varphi_{11}^*(z)}{\partial\tau_{21}} \leq 0$ and $\frac{\partial\varphi_{22}^*(z)}{\partial\tau_{21}} \leq 0$ if

$$\left(\frac{\sigma-1}{\sigma}\right)\left(\frac{\varphi_{21}^*(z)}{\varphi_{21}^*(z)+C_2(z)\varphi_{22}^*(z)}\right) \leq \frac{\tau_{21}}{w_1}\frac{\partial w_1}{\partial\tau_{21}} \leq \left(\frac{\sigma-1}{\sigma}\right)\left(\frac{C_1(z)\varphi_{11}^*(z)}{\varphi_{12}^*(z)+C_1(z)\varphi_{11}^*(z)}\right).$$

From (A.7) the term in the left-hand side term is smaller than that in the right-hand side, and there is a range of parameterizations such that $\frac{\partial\varphi_{11}^*(z)}{\partial\tau_{21}} \leq 0, \frac{\partial\varphi_{22}^*(z)}{\partial\tau_{21}} \leq 0$ and, from the free entry condition, $\frac{\partial\varphi_{12}^*(z)}{\partial\tau_{21}} \geq 0, \frac{\partial\varphi_{21}^*(z)}{\partial\tau_{21}} \geq 0$. This proves that $\frac{\partial\varphi_{11}^*(z)}{\partial\tau} \leq 0$, $\frac{\partial\varphi_{22}^*(z)}{\partial\tau} \leq 0$, $\frac{\partial\varphi_{12}^*(z)}{\partial\tau} \geq 0$, $\frac{\partial\varphi_{21}^*(z)}{\partial\tau} \geq 0$ within some range of parameterizations. Note finally that if wages are exogenously fixed, (A.29) and (A.30) imply that $\frac{\partial\varphi_{11}^*(z)}{\partial\tau} \geq 0, \frac{\partial\varphi_{22}^*(z)}{\partial\tau} \leq 0$, and hence the liberalizing country always loses from a reduction in τ, as noted in Section 4.1.

Appendix B: Zero Gravity

In this Appendix, we provide the detailed analysis of zero gravity ($\tau_{ij} = \tau_{ji} = 1$) in Section 4.1, and relate it to the general case with the variable trade cost.

Following the literature, we say that country i has a comparative advantage in producing goods in sector z if country i's unit labor costs are less than or equal to country j's unit labor costs:

$$w_i a_i(z) \leq w_j a_j(z) \quad \Longleftrightarrow \quad \frac{w_i}{w_j} \leq \frac{\mu_i(z)}{\mu_j(z)}.$$

It follows immediately that country 1 (country 2) has a comparative advantage in high-z (low-z) sectors $\bar{z} \leq z \leq 1$ ($0 \leq z \leq \bar{z}$), where

$$\bar{z} \equiv \mu^{-1}(\omega).$$

As in the main text, the sectoral equilibrium is characterized by (4), (5) and (6) with $\tau_{ij} = \tau_{ji} = 1$. From the fact that the cutoff sector $\bar{z}$ is unique, the relative market demand $B(z) \equiv B_1(z)/B_2(z)$ is *strictly* log-submodular in Lemma 1. Formally, for $z < z'$,

$$\frac{B_1(z)}{B_2(z)} > \frac{B_1(z')}{B_2(z')}.$$

Since $B(z)$ is strictly log-submodular, $B(z)$ is strictly decreasing in z. The first quadrant of Figure B.1 depicts the relationship in (z, B) space.

Next, from the above result on $B(z)$, it also follows that the relative domestic productivity cutoff $\varphi^{*d}(z) \equiv \varphi^*_{11}(z)/\varphi^*_{22}(z)$ is *strictly* log-supermodular, whereas the relative export productivity cutoff $\varphi^{*x}(z) \equiv \varphi^*_{12}(z)/\varphi^*_{21}(z)$ is *strictly* log-submodular in Lemma 2. For $z < z'$,

$$\frac{\varphi^*_{11}(z)}{\varphi^*_{22}(z)} < \frac{\varphi^*_{11}(z')}{\varphi^*_{22}(z')}, \quad \frac{\varphi^*_{12}(z)}{\varphi^*_{21}(z)} > \frac{\varphi^*_{12}(z')}{\varphi^*_{21}(z')}.$$

In addition, it can be shown that $\varphi^{*d}(z) = \varphi^{*x}(z)$ and $B(z) = \tilde{\Gamma}$ for $z = \bar{z}$ where $\tilde{\Gamma} \equiv \sqrt{\frac{f_{11} f_{21}}{f_{22} f_{12}}}$. The second quadrant of Figure B.1 depicts the relationship between (B, φ^*) space.

Finally, combining the first and second quadrants of Figure B.1, we obtain the sectoral equilibrium characterized in terms of the relative market demand and relative productivity cutoffs:

$$\begin{aligned} 0 \leq z \leq \bar{z} \quad &\Longleftrightarrow \quad B(z) \geq \tilde{\Gamma} \quad \Longleftrightarrow \quad \varphi^{*x}(z) \geq \varphi^{*d}(z), \\ \bar{z} \leq z \leq 1 \quad &\Longleftrightarrow \quad B(z) \leq \tilde{\Gamma} \quad \Longleftrightarrow \quad \varphi^{*x}(z) \leq \varphi^{*d}(z). \end{aligned}$$

As in the case with $\tau_{ij} \neq 1, \tau_{ji} \neq 1$, the gap between $\varphi^*_{ij}(z)$ and $\varphi^*_{ii}(z)$ is relatively narrower than the gap between $\varphi^*_{ji}(z)$ and $\varphi^*_{jj}(z)$ in country i's comparative advantage sectors (see Figure 3).

As described above, if $\tau_{ij} = \tau_{ji} = 1$, the relative equilibrium variables $\{B(z), \varphi^{*d}(z), \varphi^{*x}(z)\}$ are *strictly* increasing or decreasing in z. Since the aggregate variables in Lemma 3 can be written as a function of $\{B(z), \varphi^{*d}(z), \varphi^{*x}(z)\}$, these aggregate variables are also *strictly* increasing or decreasing in z. In particular, $R_{12}(z)$ is *strictly* increasing in z whereas $R_{21}(z)$ is *strictly* decreasing in z.

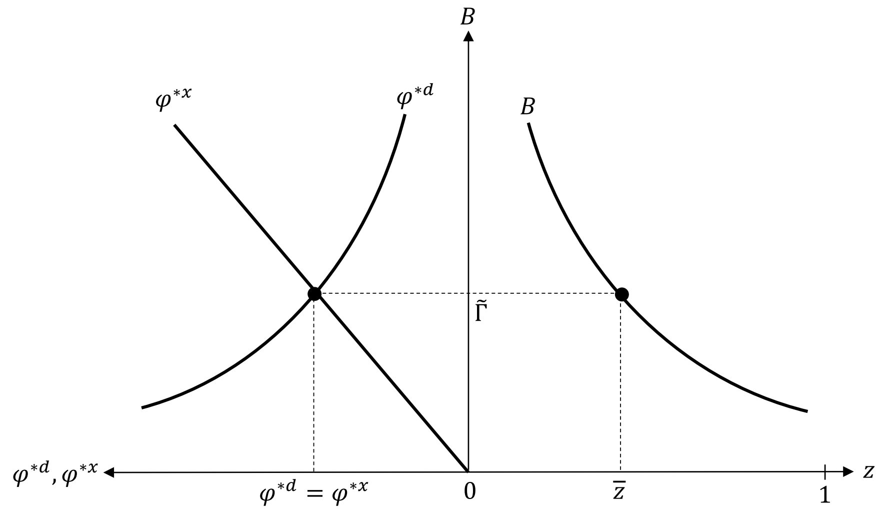


FIGURE B.1 – Market demand and productivity cutoffs in zero gravity

We next embed the sectoral equilibrium into general equilibrium. Let us define net exports from country 1 to country 2 as

$$NEXP(z) = R_{12}(z) - R_{21}(z).$$

If $\tau_{ij} = \tau_{ji} = 1$, $NEXP(z)$ is strictly increasing in z since $R_{12}(z)$ is strictly increasing in z and $R_{21}(z)$ is strictly decreasing in z. Let $\bar{z}$ denote the *hypothetical* cutoff sector in which $NEXP(\bar{z}) = 0$. From the fact that $\bar{z}$ is the unique cutoff sector in which net exports are zero in intra-industry trade, country 1 runs trade surplus (deficit) in $z > (<)\bar{z}$. Then, using $R_{ij}(z) - R_{ji}(z) = w_i L_i(z) - R_i(z)$,

$$\int_{\bar{z}}^{1} \left(w_1 L_1(z) - R_1(z)\right) dz = \int_{0}^{\bar{z}} \left(w_2 L_2(z) - R_2(z)\right) dz,$$

which can be solved for

$$\omega = \frac{\kappa_2(\bar{z}) - \lambda_2(\bar{z})}{\kappa_1(\bar{z}) - \lambda_1(\bar{z})} \left(\frac{\overline{L}_2}{\overline{L}_1}\right). \tag{B.1}$$

Another condition that pins down the equilibrium is

$$\omega = \mu(\bar{z}). \tag{B.2}$$

An in the main analysis, conditions (B.1) and (B.2) jointly determine the equilibrium variables $\{\bar{z}, \omega\}$, where $\bar{z}$ defines the equilibrium relative wage ω, and the cutoff sector $\bar{z}$ is special in that net exports are zero in intra-industry trade. Note that the logic is borrowed from DFS (1977, p.825-826); please refer to equations (10') and (11) in their paper. If we assume perfect competition, international trade allows all laborers to be allocated to comparative advantage sectors. This implies that $\kappa_i(\bar{z}) = 1$ and the equilibrium characterized by (B.1) and (B.2) is exactly the same as DFS (1977).

If $\tau_{ij} \neq 1, \tau_{ji} \neq 1$, there are the two cutoff sectors $\bar{z}_1, \bar{z}_2$ and $B(\bar{z}_1) = B(\bar{z}_2) = \Gamma$. Since $B(z)$ is decreasing in z (from Lemma 1), $B(z)$ must be *weakly* decreasing in z where $B(z) = \Gamma$ for $z \in [\bar{z}_1, \bar{z}_2]$.

Further, not only is $B(z)$ but also $\varphi^{*d}(z)$ and $\varphi^{*x}(z)$ are *weakly* increasing or decreasing in z where $B(z), \varphi^{*d}(z)$ and $\varphi^{*x}(z)$ are flat for $z \in [\bar{z}_1, \bar{z}_2]$ (see Figure 2). As a result, $R_{12}(z)$ is *weakly* increasing in z and $R_{21}(z)$ is *weakly* decreasing in z, and $NEXP(z) = R_{12}(z) - R_{21}(z)$ is *weakly* increasing in z where $NEXP(z)$ is flat for $z \in [\bar{z}_1, \bar{z}_2]$.

When embedding the sectoral equilibrium into general equilibrium, a similar argument with the zero-gravity case applies to the general case with variable trade costs. In particular, as shown in Section 3.3, the corresponding equations to (B.1) and (B.2) are

$$\omega = \frac{\kappa_2(\bar{z}_2) - \lambda_2(\bar{z}_2)}{\kappa_1(\bar{z}_1) - \lambda_1(\bar{z}_1)} \left(\frac{\overline{L}_2}{\overline{L}_1} \right),$$
$$\omega = \tau_{21}\mu(\bar{z}_1),$$
$$\omega = \frac{\mu(\bar{z}_2)}{\tau_{12}}.$$

These three equations jointly determine the equilibrium variables $\{\bar{z}_1, \bar{z}_2, \omega\}$, where $\bar{z}_1, \bar{z}_2$ define the equilibrium relative wage ω, and the interval sectors $z \in [\bar{z}_1, \bar{z}_2]$ are special in that net exports are zero in intra-industry trade. The logic is borrowed from DFS (1977, p.829-830); please refer to equations (19') and (21) in their paper. If we assume perfect competition, $\kappa_i(\bar{z}_i) = 1$ and the equilibrium characterized by the three equations is exactly the same as DFS (1977) as noted in Section 4.2.

Appendix C: Wages

The main analysis assumes that wages are the same across sectors but different across countries. In reality, however, there are a lot of empirical data available to suggest that wages differ across sectors. In order to justify the assumption, this Appendix shows that wages vary more across countries than across sectors within a country in empirical data.

To compare wages across countries and sectors, we use the World Input-Output Database (WIOD).[11] The WIOD contains the information on total labor compensation and the number of employees in different sectors across different countries, and we consider the U.S. and China as representatives of large countries that are applicable to our model, as in Section 3.7. We make use of the data denoted by LAB and EMP in the database for total labor compensation and the number of employees, mainly due to the data availability for the U.S. and China in 2014, which is the latest year of the database. Dividing LAB by EMP allows us to calculate per-capita wages (per-capita annual labor incomes more precisely). Furthermore, since LAB is measured in national currency, we use the average exchange rate of yuans to dollars in 2014, which is approximately 6.2, so that per-capita wages are measured in dollars for both countries. For simplicity, we focus on the following manufacturing sectors – textile (C13-C15), paper (C17), chemical (C20), and computer (C26) – within each country in 2014, where the number in brackets is the industry code of the WIOD, but the result continues to hold even if we consider different manufacturing sectors.

[11]The author is grateful to Hongyong Zhang for suggesting to refer to the WIOD.

Table C.1 – Per-capita wages between the U.S. and China in 2014

	Textile (C13-C15)	Paper (C17)	Chemical (C20)	Computer (C26)
U.S.	49,812	61,568	122,975	125,465
China	4,340	5,525	9,057	11,798

Table C.1 shows per-capita wages (measured in dollars) across different sectors between the U.S. and China. Contrary to our assumption, the wages are drastically different not only across countries but also across sectors. It is important to note, however, that these wages vary more across countries than across sectors within a country. For example, compare the wages in the textile and computer sectors in the U.S. and China. While the wages in the U.S. are approximately ten times bigger than the wages in China in both sectors, the wages in the computer sector are approximately three times bigger than the wages in the textile sector in both countries. This implies that the wage differences across countries are more significant than those across sectors within each country, rationalizing our assumption on the wages in the empirical data.

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