Global Firms: New Welfare Implications from Importing-Exporting*

Tomohiro Ara

Fukushima University

July 29, 2022

Abstract

This paper examines the role played by global firms that simultaneously import and export in welfare gains. In a setting of sequential production where final goods are produced with intermediate goods from different stages of production subject to selection into importing and exporting, we show that the presence of importing-exporting amplifies welfare gains from trade under an empirically observable condition: the market share of exporters conditional on also importers is greater than the market share of exporters in a general population. Under the condition that holds when importing and exporting exhibit complementarity, the standard effects of trade liberalization on aggregate outcomes are magnified through disproportionate share reallocations toward most efficient global firms.

Keywords: Global firms; welfare gains from trade; selection into importing and exporting; input-output linkages

JEL Classification Numbers: F12, F13, F16

*This study is conducted as a part of the Project "Economic Policy Issues in the Global Economy" undertaken at the Research Institute of Economy, Trade and Industry (RIETI). The author would like to thank the participants at the 80th Annual Meeting of the Japan Society of International Economics, RIETI and 2nd KAKENHI-NIESG Joint Workshop for useful comments and suggestions on an earlier version of the manuscript. Financial support from the Japan Society for the Promotion of Science under grant numbers 19K01599, 20H01492 and 20H01498 is gratefully acknowledged.

1 Introduction

Firms participate in the international economy not only as exporters of final goods but also as importers of intermediate goods from abroad, and understanding firms' joint import and export decisions has an important consequence for salient empirical patterns observed in most firm-level data. One of these patterns is *selection* into importing and exporting: not all exporters import and not all importers export, and aggregate trade is concentrated in a few number of global firms that simultaneously import and export (e.g., Bernard et al., 2018). Other studies exploring firms' global behavior as importers and exporters often document *complementarity* between importing and exporting: intensive exporters are also intensive importers, and they gain market share disproportionately to firms that export only or import only after trade shocks. The same line of research reports that such trade shocks are characterized by an increase in the import share of inputs in total input spending, enhancing the overall production efficiency in the economy. For example, Blaum (2019) finds evidence that, despite an adverse effect from the real exchange rate depreciation that makes foreign inputs expensive, large devaluations lead to a significant increase in the aggregate imported input share in Mexico, which can be fully explained by the fact that firms intensively engaging in both importing and exporting gain the market share by such trade shocks.

This paper examines the role played by global firms that simultaneously import and export in generating welfare gains from trade. We develop a model of sequential production in which firms use inputs from different stages in production by choosing markets from which to source inputs as well as to which to provide their final goods. We follow the previous literature on firm importing in assuming that intermediate inputs are produced under conditions of perfect competition, where each country produces a distinct variety of inputs.[1] Combining these inputs and labor, firms produce final goods under conditions of monopolistic competition with free entry. They are heterogeneous in productivity and firms' export decision is made based upon bearing a fixed export cost and an output transport cost per foreign market as in Melitz (2003), while firms' import decision is made upon bearing a fixed import cost and an input transport cost per foreign market as in Antràs et al. (2017). As a result, only a portion of more efficient firms engage in importing and exporting simultaneously. In this environment, we study the impact of endogenous selection into importing and exporting on the equilibrium, paying special attention to conditions under which the welfare gains from trade can be amplified in the model of importing-exporting relative to the model of exporting-only or importing-only.

We show that the presence of importing-exporting can magnify the impact on the welfare gains from trade, but this needs empirically observable moments that hold if importing and exporting exhibit complementarity: the market share of exporters conditional on also importers is greater than the market share of exporters in a general population.[2] Exporting allows firms to provide final goods to abroad which directly increases revenues, while importing allows firms to source inputs from abroad which indirectly increases revenues by improving output production efficiency. In this sense, there is complementarity between importing and exporting, which naturally raises the market share of exporters who are also importers relative to that of exporters who are not importers. Since the market share with different global status of firms is observable in the data, it is possible to check whether the market share condition is satisfied or not. We find, so long as our complementarity holds, that the standard effects of trade liberalization on the welfare gains are amplified by allowing most efficient global firms to gain the market share disproportionately. Such reallocations give rise to the magnification effect

[1] See for example Antràs et al. (2017), Bernard et al. (2018), and Tintelnot (2017) among others. The literature typically uses the Eaton-Kortum (2002) framework for sourcing inputs in a world of many potentially asymmetric countries, which means that intermediate inputs are produced competitively in different stages of production.

[2] This market share condition is shown to be a specific example of the more general case of "log-supermodularity" analyzed by Costinot (2009).

on welfare not only by increasing final goods but also by increasing inputs used in their output production. As a result, changes in the market share by trade liberalization lead to an increase in the aggregate imported input share as well as the aggregate imported output share.

In the model with importing-exporting where firms decide whether or not to import inputs and whether or not to export final goods, there are four types of global status of firms: purely-domestic firms, importers-only, exporters-only, and importer-exporters (often referred to as global firms). If trade costs are so high that there is selection into importing and exporting, our model highlights the impact of trade liberalization that has not been examined before. First, trade liberalization may have a negative impact on some of international firms. Specifically, when the number of trading countries is large enough, importers-only and exporters-only could suffer from trade liberalization, and the benefit of globalization is skewed toward importer-exporters. Due to this non-uniform effect across international firms, importer-exporters may also gain the profit share relatively more than importers-only and exporters-only. Second, when the market share satisfies the complementarity condition above, the trade elasticity becomes greater than that in the model of exporting-only or importing-only. Trade liberalization not only makes it easier for all exporters to ship final goods, but also makes it easier for all importers to source inputs from abroad. With the Armington-style input differentiation, such increased foreign inputs help improve output production efficiency by reducing importers' unit costs, which magnifies trade flows relative to the case where firms have no choice to import.

Our model also suggests that the modeling of input-output linkages give rise to quantitatively important implications for the welfare gains from trade. When such linkages are modeled with "roundabout" production – the modeling approach often employed in the literature to analyze welfare implications of tradable inputs, including Arkolakis et al. (2012), firms use the output of all other firms as intermediate inputs in production. Since final goods and intermediate goods are interchangeable, the domestic share includes these two types of goods and thus the welfare gains can be captured by that share and the trade elasticity derived from it. Then, relative to the baseline model without tradable inputs, the amplified effect on the welfare gains is captured by *exogenous* parameters governing the share of intermediate goods in production and entry costs. When input-output linkages are modeled with "sequential" production (e.g., Melitz and Redding, 2014b), in contrast, firms use inputs from different stages in production, and the domestic share is generally different in each production stage. Thus we need to distinguish the domestic share as well as the trade elasticity between input and output, both are *endogenously* different between the two types of goods. Then, the amplified effect on welfare changes is captured by the domestic share and the trade elasticity of input trade that independently enter the welfare expression. Together with complementarity between importing and exporting, this difference plays a key role in addressing whether welfare gains from trade are greater between these two production systems.

We develop a quantitative general equilibrium model that explains a firm's decision to serve foreign markets through different modes of market access. Our model builds on seminal work of Helpman et al. (2004) who study the firm's choice between exporting and foreign direct investment (FDI). Adding firm heterogeneity into a proximity-concentration tradeoff between these two choices, they find that only more productive firms find it profitable to engage in FDI while less productive firms choose exporting to serve foreign markets. This paper examines a similar tradeoff between exporting and importing-exporting where importing-exporting involves a higher fixed cost (associated with multiple international activities) but a lower variable cost (exploiting "love-of-variety" production). In doing so, we show not only that more productive firms find it profitable to engage in both importing and exporting while less productive firms choose either importing-only or exporting-only, but also that trade liberalization may induce importer-exporters to gain the market share disproportionately, which is crucial for understanding why welfare gains are amplified with importing-exporting.

A number of papers have analyzed a joint intersection of importing and exporting. Bernard et al. (2018) develop a model of importing-exporting by adopting the Eaton-Kortum (2002) framework for sourcing inputs. They find that, due to reinforcing connections between these activities, global firms have a large set of countries from which to import as in Antràs et al. (2017) and to which to export as in Eaton et al. (2011). While their model helps better understand why aggregate trade is dominated by a few number of global firms, they do not analyze any welfare implications that arise from the presence of these firms with multiple margins of trade.[3] The present paper instead investigates the mechanism through which endogenous selection into importing and exporting can magnify the welfare gains from trade.

In a setting of roundabout production, Blaum (2019) and Fieler et al. (2018) find that standard effects of trade on the economic outcomes are magnified through firms' joint import and export decisions. In particular, Blaum (2019) shows that trade shocks (devaluations) can have a positive impact on productivity and welfare in the model of importing-exporting, which stands in sharp contrast to the result in the model of importing-only. However, he considers a special case in which final goods are not traded and all firms' output is costly traded as intermediate goods. We employ sequential production in which trade in intermediate goods endogenously interacts with trade in final goods, which allows us to show selection into importing of intermediate goods and selection into exporting of final goods can jointly amplify the welfare gains from trade (relative to either one set of selection is available in the economy) in very clear manner.

This paper is also related to the welfare gains initiated by Arkolakis et al. (2012) with intermediate inputs. Using roundabout production, they analyze the welfare implication for tradable intermediate goods in which the term associated input trade are captured by some parameters governing the share of intermediate goods in production and entry costs. Their analytical results are extended by Costinot and Rodríguez-Clare (2014) to quantitatively address the welfare impact of tradable intermediates. Our welfare result can be thought of as the welfare formula of Arkolakis et al. (2012) in the model of importing-exporting with sequential production, in the sense that the welfare changes associated with trade costs are captured only by the domestic share and the trade elasticity. There is however one crucial difference. Since all firms are able to access imported inputs from abroad in their analysis, the "love-of-variety" production solely explains why the welfare gains from trade are greater in a world with trade in intermediate goods than in a world with only trade in final goods. Here, on top of that effect, there is selection into importing which allows a subset of more efficient firms to profitably import inputs from abroad. In other words, we emphasize that just like selection into exporting matters for welfare, selection into importing could also matter for welfare.[4] This makes the domestic share lower while the trade elasticity higher, so that the welfare gains from trade can be greater in a world with selection into importing than in a world with only selection into exporting.

Finally, our magnification effect applies to sequential production in which all goods markets are perfectly competitive. For example, using an Armington setting with sequential production, Melitz and Redding (2014b) show that the welfare gains become arbitrarily large as the number of production stages is arbitrarily large. The Armington assumption implies however that all firms import intermediate inputs, which generates the trade elasticity that consists only of the intensive margin. In contrast, our model with selection into importing generates the trade elasticity that consists also of the extensive margin, with different magnitudes of the two sufficient statistics. Hence, even if we consider sequential production, the welfare gains from trade would be different depending on whether there is selection among operating firms.

[3] This would be at least partly because trade liberalization could reduce the degree of competition by causing sales concentration toward such large firms, which makes welfare analysis more nuanced when there is strategic market power across a small number of global firms like theirs. In contrast, we shut down this channel by analyzing measure-zero firms in monopolistic competition.

[4] Antràs and Chor (2022) review the quantitative results in sequential production, but it relies on perfectly competitive models and the same disclaimer applies there.

2 Model

There are N potentially asymmetric countries indexed by i, j that use labor as a unique factor of production to produce goods in a number of sectors $s \in \{0, 1, ..., S\}$. Sector 0 produces a homogeneous good, which is produced with one unit of labor per unit output and freely traded across countries. The remaining sectors $s \geq 1$ produce differentiated goods using labor and intermediate inputs that are sourced from domestic and foreign markets within each sector. All inputs are produced with labor under constant-returns-to-scale technology in a perfectly competitive environment, and hence the input price equals the marginal cost of input production. Moreover, the unit labor requirement is normalized to 1 for every country.[5] Combining inputs with labor, all final goods are produced under increasing-returns-to-scale technology in a monopolistically competitive environment, and hence firms set the output price with markups over the marginal cost of output production. Moreover, the unit labor requirement (an inverse of firms' productivity introduced later) differs across firms. Each country produces a distinct variety of goods so that both firms and consumers benefit from accessing more intermediate goods and more final goods differentiated by country of origin.

A representative consumer in country i provides $\overline{L}_i$ units of labor. This consumer has a two-tier utility function, where the upper tier is Cobb-Douglas with a consumption share $0 \leq \beta_s \leq 1$ in sector s and the lower tier is Dixit-Stiglitz with elasticity of substitution $\varepsilon_s = 1/(1 - \alpha_s) > 1$ in sector $s \geq 1$.[6] We require that β_0 is large enough and the difference in $\overline{L}_i$ is small enough that all countries produce the homogeneous good and wages are equalized across countries. Choosing this good as a numeraire of the model, the common wage rate equals one $w_i = w = 1$, and labor income in country i coincides with labor endowments $\overline{L}_i$. For the moment, consider a particular sector that produces differentiated goods, and drop the sector index s from all relevant variables (unless needed) for notational simplicity.

To enter the sector in country i, a mass M_{Ei} of firms bear a fixed cost of entry f_E, upon which each firm draws productivity φ from a distribution $G_i(\varphi)$ with support in $[\underline{\varphi}_i, \infty)$. After observing this productivity, a firm decides whether to enter the sector by choosing markets from which to source inputs as well as to which to provide output. If the firm provides output to the domestic market using only domestic inputs, it bears a fixed overhead cost f_D. If it does so using both domestic and foreign inputs, it bears an additional fixed cost f_{DM} for searching foreign suppliers. The firm may also choose to serve the foreign market. If the firm exports using only domestic inputs, it bears an additional fixed cost f_X for forming a foreign network. If it does so using both domestic and foreign inputs, it bears an additional fixed cost f_{XM} for coordinating importing and exporting activities. All of these fixed costs are assumed to be the same across countries, but it is straightforward to allow for the difference. In addition to the fixed costs, the firm bears a melting-iceberg transport cost. We allow the transport cost to differ between inputs and outputs and to vary across country pairs. Importing inputs from country j to country i is subject to an input transport cost $\tau_{Mji} > 1$ for $j \neq i$ and $\tau_{Mii} = 1$; similarly, exporting output from country i to country j is subject to an output transport cost $\tau_{Xij} > 1$ for $i \neq j$ and $\tau_{Xii} = 1$. After entry, firms engage in monopolistic competition.

[5] This assumption is made for tractability. Antràs et al. (2017) assume that each country draws input production efficiency (an inverse of the unit labor requirement defined at country level) from the Fréchet distribution as in Eaton and Kortum (2002). As they stress, however, there is some isomorphism between their approach and the love-for-variety approach, where the latter is employed in this paper.

[6] Specifically, the representative consumer's preferences are given by

$$U_i = \sum_{s=0}^{S} \ln Q_{is}, \quad \sum_{s=0}^{S} \beta_s = 1,$$

where $Q_{is} = \left(\int_v q_{isv}^{\alpha_s} dv\right)^{1/\alpha_s}$ in sector $s \geq 1$. The associated price index is $P_{is}^{1-\varepsilon} = \int_v p_{isv}^{1-\varepsilon} dv$ and aggregate output expenditure is $R_{is} = \int_v p_{isv} q_{isv} dv$.

Production of output requires the assembly of a bundle of inputs. A firm's technology with productivity φ in country i is $q_i = \varphi x_i$ where q_i is the output quantity and x_i is the quantity of the input bundle, combining domestic inputs and foreign inputs under a CES production function with an elasticity of $\varepsilon = 1/(1-\alpha) > 1$:[7]

$$x_i = \left(z_{Di}^{\alpha} + x_{Mi}^{\alpha}\right)^{1/\alpha},$$

where z_{Di} is the domestic input quantity sourced from country i and x_{Mi} is the bundle of foreign inputs in country i which are combined under a CES production function with the same elasticity ε:

$$x_{Mi} = \left(\sum_{j\in n} z_{Mji}^{\alpha}\right)^{1/\alpha},$$

where z_{Mji} is the foreign input quantity sourced from country j and n is set of countries from which the firm sources inputs. We refer to this set as the firm's *sourcing strategy* hereafter. When the firm chooses markets from which to source inputs, it may not use foreign inputs because these inputs can be sourced only after bearing the fixed sourcing cost. As a result, the firm's sourcing strategy depends on the firm's productivity: the more productive the firm, the larger the firm's sourcing strategy (Antràs et al., 2017). Moreover, the sunk nature of the sourcing cost dictates the firm to adopt the same sourcing strategy across markets. If the firm uses both domestic and foreign inputs to serve the domestic market, for example, it uses both kinds of inputs to serve the foreign market. When the firm chooses markets to which to provide output, on the other hand, it may not export output to a foreign country due to the fixed export cost and the set of countries to which the firm provides output depends similarly on the firm's productivity. Since the firm decides whether or not to import inputs and whether or not to export output, there are four types of the firm's global status in this setting: a purely-domestic firm, an importer-only, an exporter-only, and an importer-exporter.

Consider now a firm that uses both domestic and foreign inputs. From input production technology and our choice of the numeraire, the domestic input price in every country equals 1, while the foreign input price imported from country j to country i equals τ_{Mji}, and thus the firm's expenditure to purchase input quantities is $e_i = z_{Di} + \sum_{j\in n} \tau_{Mji} z_{Mji}$ for a given sourcing strategy. Cost minimization subject to output production technology yields optimal input quantities from each country, which in turn yields the firm's marginal cost:

$$c_i = \frac{1}{\varphi}\left(1 + \sum_{j\in n} \tau_{Mji}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}.$$

Given the Armington-style input differentiation, it is not surprising to see that the marginal cost of the firm is lower than that of another firm that uses only domestic inputs. This love-of-variety effect, however, can be exploited by a subset of firms, which is endogenously determined in the model. It is useful for our analysis to define the firm's input expenditure share allocated to domestic inputs. Let $s_{Di} = e_{Di}/e_i$ denote the domestic input share defined at firm level where e_{Di} is the domestic input expenditure of that firm. From the optimal input quantities, this input share is given by

$$s_{Di} = \frac{1}{1 + \sum_{j\in n} \tau_{Mji}^{1-\varepsilon}}.$$

[7] We assume the same elasticity between firms' technology and consumers' preferences, but this would not qualitatively affect the key results of the paper.

Obviously, the domestic input share equals 1 if the firm uses only domestic inputs while the share is smaller than 1 if the firm uses both domestic and foreign inputs. Further, the firm's input share is decreasing in the set of sourcing countries n but is increasing in the input transport cost τ_{Mji}. Using this domestic input share, the firm's marginal cost can be alternatively expressed as

$$c_i = \frac{1}{\varphi}(s_{Di})^{\frac{1}{\varepsilon-1}}. \tag{1}$$

This shows that the firm using both domestic and foreign inputs is more efficient than the firm using only domestic inputs because the domestic input share is lower. The cost-reducing effect is more significant, the larger the set of sourcing countries and the lower the input transport cost.

The Cobb-Douglas upper tier of utility implies that the representative consumer spends $R_i = \beta\overline{L}_i$ on goods in the sector. As is well-known, the lower-tier Dixit-Stiglitz CES preferences generate a demand function $A_i p_i^{-\varepsilon}$ where $A_i = R_i P_i^{1-\varepsilon}$ is the demand level. If the firm in country i serves the domestic market, it sets the price equal to c_i/α where $1/\alpha$ is markups over the marginal cost c_i which depends on the firm's sourcing strategy. From (1), it directly follows that the domestic pricing rule for the firm using only domestic inputs is $1/\varphi\alpha$, while that for the firm using both domestic inputs and imported inputs is $(s_{Di})^{1/(\varepsilon-1)}/\varphi\alpha$. If the firm also serves the foreign market in country j, it sets the price equal to $\tau_{Xij}c_i/\alpha$ which is higher than the domestic pricing rule due to the output transport cost τ_{Xij}. The marginal cost depends on the firm's sourcing strategy in serving the foreign market as in the domestic market, but the firm is assumed to adopt the same sourcing strategy between the domestic and foreign markets, implying that the marginal cost at the factory gate is the same between domestic output and exported output. Under this assumption, the foreign pricing rule for the firm using only domestic inputs is $\tau_{Xij}/\varphi\alpha$, while that for the firm using both domestic and imported inputs is $\tau_{Xij}(s_{Di})^{1/(\varepsilon-1)}/\varphi\alpha$.

From the firm's domestic pricing rule, its domestic revenue is $B_i c_i^{1-\varepsilon}/(1-\alpha)$ where $B_i = (1-\alpha)A_i/\alpha^{1-\varepsilon}$ is the index of the market demand in country i which is composed of the price index and aggregate expenditure. Denoting this domestic revenue by r_{Di}, the firm's variable operating profits earned from the domestic market are $(1-\alpha)r_{Di}$. Further, noting the marginal cost (1) and subtracting the fixed cost, operating profits of the firm using only domestic inputs from the domestic market are

$$\pi_{Di} = B_i\varphi^{\varepsilon-1} - f_D.$$

If the firm uses foreign inputs, it incurs *additional* fixed costs nf_{DM} but reduces marginal cost (1) with $s_{Di} < 1$ (for a given n). Thus operating profits of the firm using both domestic and imported inputs are

$$\pi_{DMi} = \frac{B_i}{s_{Di}}\varphi^{\varepsilon-1} - f_D - nf_{DM}.$$

Among operating firms in the domestic market, some of more efficient firms serve the foreign market. If the firm exports, *additional* operating profits of the firm using only domestic inputs from the foreign market are

$$\pi_{Xij} = B_j\tau_{Xij}^{1-\varepsilon}\varphi^{\varepsilon-1} - f_X,$$

while *additional* operating profits of the firm using both domestic and foreign inputs (for a given n) are

$$\pi_{XMij} = \frac{B_j\tau_{Xij}^{1-\varepsilon}}{s_{Di}}\varphi^{\varepsilon-1} - f_X - nf_{XM}.$$

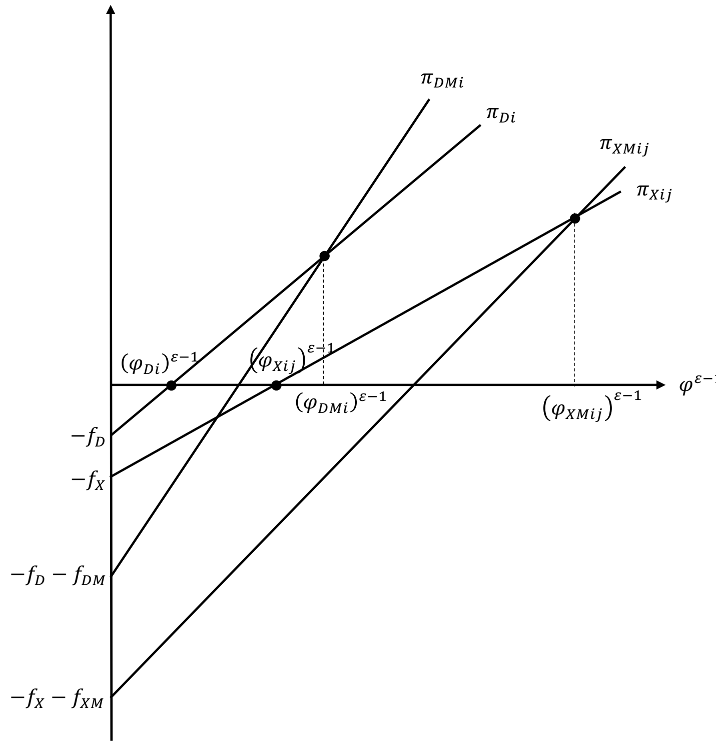

Figure 1 – Operating profits from domestic and foreign markets

While operating profits from the domestic market π_{Di}, π_{DMi} are measured by country i's market demand B_i, those from the foreign market π_{Xij}, π_{XMij} are measured by country j's market demand B_j. The difference in the market demand levels stems from the difference in country size (in terms of the labor endowments $\overline{L}_i, \overline{L}_j$) that endogenously affects both the price index and aggregate expenditure. Note that the domestic input share s_{Di} is the same between the domestic and foreign markets, reflecting the fact that the firm takes the same sourcing strategy between the different markets.

Figure 1 depicts the profit functions for the case in which there are two countries (and hence the sourcing strategy is limited to $n = 1$) and the market demand levels are the same between countries i and j. In the figure, the horizontal axis measures productivity while the vertical axis measures operating profits for different global status of firms. The slopes of π_{Di} and π_{DMi} are B_i and B_i/s_{Di} respectively, which reflects that variable operating profits are greater for importers than for non-importers. At the same time, the intercepts of π_{Di} and π_{DMi} are $-f_D$ and $-f_D - f_{DM}$ respectively, and the fixed cost is greater for importers than for non-importers. From this tradeoff, there is a productivity cutoff $\varphi_{DMi}^{\varepsilon-1}$ at which operating profits of importers exceed those of non-importers. There is another cutoff $\varphi_{Di}^{\varepsilon-1}$ at which the least productive firm finds it profitable to produce for the domestic market. These cutoffs show the sorting pattern such that more productive firms above $\varphi_{DMi}^{\varepsilon-1}$ use both domestic and foreign inputs while less productive others between $\varphi_{Di}^{\varepsilon-1}$ and $\varphi_{DMi}^{\varepsilon-1}$ use only domestic inputs to serve the domestic market in country i. In a similar fashion, the slopes of π_{Xij} and π_{XMij} are $B_j\tau_{ij}^{1-\varepsilon}$ and $B_j\tau_{ij}^{1-\varepsilon}/s_{Di}$ respectively, which are smaller than the slopes of π_{Di} and π_{DMi} under $B_i = B_j$. There are also productivity cutoffs, $\varphi_{Xij}^{\varepsilon-1}, \varphi_{XMij}^{\varepsilon-1}$, such that more productive exporters above $\varphi_{XMij}^{\varepsilon-1}$ use both domestic and foreign inputs while less productive exporters between $\varphi_{Xij}^{\varepsilon-1}$ and $\varphi_{XMij}^{\varepsilon-1}$ use only domestic inputs to serve the foreign market in country j.

It follows from Figure 1 that the domestic productivity cutoffs $\varphi_{Di}^{\varepsilon-1}$ and $\varphi_{DMi}^{\varepsilon-1}$ are determined at which $\pi_{Di} = 0$ and $\pi_{Di} = \pi_{DMi}$ in the domestic market in country i, while the export productivity cutoffs $\varphi_{Xij}^{\varepsilon-1}$ and $\varphi_{XMij}^{\varepsilon-1}$ are determined at which $\pi_{Xij} = 0$ and $\pi_{Xij} = \pi_{XMij}$ in the foreign market in country j. Using the domestic input share s_{Di} in the operating profits above, these productivity cutoffs are determined by

$$\begin{aligned} B_i \varphi_{Di}^{\varepsilon-1} &= f_D, \\ \left(\frac{1-s_{Di}}{ns_{Di}}\right) B_i \varphi_{DMi}^{\varepsilon-1} &= f_{DM}, \\ B_j \tau_{Xij}^{1-\varepsilon} \varphi_{Xij}^{\varepsilon-1} &= f_X, \\ \left(\frac{1-s_{Di}}{ns_{Di}}\right) B_j \tau_{Xij}^{1-\varepsilon} \varphi_{XMij}^{\varepsilon-1} &= f_{XM}. \end{aligned} \tag{2}$$

The first and third conditions that pin down φ_{Di} and φ_{Xij} are similar to those in the heterogeneous firm model with exporting-only, but the second and forth conditions that pin down φ_{DMi} and φ_{XMij} are specific to the current model with importing-exporting, assigning the value of the productivity cutoffs for a *given* sourcing strategy n. If there are more than two countries (and hence the firm's sourcing strategy is $n \geq 2$), there are in general multiple sets of φ_{DMi} and φ_{XMij}. For example, if there are three countries (and the sourcing strategy can be $n = 1, 2$), some less efficient firms would source foreign inputs from only one foreign country ($n = 1$), while other more efficient firms would source foreign inputs from all foreign countries ($n = 2$). In that sense, the firm's sourcing strategy is endogenously determined by the firm's productivity level.

To analyze the equilibrium of the model of importing-exporting, we need to make some restrictions on the exogenous variables that affect the productivity cutoffs. First, from empirical evidence that a small fraction of firms can access imported inputs (e.g., Kasahara and Lapham 2013; Halpern et al. 2015), we assume that the import productivity cutoffs are larger than the non-import productivity cutoffs ($\varphi_{DMi}^{\varepsilon-1} > \varphi_{Di}^{\varepsilon-1}, \varphi_{XMij}^{\varepsilon-1} > \varphi_{Xij}^{\varepsilon-1}$). Noting (2), these inequalities hold if and only if

$$\min\left\{\frac{nf_{DM}}{f_D}, \frac{nf_{XM}}{f_X}\right\} > \frac{1-s_{Di}}{s_{Di}}. \tag{3}$$

(3) is selection into importing which holds for the domestic and foreign markets; thus not all exporters import. We further assume that the export productivity cutoffs are larger than the non-export productivity cutoffs ($\varphi_{Xij}^{\varepsilon-1} > \varphi_{Di}^{\varepsilon-1}, \varphi_{XMij}^{\varepsilon-1} > \varphi_{DMi}^{\varepsilon-1}$), which hold if and only if

$$\min\left\{\frac{\tau_{Xij}^{\varepsilon-1} f_X}{f_D}, \frac{\tau_{Xij}^{\varepsilon-1} f_{XM}}{f_{DM}}\right\} > \frac{B_j}{B_i}. \tag{4}$$

(4) is selection into exporting which holds regardless of import status of firms; thus not all importers export. These selection patterns accord with empirical evidence that selection is ubiquitous for importing as well as exporting (Bernard et al., 2007, 2012, 2018). These two conditions are satisfied when trade costs are sufficiently large and country size – which endogenously affects the right-hand side of (3) and (4) – is sufficiently similar. Under these conditions, the productivity cutoffs satisfy $\varphi_{Di} < \min\{\varphi_{DMi}, \varphi_{Xij}\} < \varphi_{XMij}$, which shows that among operating firms in country i, the least productive firms with $\varphi < \min\{\varphi_{DMi}, \varphi_{Xij}\}$ are purely-domestic while the most productive firms with $\varphi > \varphi_{XMij}$ simultaneously import and export. The remaining firms with intermediate productivity engage in either importing-only or exporting-only under the assumption that the firm's sourcing strategy is the same between the domestic and foreign markets.

Free entry requires that the expected operating profits of a potential entrant equalize the fixed entry cost. Given our assumption that firms observe their productivity only after bearing the fixed cost of entry, the free entry condition in country i is defined as

$$\int_{\varphi_{Di}}^{\infty}\left(\frac{B_i}{s_{Di}}\varphi^{\varepsilon-1}-f_D-nf_{DM}\right)dG_i(\varphi)+\sum_{j\neq i}\int_{\varphi_{Xij}}^{\infty}\left(\frac{B_j\tau_{Xij}^{1-\varepsilon}}{s_{Di}}\varphi^{\varepsilon-1}-f_X-nf_{XM}\right)dG_i(\varphi)=f_E, \tag{5}$$

where the first (second) term denotes the expected profits from the domestic (foreign) market. To understand the first term, firms with productivity $\varphi<\varphi_{Di}$ cannot earn positive profits by choosing any sourcing strategy and exit immediately without producing. Among domestic firms, those with productivity $\varphi_{Di}<\varphi<\varphi_{DMi}$ serve the domestic market by using only domestic inputs ($n=0, s_{Di}=1$), while those with $\varphi>\varphi_{DMi}$ do so by using both domestic and foreign inputs ($n\geq 1, s_{Di}<1$); further, more efficient firms choose the larger set of importing countries, i.e., the sourcing strategy n (and the domestic input share s_{Di}) depends on productivity. Under selection into exporting (4), a similar interpretation applies to the second term in (5).

The equilibrium is defined by the set of the zero cutoff profit condition (2) and the free entry condition (5), which jointly provide implicit solutions of the productivity cutoffs $\varphi_{Di},\varphi_{DMi},\varphi_{Xij},\varphi_{XMij}$ and the market demand B_i in every country. Once these unknowns are determined, other endogenous variables can be written as a function of them. In particular, welfare per worker in country i is given by (see Appendix A.1):

$$W_i=\prod_{s=0}^{S}\left(\left(\frac{(1-\alpha_s)f_{Ds}}{\beta_s\overline{L}_i}\right)^{\frac{1}{\varepsilon_s-1}}\frac{1}{\alpha_s\varphi_{Dis}}\right)^{-\beta_s}. \tag{6}$$

This welfare expression shows that the domestic productivity cutoff is a single sufficient statistic for welfare in the sense that changes in welfare can be inferred only from changes in that cutoff which is directly related to resource reallocations by Melitz (2003). In other words, the welfare property of the heterogeneous firm model holds true even in the model with importing-exporting just like the standard trade model with exporting-only. Later we relate this unobservable cutoff with empirically observable objects, which help quantify the difference in the welfare gains from trade between the different trade models.

Note that if inputs are prohibitively costly and no firm profitably imports, the import productivity cutoffs approach to $\varphi_{DMi}=\varphi_{XMi}\to\infty$ (from $\tau_{Mji}\to\infty$ in (2)) and our model collapses to an asymmetric-country version of Melitz (2003). On the other hand, if outputs are prohibitively costly and no firm profitably exports, the export productivity cutoffs approach to $\varphi_{Xi}=\varphi_{XMi}\to\infty$ (from $\tau_{Xij}\to\infty$) and our model collapses to a simplified version of Antràs et al. (2017) where all countries have the same unit labor requirement for input production (and the fixed sourcing costs are common for all foreign countries).

2.1 Special Case: Symmetric Countries

To provide intuition for new mechanisms in the model, we consider a special case of country symmetry. Assume that all production and trade costs are the same between every country-pair within each sector. Every country has the same labor endowment $\overline{L}$, though this assumption can be relaxed and the following results would hold in the presence of country-size differences.[8] Finally, the productivity distribution is the same across countries with common support in $[\underline{\varphi},\infty)$.

[8]Under CES preferences that generate exogenously fixed markups, the differences in country size affect only the mass of entrants (and thus the number of varieties consumed) but does not affect any firm-level variables at all, including the productivity cutoffs. Allowing for such differences leads to the home market effect in entry pattens so that large countries entail disproportionate entry; see Helpman et al. (2004) for this kind of analysis.

Imposing these restrictions on the equilibrium system of (2) and (5), we have the same productivity cutoffs $\varphi_{Di} = \varphi_D, \varphi_{DMi} = \varphi_{DM}, \varphi_{Xij} = \varphi_X, \varphi_{XMij} = \varphi_{XM}$ and the same market demand level $B_i = B$ in i, j. In this special case, the marginal cost of a firm is the same in every country $c = (s_D)^{\frac{1}{\varepsilon-1}}/\varphi$. Moreover, the firm imports all foreign countries, the firm's sourcing strategy is $n = N-1$ and hence the domestic input share is $s_D = 1/(1+(N-1)\tau_M^{1-\varepsilon})$. Finally, since the sourcing strategy is made after incurring the fixed sourcing costs $(N-1)f_{DM}, (N-1)f_{XM}$, condition (3) that requires selection into importing becomes

$$\min\left\{\frac{\tau_M^{\varepsilon-1}f_{DM}}{f_D}, \frac{\tau_M^{\varepsilon-1}f_{XM}}{f_X}\right\} > 1,$$

which is very similar to condition (4) that requires selection into exporting under $B_i = B_j$. We assume that trade costs are high enough to satisfy new condition (3) as well as (4).

The equilibrium is defined by the set of new zero cutoff profit conditions and a new free entry condition. Imposing the symmetry of the variables, the zero profit cutoff conditions are now expressed as

$$\begin{aligned} B\varphi_D^{\varepsilon-1} &= f_D, \\ B\tau_M^{1-\varepsilon}\varphi_{DM}^{\varepsilon-1} &= f_{DM}, \\ B\tau_X^{1-\varepsilon}\varphi_X^{\varepsilon-1} &= f_X, \\ B(\tau_X\tau_M)^{1-\varepsilon}\varphi_{XM}^{\varepsilon-1} &= f_{XM}. \end{aligned} \tag{7}$$

Further, since all firms with productivity $\varphi > \varphi_{DM}$ and $\varphi > \varphi_X$ import from and export to all $(N-1)$ foreign countries in this special case, the free entry condition is expressed as (see Appendix A.2):

$$f_D J(\varphi_D) + (N-1)f_{DM}J(\varphi_{DM}) + (N-1)f_X J(\varphi_X) + (N-1)^2 f_{XM}J(\varphi_{XM}) = f_E, \tag{8}$$

where $J(\varphi_c) = \int_{\varphi_c}^{\infty}\left[\left(\frac{\varphi}{\varphi_c}\right)^{\varepsilon-1} - 1\right]dG(\varphi)$ is a monotonically decreasing function of φ_c. These five equilibrium conditions in (7)-(8) jointly pin down the five unknowns in the country-symmetric case: four symmetric cutoffs $\varphi_D, \varphi_{DM}, \varphi_X, \varphi_{XM}$ and the one symmetric market demand B. Having solved for the productivity cutoffs and the market demand level in every country, we can then determine other endogenous variables of the model. In this environment, the welfare property in (6) continues to hold in that the domestic productivity cutoff φ_D is single sufficient statistic for welfare.

Below we calculate the revenue shares in the domestic market. These shares not only are written in terms of the five unknowns, but also are classified in terms of global status of firms. We find it useful to derive such shares, because they are observable objects in the data and thus can be used to empirically test the impact of trade liberalization in the model of importing-exporting. For that purpose, let r_D/R denote a domestic firm's revenue share in the domestic market, and let σ_D (σ_{DM}) denote all non-importers' (all importers') revenue share in the same market. From the sorting pattern of domestic firms in the domestic market, σ_D (σ_{DM}) is obtained by aggregating r_D/R over the productivity range between φ_D and φ_{DM} (above φ_{DM}).[9] Similarly, let r_X/R denote an exporter's revenue share in the domestic market, and let σ_X (σ_{XM}) denote all exporters' (all exporters-importers') revenue share in that domestic market, which can be also calculated from the souring patterns of exporters in the domestic market. Noting that aggregate output expenditure equals labor income

[9] From the domestic revenues, each firm's revenue share is $B\varphi^{\varepsilon-1}/(1-\alpha)R$ for those with productivity $\varphi \in (\varphi_D, \varphi_{DM})$ and $B(1+(N-1)\tau_M^{1-\varepsilon})\varphi^{\varepsilon-1}/(1-\alpha)R$ for those with productivity $\varphi \in (\varphi_{DM}, \infty)$ in every country. Aggregating these revenues over the respective productivity range and noting that there is a mass M_E of entrants, $\sigma_D = M_E\int_{\varphi_D}^{\varphi_{DM}} B\varphi^{\varepsilon-1}dG(\varphi)/(1-\alpha)R$ and $\sigma_{DM} = M_E\int_{\varphi_{DM}}^{\infty} B(1+(N-1)\tau_M^{1-\varepsilon})\varphi^{\varepsilon-1}dG(\varphi)/(1-\alpha)R$.

$R = \beta\overline{L}$ under free entry and using the productivity cutoff φ_c in (7), we get

$$\begin{aligned}
\sigma_D &= \frac{B[V(\varphi_D) - V(\varphi_{DM})]}{(1-\alpha)\beta}\frac{M_E}{\overline{L}}, \\
\sigma_{DM} &= \frac{B(1+(N-1)\tau_M^{1-\varepsilon})V(\varphi_{DM})}{(1-\alpha)\beta}\frac{M_E}{\overline{L}}, \\
\sigma_X &= \frac{B\tau_X^{1-\varepsilon}[V(\varphi_X) - V(\varphi_{XM})]}{(1-\alpha)\beta}\frac{(N-1)M_E}{\overline{L}}, \\
\sigma_{XM} &= \frac{B\tau_X^{1-\varepsilon}(1+(N-1)\tau_M^{1-\varepsilon})V(\varphi_{XM})}{(1-\alpha)\beta}\frac{(N-1)M_E}{\overline{L}},
\end{aligned} \tag{9}$$

where $V(\varphi_c) = \int_{\varphi_c}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)$ is a monotonically decreasing function of φ_c. It is verified that these shares sum up to 1 and σ_c is thought of as the revenue share in the domestic market where subscript $c \in \{D, DM, X, XM\}$ denotes the firms' global status.

We are interested in how the revenue shares in (9) are affected by trade shocks in the presence of importing-exporting. While the impact of trade liberalization on these shares will be examined in the next section, the following property of these shares is worth observing for now. Using (9), the relative revenue share of importers among domestic firms is given by σ_{DM}/σ_D, and that share among exporters is given by σ_{XM}/σ_X. From their observability, it is possible to compare the relative magnitude of these relative revenue shares in the data. For example, Blaum (2019) finds that firms intensively engaging in both importing and exporting gain the market share by trade shocks (devaluations). In terms of the relative revenue shares, this finding suggests that firms engaging in both importing and exporting have a relatively larger revenue share than firms engaging in one of these activities. We thus assume for any level of trade costs that

$$\frac{\sigma_{XM}}{\sigma_X} > \frac{\sigma_{DM}}{\sigma_D}. \tag{10}$$

Substituting (9) into the inequality, we know that (10) holds if and only if $V(\varphi_{XM})/V(\varphi_X) > V(\varphi_{DM})/V(\varphi_D)$. This condition would arise when importing and exporting exhibit complementarity: firms that simultaneously import and export have the revenue share relatively more than firms that either import or export.

To help appreciate this meaning, we show that complementarity in (10) is the same as "log-supermodularity," one particular form of complementarity introduced by Costinot (2009) into the trade literature. Formally, for $x_1' \geq x_1''$, $x_2' \geq x_2''$, log-supermodularity of g in (x_1, x_2) implies that $g(x_1', x_2') \cdot g(x_1'', x_2'') > g(x_1', x_2'') \cdot g(x_1'', x_2')$.[10] If g is strictly positive, this inequality can be rearranged as

$$\frac{g(x_1', x_2')}{g(x_1'', x_2')} > \frac{g(x_1', x_2'')}{g(x_1'', x_2'')}.$$

The concept is directly applicable to our setting. Let the revenue shares with different global status of firms be $\sigma_{XM} = \sigma(\tau_M, \tau_X)$, $\sigma_X = \sigma(0, \tau_X)$, $\sigma_{DM} = \sigma(\tau_M, 0)$, $\sigma_D = \sigma(0, 0)$ where the first argument denotes whether firms use imported input by incurring τ_M and the second argument denotes whether firms ship their output to abroad by incurring τ_X (for given firms' global status). Using this notation, we find that the revenue shares in (10) satisfy the log-supermodularity condition:

$$\frac{\sigma(\tau_M, \tau_X)}{\sigma(0, \tau_X)} > \frac{\sigma(\tau_M, 0)}{\sigma(0, 0)}.$$

[10]See Costinot (2009, pp.1169). Throughout the analysis, we assume *strict* log-supermodularity with a *strict* inequality.

Thus, (10) implies that firms that simultaneously import and export have the revenue share relatively more than those that do either one of these activities; the formal counterpart to this statement is that the revenue share is log-supermodular in importing and exporting. For comparison, we refer to $\sigma_{XM}/\sigma_X = \sigma_{DM}/\sigma_D$ as *no complementarity* between importing and exporting, which means that these two international activities have independent effects on the revenue share.

We often assume that firm productivity φ is distributed Pareto with a shape parameter k. Under such a specific parameterization, (10) has another implication. Recall that the inequality in (10) holds if and only if $V(\varphi_{XM})/V(\varphi_X) > V(\varphi_{DM})/V(\varphi_D)$. Since $V(\varphi_c)/V(\varphi_{c'}) = (\varphi_{c'}/\varphi_c)^{k-(\varepsilon-1)}$ for any productivity cutoffs φ_c, φ_c' which are given in (7), the condition can be expressed as

$$\frac{f_{DM}}{f_D} > \frac{f_{XM}}{f_X}.$$

This means that when a firm is already an importer, becoming an exporter does not entail full payments of the fixed export costs, but rather a smaller amount of such costs, i.e., complementarity between importing and exporting that happens via a reduction in the fixed costs.

We can express the domestic output share, denoted by λ_D, in terms of the revenue shares. This share is a sum of the revenue shares of all domestic firms in the domestic market, aggregating r_D/R over the productivity range above φ_D. By definition, $\lambda_D = \sigma_D + \sigma_{DM}$. Dividing by $\sum_c \sigma_c = 1$ and rearranging,

$$\lambda_D = \frac{1}{1 + \left(\frac{\sigma_X(1+\sigma_{XM}/\sigma_X)}{\sigma_D(1+\sigma_{DM}/\sigma_D)}\right)}. \tag{11}$$

Inspection of (9) and (11) reveals immediately that the domestic share λ_D depends only on the productivity cutoff φ_c and is independent of other key endogenous variables (such as the market demand B and the mass of entrants M_E), which proves convenient for the equilibrium analysis. In addition, if there is complementarity in (10), the domestic share is lower in the economy with importing-exporting than with exporting-only. If there is no complementarity, however, the domestic share collapses to that in the standard heterogeneous firm model without importing.[11]

It is also possible to calculate the input expenditure shares in the domestic market. Let $\tilde{\sigma}_D$ ($\tilde{\sigma}_X$) denote all non-exporters' (all exporters') expenditure share in aggregate expenditure in the domestic market, while let $\tilde{\sigma}_{DM}$ ($\tilde{\sigma}_{XM}$) denote all importers' (all importer-exporters') expenditure share in aggregate expenditure in that market. From firms' input demand z_D, z_M, this share is derived similarly to the revenue share σ_c. Noting that aggregate expenditure is a fraction of labor income $E = \alpha\beta\overline{L}$ under free entry, we get (see Appendix A.3)

$$\begin{aligned}
\tilde{\sigma}_D &= \frac{BV(\varphi_D)}{(1-\alpha)\beta}\frac{M_E}{\overline{L}}, \\
\tilde{\sigma}_X &= \frac{B(N-1)\tau_X^{1-\varepsilon}V(\varphi_X)}{(1-\alpha)\beta}\frac{M_E}{\overline{L}}, \\
\tilde{\sigma}_{DM} &= \frac{B(N-1)\tau_M^{1-\varepsilon}V(\varphi_{DM})}{(1-\alpha)\beta}\frac{M_E}{\overline{L}}, \\
\tilde{\sigma}_{XM} &= \frac{B(N-1)^2(\tau_X\tau_M)^{1-\varepsilon}V(\varphi_{XM})}{(1-\alpha)\beta}\frac{M_E}{\overline{L}},
\end{aligned} \tag{12}$$

[11] This follows from that $\lambda_D = \frac{1}{1+\frac{\sigma_X}{\sigma_D}}$ with no complementarity. See, for example, equation (18) in Melitz and Redding (2015) where no firm can access foreign inputs ($\tau_M \to \infty$ and hence $\varphi_{DM} = \varphi_{XM} \to \infty$).

where $\sum_c \tilde{\sigma}_c = 1$. This expenditure share can be shown to satisfy the same property as the revenue share σ_c, in the sense that $\tilde{\sigma}_{XM}/\tilde{\sigma}_{DM} > \tilde{\sigma}_X/\tilde{\sigma}_D$ if and only if $V(\varphi_{XM})/V(\varphi_{DM}) > V(\varphi_X)/V(\varphi_D)$. Thus, so long as (10) holds, there is also complementarity between importing and exporting in the expenditure share. From this reason, we refer to $\tilde{\sigma}_{XM}/\tilde{\sigma}_{DM} = \tilde{\sigma}_X/\tilde{\sigma}_D$ as no complementarity. Moreover, the domestic input share is $\tilde{\sigma}_D + \tilde{\sigma}_X$. After some manipulation, the domestic input share is expressed in terms of $\tilde{\sigma}_c$:

$$\delta_D = \frac{1}{1 + \left(\frac{\tilde{\sigma}_{DM}(1+\tilde{\sigma}_{XM}/\tilde{\sigma}_{DM})}{\tilde{\sigma}_D(1+\tilde{\sigma}_X/\tilde{\sigma}_D)}\right)}. \tag{13}$$

In contrast to s_D defined at firm level, this domestic input share is defined at country level. From the similarity between (11) and (13), it follows that if there is complementarity, the domestic input share is lower in the economy with importing-exporting than with importing-only.

3 Impact of Trade

This section examines the impact of trade liberalization on equilibrium in the model of importing-exporting. Throughout the section, we focus on reductions in variable trade costs that take place proportionately between input trade and output trade (i.e., $d\tau_M = d\tau_X \equiv d\tau$), which would be suitable to the analysis of transport costs where globalization equally reduces trade costs of the two types of goods. Although not presented below, we can explore the impact of trade liberalization of input and output separately, which would be more suitable to the analysis of tariffs.

3.1 Resource Reallocations

We start the analysis by examining the impact on the productivity cutoffs. Differentiating the two equilibrium conditions (7) and (8) with respect to variable trade costs and solving these simultaneously yields the following relationships between the productivity cutoffs and variable trade costs without restricting the firm distribution function to a particular parameterization (see Appendix A.4):

$$\begin{aligned} d\ln\varphi_D &= -\Big[(1-\lambda_D) + (1-\delta_D)\Big] d\ln\tau, \\ d\ln\varphi_{DM} = d\ln\varphi_X &= (-1+\lambda_D+\delta_D) d\ln\tau, \\ d\ln\varphi_{XM} &= (\lambda_D+\delta_D) d\ln\tau, \end{aligned} \tag{14}$$

where λ_D and δ_D are the domestic shares in (11) and (13) respectively. As they take values between 0 and 1, (14) implies that φ_D is decreasing in τ but φ_{XM} is increasing in τ. Thus trade liberalization forces purely-domestic firms to stop producing in the domestic market, while it allows importer-exporters to import inputs and export output from/to abroad more easily, thereby reallocating resources from less efficient firms to more efficient firms. In contrast, whether φ_{DM} and φ_X are increasing or decreasing in τ depends on the number of trading countries N. We find, for example, that when there are only two countries ($N = 2$), the domestic shares are relatively large so that $-1 + \lambda_D + \delta_D$ is always positive. More generally, there exists N^* such that

$$\frac{d\ln\varphi_D}{d\ln\tau} < 0 < \frac{d\ln\varphi_{DM}}{d\ln\tau} = \frac{d\ln\varphi_X}{d\ln\tau} < \frac{d\ln\varphi_{XM}}{d\ln\tau} \quad \text{for} \quad N < N^*.$$

Thus, trade liberalization decreases the productivity cutoffs of all firms engaging in any international activities, and resources are reallocated to international firms who source inputs and/or provide output in the multiple markets. While the mechanism is akin to the standard heterogeneous firm model with exporting-only, such resource reallocation effects are most significant for firms that simultaneously import and export in the model of importing-exporting. The result itself is not very surprising: changes in the cutoffs are greater for φ_{XM} than for φ_{DM}, φ_X because firms above φ_{XM} incur two types of trade costs τ_M, τ_X, while remaining international firms incur either τ_M or τ_X. When the number of trading countries is larger than the threshold N^*, however, the domestic shares become low enough that $\lambda_D + \delta_D < 1$, in which case φ_{DM} and φ_X are decreasing in τ as in φ_D (though the decline in φ_{DM} and φ_X is relatively smaller than φ_D) and

$$\frac{d\ln\varphi_D}{d\ln\tau} < \frac{d\ln\varphi_{DM}}{d\ln\tau} = \frac{d\ln\varphi_X}{d\ln\tau} < 0 < \frac{d\ln\varphi_{XM}}{d\ln\tau} \quad \text{for} \quad N > N^*.$$

In this case, importers-only and exporters-only suffer from trade liberalization, and the benefit of globalization is skewed toward firms that simultaneously import and export. Intuitively, when N is sufficiently larger, not only does the domestic market become more competitive through a sharper decline of the price index, but also the merit of being importers-only or exporters-only could not outweigh the loss that arises from competitive pressures by making importer-exporters stronger.

It is worth considering the elasticity of the productivity cutoffs to variable trade costs in the model with exporting-only. Suppose that input transport costs τ_M are sufficiently high that no firm profitably can import. Then all inputs must be procured in the domestic market and hence the domestic input share satisfies $\delta_D = 1$. In addition, when $\tau_M \to \infty$, the import productivity cutoffs are $\varphi_{DM} = \varphi_{XM} \to \infty$ where only the first and third equalities in the zero cutoff productivity condition (7) are relevant.[12] Since $J(\varphi_c)$ is strictly decreasing in φ_c, these changes imply that the free entry condition (8) reduces to $f_D J(\varphi_D) + (N-1) f_X J(\varphi_X) = f_E$. Solving these equilibrium conditions for the three unknowns φ_D, φ_X, B yields the elasticity in (14) evaluated at $\delta_D = 1$, i.e., $d\ln\varphi_D = -(1-\lambda_D)\, d\ln\tau$, $d\ln\varphi_X = \lambda_D d\ln\tau$. Noting that the domestic productivity cutoff φ_D is a single sufficient statistic for welfare within this class of the models, we find that trade liberalization can entail greater welfare effects in the model with importing-exporting than in the model with exporting-only, due to the additional channel $-(1-\delta_D) d\ln\tau$ in (14). This term captures the welfare gains from input trade as importers can reduce their production costs by exploiting the love-for-variety effect which in turn reduces the price index.

Two points deserve mentioning here. First, the same reasoning applies when considering the impact in the model with importing-only (e.g., Antràs et al., 2017). It is easy to see that the equilibrium is characterized by (i) all outputs are procured in the domestic market ($\lambda_D = 1$); (ii) the export productivity cutoffs are infinity ($\varphi_X = \varphi_{XM} \to \infty$); and (iii) the free entry condition reduces to $f_D J(\varphi_D) + (N-1) f_{DM} J(\varphi_{DM}) = f_E$. As a result of these, we get the elasticity as a special case of (14): $d\ln\varphi_D = -(1-\delta_D)\, d\ln\tau$, $d\ln\varphi_{DM} = \delta_D d\ln\tau$. Thus the welfare gains from trade liberalization can be greater in the model with importing-exporting than in the model with importing-only, due to the additional channel $-(1-\lambda_D) d\ln\tau$ in (14) which captures the welfare gains from output trade (in form of increased output variety). Second, the level of the domestic shares is generally different between the different models. As we have seen in the previous section, if importing and exporting exhibit complementarity (10), the domestic shares are lower in the model with importing-exporting than in the model with exporting-only or importing-only. Thus trade liberalization can entail greater welfare effects in the model with importing-exporting than in the model with exporting-only or importing-only, because the domestic shares are lower *within* each channel: $-(1-\lambda_D) d\ln\tau$, $-(1-\delta_D) d\ln\tau$ in (14). We will elaborate on these welfare effects in Section 3.3.

[12] While we mainly focus on the variable import costs, the same logic applies to the fixed import costs in the sense that when $f_{DM} = f_{XM} \to \infty$, then $\varphi_{DM} = \varphi_{XM} \to \infty$ (from (7)).

The impact on the productivity cutoffs allows us to explore the impact on the revenue/expenditure shares among different global status of firms. Consider first the revenue share σ_c in (9). From the changes in (14), it is obvious that trade liberalization increases the revenue share of importer-exporters σ_{XM} but decreases the revenue share of purely-domestic firms σ_D. (The revenue share of importers-only or exporters-only σ_{DM}, σ_X can increase or decrease, depending on the number of trading countries.) It is not so obvious, however, whether such trade liberalization increases the revenue share disproportionately among exporters and non-exporters. From (9), it follows that the relative revenue share among exporters σ_{XM}/σ_X and that among non-exporters σ_{DM}/σ_D depend only on the productivity cutoffs, which in turn yields

$$\frac{\sigma_{XM}/\sigma_X}{\sigma_{DM}/\sigma_D} = \frac{\frac{V(\varphi_D)}{V(\varphi_{DM})} - 1}{\frac{V(\varphi_X)}{V(\varphi_{XM})} - 1}.$$

While the revenue share σ_c depends on many variables, the ratio of the relative revenue shares is expressed in terms of the relative value of $V(\varphi_c)/V(\varphi_{c'})$ only, which summarizes the distribution of the relative firm sales above the productivity cutoffs φ_c, $\varphi_{c'}$ (Helpman et al., 2004). Simple inspection of changes in the productivity cutoffs reveals that it is ambiguous to see whether these relative shares rise as a result of trade liberalization. In fact, differentiating this equality with respect to variable trade costs, changes in the relative revenue shares depend on the elasticity of $V(\varphi_c)$ with respect to φ_c, denoted by $\gamma_c \equiv -d \ln V(\varphi_c)/d \ln \varphi_c$.[13] This reflects that trade liberalization impacts on firm sales $V(\varphi_c)$ by allowing for entry and exit of firms (i.e., changes in φ_c). Following Arkolakis et al. (2012), we refer to γ_c as the *extensive margin elasticity* below. Clearly, we cannot figure out whether the ratio of the relative revenue shares rises or not without knowing the realization of γ_c, which is determined by the firm distribution function $G(\varphi)$.

As a benchmark, we assume that productivity is distributed Pareto with a shape parameter k. Under this parameterization, the relative firm sales $V(\varphi_c)/V(\varphi_{c'})$ is distributed Pareto with a shape parameter $k-(\varepsilon-1)$, which means that the extensive margin elasticity is constant at $\gamma_c = k - (\varepsilon - 1)$ for any productivity cutoff c. Denoting this unique elasticity by γ and and using the impact on the productivity cutoffs in (14), changes in the relative shares are simply expressed as

$$d \ln \left(\frac{\sigma_{XM}}{\sigma_X} \right) - d \ln \left(\frac{\sigma_{DM}}{\sigma_D} \right) = -\gamma \left[\frac{\frac{V(\varphi_D)}{V(\varphi_{DM})} - \frac{V(\varphi_X)}{V(\varphi_{XM})}}{\left(\frac{V(\varphi_X)}{V(\varphi_{XM})} - 1 \right) \left(\frac{V(\varphi_X)}{V(\varphi_{XM})} - 1 \right)} \right] d \ln \tau. \tag{15}$$

The value in the brackets of (15) is positive if and only if complementarity in (10) holds. Therefore, while all international firms can gain the revenue share by trade liberalization, importer-exporters gain the revenue share disproportionately relative to importers-only or exporters-only under complementarity between importing and exporting. In other words, the difference in the revenue shares that exists ex ante can be magnified by trade liberalization ex post. This result is consistent with the existing findings in the literature that investigates the firm's joint interaction between importing and exporting. For example, Bernard et al. (2018) find that trade liberalization amplifies an exogenous difference in productivity between firms through endogenous differences in firm performance, leading to the skewed size distribution across firms. Similarly, Blaum (2019) finds that

[13] Specifically, using the equilibrium result in (14), such changes are expressed as

$$d \ln \left(\frac{\sigma_{XM}}{\sigma_X} \right) - d \ln \left(\frac{\sigma_{DM}}{\sigma_D} \right) = - \left[\frac{[(\gamma_{XM} - \gamma_X)(\lambda_D + \delta_D) + \gamma_X]}{\frac{V(\varphi_X)}{V(\varphi_{XM})} - 1} - \frac{[(\gamma_{DM} - \gamma_D)(-1 + \lambda_D + \delta_D) + \gamma_D]}{\frac{V(\varphi_D)}{V(\varphi_{DM})} - 1} \right] d \ln \tau,$$

which depends on the difference in the elasticities associated with changes in $V(\varphi_c)/V(\varphi_{c'})$, i.e., $\gamma_c - \gamma_{c'}$.

intense importers tend to be also intense exporters, and firms intensively engaging in both importing and exporting tend to gain the market share relatively more than firms engaging on either one of the international activities by trade shocks (devaluations). Our novelty is that such magnification effects can be measured by the revenue share between different global status of firms that is observable in the data.

Next consider the impact on the expenditure share. Using the expression of $\tilde{\sigma}_c$ shown in (12), the relative expenditure shares $\tilde{\sigma}_X/\tilde{\sigma}_D, \tilde{\sigma}_{XM}/\tilde{\sigma}_{DM}$ also depend only on the productivity cutoffs:

$$\frac{\tilde{\sigma}_{XM}/\tilde{\sigma}_{DM}}{\tilde{\sigma}_X/\tilde{\sigma}_D} = \frac{\frac{V(\varphi_D)}{V(\varphi_X)}}{\frac{V(\varphi_{DM})}{V(\varphi_{XM})}}.$$

The ratio of the relative expenditure shares is expressed in terms of the relative value of $V(\varphi_c)/V(\varphi_{c'})$ only as before. Given the impact on φ_c in (14), it is generally ambiguous to see whether these relative shares rise as a result of trade liberalization. However, under the Pareto benchmark case where the extensive margin constant at $\gamma = k - (\varepsilon - 1)$, the impact is uniquely determined. Differentiating this equality with respect to variable trade costs and using (14), changes in the relative revenue shares are given by

$$d\ln\left(\frac{\tilde{\sigma}_{XM}}{\tilde{\sigma}_{DM}}\right) - d\ln\left(\frac{\tilde{\sigma}_X}{\tilde{\sigma}_D}\right) = 0. \tag{16}$$

In contrast to the revenue share that importer-exporters gain disproportionately to importers-only as in (15), the expenditure share changes proportionately among importers and non-importers after trade liberalization. Recalling that complementarity in (10) can be written as $V(\varphi_{XM})/V(\varphi_X) > V(\varphi_{DM})/V(\varphi_D)$, we know that the expenditure share also satisfies complementarity. The reason goes as follows. Trade liberalization increases foreign input demand of importer-exporters relatively more than importers-only, which raises $\tilde{\sigma}_{XM}/\tilde{\sigma}_{DM}$. At the same time, however, such liberalization decreases the productivity cutoff of importer-exporters relatively more than importers-only. Since less efficient firms become importer-exporters, this change reduces $\tilde{\sigma}_{XM}/\tilde{\sigma}_{DM}$. Under the Pareto distribution, these two forces exactly offset one another (the same logic applies to $\tilde{\sigma}_{DM}/\tilde{\sigma}_D$). This asymmetric response in (15) and (16) means that importer-exporters gain the profit share relatively more than other types of international firms.

Finally, we briefly note the case where there is no complementarity between importing and exporting. Since $V(\varphi_{XM})/V(\varphi_X) = V(\varphi_{DM})/V(\varphi_D)$ in this case, the *levels* of the revenue/expenditure shares are proportional *before* trade liberalization and the *changes* in these shares are also proportional *after* trade liberalization. The result would be less likely to hold in reality, but this highlights the importance of the complementarity condition in the model of importing-exporting, which can be empirically investigated by exploiting the observability of these shares in the data.

3.2 Trade Flows

Using the impact on the productivity cutoffs and revenue/expenditure shares, we next examine the impact on trade flows. In particular, we explore the channel through which trade liberalization has magnification effects on trade flows in the model of importing-exporting relative to the standard trade model with exporting-only or importing-only. Since there are the two types of goods that are costly traded between borders, it is necessary to study the impact of trade liberalization on output trade flows and input trade flows separately. As before, we mainly analyze trade liberalization that reduces variable trade costs proportionately between input trade and output trade (i.e., $d\tau_M = d\tau_X \equiv d\tau$).

Following Arkolakis et al. (2012), we measure the impact on trade flows in terms of the "trade elasticity," which is defined as the elasticity of imports relative to domestic sales with respect to variable trade costs, holding any indirect effects from such costs fixed. Let R_D denote aggregate domestic sales earned by all non-exporters while let R_X denote aggregate export sales earned by all exporters, satisfying $R_D + R_X = R$. Then, the output trade elasticity is given by $\theta_X \equiv -\partial \ln \left(\frac{R_X/(N-1)}{R_D} \right) / \partial \ln \tau$. From $R_D = \lambda_D R$ and $R_X = (1-\lambda_D)R$, the elasticity is alternatively written as $\theta_X = -\partial \ln \left(\frac{(1-\lambda_D)/(N-1)}{\lambda_D} \right) / \partial \ln \tau$, where the value in the brackets is

$$\frac{\sigma_X + \sigma_{XM}}{\sigma_D + \sigma_{DM}} = \tau_X^{1-\varepsilon} \left(\frac{V(\varphi_X) + (N-1)\tau_M^{1-\varepsilon} V(\varphi_{XM})}{V(\varphi_D) + (N-1)\tau_M^{1-\varepsilon} V(\varphi_{DM})} \right).$$

The expression shows that θ_X consists of three components. First is the elasticity of the intensive margin from changes in $\tau_X^{1-\varepsilon}$. This captures an increase of output trade flows due to reductions in trade cost among all exporters, with elasticity of $\varepsilon - 1$ under CES preferences. Second is the elasticity of the intensive and extensive margins from changes in $V(\varphi_X) + (N-1)\tau_M^{1-\varepsilon} V(\varphi_{XM})$. This captures an increase in output trade flows due to changes in the revenue shares among exporters. Trade liberalization induces importer-exporters to gain the revenue share relatively more than exporters-only in the domestic market, because such liberalization allows importer-exporters not only to access foreign inputs more easily (by lowering τ_M) but also to export output more easily (by lowering φ_{XM} more than φ_X from (14)). These effects reallocate the revenue share toward more efficient importer-exporters which leads to an increase in output trade flows. Third is the elasticity of the intensive and extensive margins from changes in $V(\varphi_D) + (N-1)\tau_M^{1-\varepsilon} V(\varphi_{DM})$. This captures a *decrease* in output trade flows due to changes in the revenue shares among non-exporters, because trade liberalization also induces importers-only to gain the revenue share relatively more than purely-domestic firms in the domestic market. Such liberalization expands the revenue share of importers-only who do not export, which negatively affects output trade flows. From the analysis in the last section, whether the second positive effect dominates the third negative effect depends on the firm distribution.

Before parameterizing the distribution, the following observations stand out. First, not surprisingly, when firms cannot import foreign inputs from abroad, the revenue shares of importers-only and importer-exporters are zero ($\sigma_{DM} = \sigma_{XM} = 0$ or equivalently $V(\varphi_{DM}) = V(\varphi_{XM}) = 0$). Then, the trade elasticity collapses to that in the standard heterogeneous firm model with exporting-only. Second, perhaps more surprisingly, the same result occurs when there is no complementarity between importing and exporting ($\sigma_{XM}/\sigma_X = \sigma_{DM}/\sigma_D$ or equivalently $V(\varphi_{XM})/V(\varphi_X) = V(\varphi_{DM})/V(\varphi_D)$). In this case, even though some firms do import inputs, the positive effect among exporters offsets the negative effect among non-exporters under any distribution, as the term capturing their relative effect reduces to $V(\varphi_X)/V(\varphi_D)$ without complementarity, so as to leave the trade elasticity identical with that in the model of exporting-only.

Now we parameterize the distribution by assuming that productivity is distributed Pareto as a benchmark. The relative market share above can be written as relative firm sales $V(\varphi_c)/V(\varphi_{c'})$ only and hence its changes are written as the shape parameter $k - (\varepsilon - 1) = \gamma$. In addition, since we consider only the direct effect of trade costs in deriving the trade elasticity, we need to control any indirect effects holding the market demand level B – which includes both the price index P and aggregate expenditure R – fixed. Then, from (7), we have $\partial \ln \varphi_D / \partial \ln \tau = 0$, $\partial \ln \varphi_X / \partial \ln \tau = \partial \ln \varphi_{DM} / \partial \ln \tau = 1$, and $\partial \ln \varphi_{XM} / \partial \ln \tau = 2$ for a fixed B. Using these elasticities of the productivity cutoffs and noting that $\gamma_c = -\partial \ln V(\varphi_c)/\partial \ln \varphi_c$ from the definition of $V(\varphi_c)$, the output trade elasticity has the following closed-form solution:

$$\theta_X = (\varepsilon - 1 + \gamma)(1 + \kappa_M), \tag{17}$$

where κ_M captures a feedback effect from input trade flows on output trade flows.[14] This trade elasticity helps to confirm the above implications for the trade elasticity in the model of importing-exporting. First, when the input transport costs are prohibitively high ($\tau_M = \infty$), no firm can access imported inputs from abroad. Hence we have $\kappa_M = 0$ and the output trade elasticity in (17) reduces to $\theta_X = \varepsilon - 1 + \gamma$ which is decomposed into the intensive margin elasticity $\varepsilon - 1$ and the extensive margin elasticity γ. This is exactly the same as that in Chaney (2008) who develops the heterogeneous firm model with exporting-only. Second, when the relative share is proportional among exporters and non-exporters, it follows from (10) that $V(\varphi_D)V(\varphi_{XM}) = V(\varphi_X)V(\varphi_{DM})$ and we again have $\kappa_M = 0$ which means that the presence of importing-exporting has no impact on output trade flows at all without complementarity between importing and exporting. However, when the expenditure share satisfies (10), $\kappa_M > 0$ and thus the output trade elasticity becomes greater than the previous cases. This is because, so long as there is complementarity, trade liberalization not only makes it easier to export output, but also makes it easier to import from abroad. With love-for-variety production, such imported inputs then improve the output production efficiency by reducing importers' unit costs, which magnifies output trade flows relative to the above cases.

Under the Pareto distribution, there is an additional implication for the impact on trade flows. We find that the trade elasticity further increases with the number of trading partners for sufficiently high trade costs. This can be shown from noting that κ_M is increasing in N if $\tau_M^{2k} \left(\frac{f_{DM} f_{XM}}{f_D f_X} \right)^{\frac{k-(\varepsilon-1)}{\varepsilon-1}} - (N-1)^2 > 0$, which implies that the number of trading countries has the two opposing effects on the trade elasticity. On the one hand, an increase in N magnifies output trade flows from the import perspective: the larger the number of trading countries, the more inputs sourced from abroad and thus the stronger importer-exporters relative to exporters-only (see Section 3.1). This makes the set of firms that export relatively more concentrated on firms that also import, which can amplify the impact on output trade flows. On the other hand, an increase in N also dampens output trade flows from the export perspective: the larger the number of trading countries, the more opportunities all types of exporters earn from foreign markets. This weakens the selection effect toward importer-exporters. The above inequality shows that the former outweighs the latter when the trade costs are sufficiently high that selection into importing and exporting is tight enough.

Next consider the impact of trade liberalization on input trade flows. Like output trade flows, we measure the impact on input trade flows in terms of the trade elasticity defined as follows. Let E_D denote aggregate domestic expenditure spent by all non-importers while let E_M denote aggregate import expenditure spent by all importers, satisfying $E_D + E_M = E$. Then the input trade elasticity in our setting is given by $\theta_M \equiv -\partial \ln \left(\frac{E_M/(N-1)}{E_D} \right) / \partial \ln \tau$. From $E_D = \delta_D E$ and $E_M = (1-\delta_D)E$, the elasticity is alternatively written as $\theta_M = -\partial \ln \left(\frac{(1-\delta_D)/(N-1)}{\delta_D} \right) / \partial \ln \tau$, where the value in the brackets is

$$\frac{\tilde{\sigma}_{DM} + \tilde{\sigma}_{XM}}{\tilde{\sigma}_D + \tilde{\sigma}_X} = \tau_M^{1-\varepsilon} \left(\frac{V(\varphi_{DM}) + (N-1)\tau_X^{1-\varepsilon} V(\varphi_{XM})}{V(\varphi_D) + (N-1)\tau_X^{1-\varepsilon} V(\varphi_X)} \right).$$

As in θ_X, this expression shows that the input trade elasticity θ_M consists of three components. This also means that the closed-form solution of θ_M is similar to that of θ_X under the Pareto distribution with $\gamma_c = \gamma$. In fact, the input trade elasticity is expressed as

$$\theta_M = (\varepsilon - 1 + \gamma)(1 + \kappa_X), \tag{18}$$

[14] $\kappa_M \equiv \frac{(N-1)\tau_M^{1-\varepsilon}[V(\varphi_D)V(\varphi_{XM}) - V(\varphi_X)V(\varphi_{DM})]}{[V(\varphi_D)+(N-1)\tau_M^{1-\varepsilon}V(\varphi_{DM})][V(\varphi_X)+(N-1)\tau_M^{1-\varepsilon}V(\varphi_{XM})]}$. (17) is the "partial" trade elasticity as in Arkolakis et al. (2012), but it is also the "full" trade elasticity $-d \ln \left(\frac{R_X/(N-1)}{R_D} \right) / d \ln \tau$ so long as $\gamma_c = \gamma$.

where κ_X captures a feedback effect from output trade flows on input trade flows.[15] The amplified effect on the input trade elasticity can be understood by considering a special case when there is neither exporting nor complementarity between importing and exporting. In such a case, $\kappa_X = 0$ and the input trade elasticity reduces to $\theta_M = \varepsilon - 1 + \gamma$. The elasticity again coincides with that in Chaney (2008), but the interpretation differs from the output trade elasticity in (17). The intensive margin elasticity is the same as firms' technology and consumers' preferences are CES with the common elasticity ε, while the extensive margin elasticity is the same as the Pareto distribution yields the common elasticity for any productivity cutoffs. From the similarity between (17) and (18), we have that selection into importing increases the input trade elasticity (relative to the absence of such selection) due to the extensive margin. When there is complementarity in (10), the input trade elasticity becomes greater further (from $\kappa_X > 0$). This is because trade liberalization not only makes it easier to import abroad, but also makes it easier to export output. With love-for-variety consumption, such increased output demand leads to increased input demand, which magnifies input trade flows relative to the economy without exporting.

3.3 Welfare Gains

We are now ready to examine the welfare gains from trade liberalization in the presence of importing-exporting. Recall that welfare per worker depends only on the domestic productivity cutoff and welfare changes associated with trade liberalization are proportional to the cutoff (i.e., $d \ln W = \sum_s \beta_s d \ln \varphi_{Ds}$) in our model with multiple sectors from (6). The problem, however, is that the cutoff is not directly observable in the data. To measure the welfare gains from trade liberalization, we thus need to replace the productivity cutoff with some empirically observable objects.

In what follows, following Arkolakis et al. (2012), we demonstrate that changes in welfare associated with trade liberalization are expressed in terms of the domestic share and the trade elasticity. First of all, note that our model satisfies their three macro-level restrictions: (i) trade is balanced; (ii) aggregate profits are constant share of aggregate expenditure; and (iii) import demand is CES with a constant trade elasticity with respect to variable trade costs. When input-output linkages are modeled by sequential production where final goods are produced with intermediate inputs from different stages (e.g., Melitz and Redding, 2014b) instead of roundabout production where output is sold as final goods and intermediate goods (Arkolakis et al., 2012), however, the domestic share is generally different between input and output, as in (11) and (13), which in turn implies that the trade elasticity is different between these two types of goods, as in (17) and (18). As a result, the model generates a separate source of welfare changes between input trade and output trade. Moreover, in contrast to their roundabout production model where all firms make use of foreign inputs, this is not the case in our sequential production model where only a subset of firms import. As seen above, such selection into importing raises the trade elasticity (and thus lowers the domestic share), which can magnify welfare changes. This channel may lead to new welfare implications that are quantitatively important.

To establish the welfare result a la Arkolakis et al. (2012) in sequential production, from the changes in the productivity cutoffs in (14), we have that the welfare changes $d \ln W = \sum_s \beta_s d \ln \varphi_{Ds}$ can be written in terms of the domestic share of the two types of goods:

$$d \ln W = -\sum_{s=0}^{S} \beta_s \Big[(1 - \lambda_{Ds}) + (1 - \delta_{Ds}) \Big] d \ln \tau_s.$$

[15] $\kappa_X \equiv \frac{(N-1)\tau_X^{1-\varepsilon}[V(\varphi_D)V(\varphi_{XM}) - V(\varphi_X)V(\varphi_{DM})]}{[V(\varphi_D)+(N-1)\tau_X^{1-\varepsilon}V(\varphi_X)][V(\varphi_{DM})+(N-1)\tau_X^{1-\varepsilon}V(\varphi_{XM})]}$. (18) is the "partial" and "full" trade elasticity under $\gamma_c = \gamma$.

As explained in Section 3.1, the first and second terms respectively capture the welfare gains associated with output trade and input trade. This means that, conditional on these domestic shares, the welfare gains from trade liberalization are greater in the model of importing-exporting than in the model with exporting-only where $\delta_{Ds} = 1$ or with importing-only where $\lambda_{Ds} = 1$. The welfare result would apply to any trade model in which final goods are produced with inputs from different stages without roundabout production structure. To express the welfare changes in terms of the domestic share and the trade elasticity, differentiating the domestic shares $\lambda_{Ds} = \sigma_{Ds} + \sigma_{DMs}, \delta_{Ds} = \tilde{\sigma}_{Ds} + \tilde{\sigma}_{Xs}$ under $\gamma_{cs} = \gamma_s$ and subsequently using (14), changes in the shares associated with trade liberalization are given by

$$d \ln \lambda_{Ds} = \theta_{Xs}(1 - \lambda_{Ds}) d \ln \tau_s + d \ln M_{Es},$$
$$d \ln \delta_{Ds} = \theta_{Ms}(1 - \delta_{Ds}) d \ln \tau_s + d \ln M_{Es},$$

where θ_{Xs} and θ_{Ms} are the output and input trade elasticities in sector s given in (17) and (18), respectively. Moreover, changes in the mass of entrants M_{Ei} are proportional to changes in sectoral labor supply L_s under $\gamma_{cs} = \gamma_s$ (see Appendix A.5).[16] Using these relationships, the welfare changes are expressed as

$$d \ln W = -\sum_{s=0}^{S} \left[\frac{\beta_s}{\theta_{Xs}} d \ln \left(\frac{\lambda_{Ds}}{L_s} \right) + \frac{\beta_s}{\theta_{Ms}} d \ln \left(\frac{\delta_{Ds}}{L_s} \right) \right].$$

Let a "hat" denote proportional changes of a variable (i.e., $\widehat{W} = dW/W$). Then integrating the above welfare changes, we finally get the following expression:

$$\widehat{W} = \prod_{s=0}^{S} \left(\frac{\widehat{\lambda}_{Ds}}{\widehat{L}_s} \right)^{-\frac{\beta_s}{\theta_{Xs}}} \left(\frac{\widehat{\delta}_{Ds}}{\widehat{L}_s} \right)^{-\frac{\beta_s}{\theta_{Ms}}}. \tag{19}$$

The welfare changes in (19) can be thought of as the welfare formula of Arkolakis et al. (2012) in the model of importing-exporting with sequential production, in the sense that the welfare changes associated with trade costs are captured only by the domestic share and the trade elasticity. In particular, if intermediate goods are prohibitively costly to trade across borders so that inputs are produced domestically in any sector ($\delta_{Ds} = 1$), the expression (19) collapses to their welfare formula with multiple sectors. If inputs are tradable ($\delta_{Ds} < 1$), in contrast, the domestic input share also matters for welfare evaluations.

The difference in welfare changes between theirs and ours comes from the difference in modeling of input-output linkages. To allow for tradable intermediate goods in their setting, Arkolakis et al. (2012) introduce roundabout production in which firms use output of all other firms as inputs in production. Since final goods and intermediate goods are interchangeable, the domestic output share includes these two types of goods and thus the welfare changes can be still captured by that share and its trade elasticity only. Then, relative to the baseline model, the amplified effect on welfare changes is captured by new parameters governing the share of intermediate goods in production and entry costs that enter the welfare expression. When input-output linkages are modeled with sequential production in which firms use inputs from different stages in production, however, the domestic share is generally different in each production stage. Thus we need to distinguish the domestic input share from the domestic output share, which yields the trade elasticity that is different between the two types of goods. Then, the amplified effect on welfare changes is captured by the domestic share and the trade elasticity of input trade that independently enter the welfare expression.

[16] Labor supply is an endogenous variable with $\sum_s L_s = \overline{L}$, while the condition $\gamma_{cs} = \gamma_s$ is related to the third macro-level restriction which is crucial to a constant trade elasticity in (17) and (18).

While (19) illustrates that the welfare changes associated with trade liberalization can be measured by two sufficient statistics – the domestic share and the trade elasticity of two types of goods that are costly traded, it would be useful to define the welfare gains from trade as the absolute value of the percentage change in the real income that would be associated with moving from autarky to costly trade. Let $G \equiv W^{\text{Open}}/W^{\text{Closed}}$ denote the welfare ratio between the open and closed economies, which is expressed in terms of the domestic shares $\lambda_{Ds}^{\text{Open}}$, $\delta_{Ds}^{\text{Open}}$ (as the domestic shares under autarky are fixed at $\lambda_{Ds}^{\text{Closed}} = \delta_{Ds}^{\text{Closed}} = 1$) and the ratio in sectoral labor allocations between the open and closed economies $L_s^{\text{Open}}/L_s^{\text{Closed}}$. Then it follows immediately from (19) that the welfare gains from trade are given by

$$G = \prod_{s=0}^{S} \left(\lambda_{Ds}^{\text{Open}}\right)^{-\frac{\beta_s}{\theta_{Xs}}} \left(\frac{L_s^{\text{Open}}}{L_s^{\text{Closed}}}\right)^{\frac{\beta_s}{\theta_{Xs}}} \left(\delta_{Ds}^{\text{Open}}\right)^{-\frac{\beta_s}{\theta_{Ms}}} \left(\frac{L_s^{\text{Open}}}{L_s^{\text{Closed}}}\right)^{\frac{\beta_s}{\theta_{Ms}}}. \tag{20}$$

The welfare gains from trade in (20) are very similar to those by Melitz and Redding (2014a) who derive such gains without input-output linkages. Our welfare gains can be thought of as extensions of their formula where input-output linkages are modeled with sequential production. If we instead consider roundabout production, the term associated with input trade would be partly subsumed into the term associated with output trade along with new parameters governing the share of intermediate goods in production and entry costs.

A natural question that arises from the argument above is then: which welfare gains from trade are greater in roundabout production and sequential production? Answering to this question would be difficult at least theoretically, because the amplified effect from tradable inputs depends on the *exogenous* parameters in roundabout production, while the effect is realized as the *endogenous* domestic share and trade elasticity that are separable between different stages in sequential production. Thus it would be necessary to resort to numerical exercises to compare the quantitative difference between these two production systems. Nonetheless, it is worth stressing the qualitative difference that could potentially give rise to different welfare implications. With roundabout production, firms use output of other domestic firms as domestic inputs and output of other foreign firms as imported inputs in production. Since all firms are able to access imported inputs from abroad, the "love-of-variety" production solely explains why the welfare gains from trade are greater in a world with trade in intermediate goods than in a world with only trade in final goods. Here, on top of that effect, there is selection into importing which allows a subset of more efficient firms to profitably import inputs from abroad. In other words, like selection into exporting matters for welfare, selection into importing matters for welfare. Formally this can be seen from changes in the domestic share and the trade elasticity. Without selection into importing, the domestic input share is $\delta_s = 1/(1 + \tau_{Ms}^{1-\varepsilon_s})$ which in turn generates the input trade elasticity $\varepsilon_s - 1$. If there is selection into importing together with complementarity between importing and exporting, in contrast, the domestic input share is lower from (13) while the input trade elasticity is higher from (18). These changes in the two sufficient statistics imply that the welfare gains from trade in (20) can be greater in a world with selection into importing than in a world with only selection into exporting.

A similar claim also applies to sequential production in which all goods markets are perfectly competitive. For example, using an Armington setting with sequential production, Melitz and Redding (2014b) show that the welfare gains from trade become arbitrarily large as the number of production stages is arbitrarily large. The Armington assumption implies however that the domestic input share is again given by $\delta_s = 1/(1+\tau_{Ms}^{1-\varepsilon_s})$ under country symmetry, which in turn generates the input trade elasticity $\varepsilon_s - 1$. Hence, even if we consider sequential production, the welfare gains from trade would be different depending on whether there is selection among operating firms. The reasoning above suggests that the welfare gains from trade can be greater in a world with selection due to differences in the sufficient statistics within each production stage.

4 Conclusion

We have developed a model of sequential production in which firms use inputs from different stages in production by choosing markets from which to source inputs as well as to which to provide their final goods. Firms that are heterogeneous in productivity choose to importing and exporting, which require different trade costs to access the foreign markets. Among operating firms, only more productive firms find it profitable to engage in both importing and exporting while less productive firms choose either importing-only or exporting-only. In addition to these sorting patterns, the model predicts that trade liberalization may have a non-uniform effect among international firms. In particular, when there is complementarity between importing and exporting, exporters who are also importers can gain the market share relatively more than exporters who are not importers. As a result, firms that simultaneously import and export are more likely to benefit from output trade liberalization, magnifying the effect of initial productivity differences and leading to sales concentration toward these most globalized firms. As they are the most efficient firms among operating firms, such concentration can contribute to increment trade flows and welfare gains relative to those in the absence of importing.

To show our welfare result, we have restricted attention to the symmetric-country case where all exporters ship final goods to all foreign markets and all importers source intermediate goods from all foreign markets. If we allow for country asymmetry in the model, the firm's sourcing strategy is affected by the firm's productivity and hence the export/import productivity cutoffs are no longer unique among exporters/importers. This makes it difficult to obtain the analytical solution, but we expect that so long as we consider measure-zero firms, the welfare result would continue to hold as the skewed effect toward most efficient global firms would still exist in such a setting. While the paper has examined the role of global firms in generating new welfare implications theoretically, it is possible to explore its quantitative relevance by calibrating the current model. Such analysis would help us quantify a potential bias of ignoring the presence of importing in estimating the welfare gains from trade when selection plays a key role. For that purpose, what we need to know is information about the market share with different global status of firms which is observable in the data. In future work, we plan to derive quantitative impacts of trade liberalization on the welfare gains from trade, and compare them with those in the model without selection into importing and/or exporting.

A Proofs

A.1 Welfare

We show the derivations of (6). Solving the market demand $B_i = (1-\alpha)A_i/\alpha^{1-\varepsilon}$ where $A_i = R_i/P_i^{\varepsilon-1}$ for the price index P_i and using $R_i = \beta\overline{L}_i$,

$$P_i^{1-\varepsilon} = (1-\alpha)\alpha^{\varepsilon-1}\frac{\beta\overline{L}_i}{B_i}.$$

Moreover, solving the first equality in (2) for B_i and substituting it into the above,

$$P_i = \left(\frac{(1-\alpha)f_D}{\beta\overline{L}_i}\right)^{\frac{1}{\varepsilon-1}}\frac{1}{\alpha\varphi_{Di}}. \tag{A.1}$$

Note that (A.1) is the Dixit-Stiglitz price index associated with varieties from a particular sector $s \geq 1$. Since the representative consumer has a two-tier utility function with the upper tier being Cobb-Douglas, the overall price index in country i is $\prod_{s=0}^{S} P_{is}^{\beta_s}$ where P_{is} is the sectoral price index above. Attaching the sector index s to (A.1), welfare per worker in country i with (common) wage $w_i = 1$ is given by

$$W_i \equiv \prod_{s=0}^{S} P_{is}^{-\beta_s}.$$

Substituting (A.1) into the above welfare expression yields the desired result.

A.2 Free Entry

We show the derivation of (8). Consider the expected domestic profits, i.e., the first term of (5). Imposing the symmetric costs, we get $\varphi_{Di} = \varphi_D$ and $\varphi_{DMi} = \varphi_{DM}$ where the firm's sourcing strategy and domestic input share reduce to $n = 0$ and $s_D = 1$ for $\varphi_D < \varphi < \varphi_{DM}$ while $n = N-1$ and $s_D = 1/(1+(N-1)\tau_M^{1-\varepsilon})$ for $\varphi_{DM} < \varphi < \infty$. Then the expected domestic profits are

$$\begin{aligned}
&\int_{\varphi_D}^{\varphi_{DM}} \pi_D dG(\varphi) + \int_{\varphi_{DM}}^{\infty} \pi_{DM} dG(\varphi)\\
&= \int_{\varphi_D}^{\varphi_{DM}} (B\varphi^{\varepsilon-1} - f_D) dG(\varphi) + \int_{\varphi_{DM}}^{\infty} \left(\frac{B}{s_D}\varphi^{\varepsilon-1} - f_D - (N-1)f_{DM}\right) dG(\varphi),
\end{aligned}$$

which can be rearranged as

$$\int_{\varphi_D}^{\infty} (B\varphi^{\varepsilon-1} - f_D) dG(\varphi) + \int_{\varphi_{DM}}^{\infty} \left(B\left(\frac{1-s_D}{s_D}\right)\varphi^{\varepsilon-1} - (N-1)f_{DM}\right) dG(\varphi).$$

Further, using the productivity cutoffs φ_D and φ_{DM} in (7) as well as s_D seen above,

$$f_D \int_{\varphi_D}^{\infty} \left[\left(\frac{\varphi}{\varphi_D}\right)^{\varepsilon-1} - 1\right] dG(\varphi) + (N-1)f_{DM} \int_{\varphi_{DM}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{DM}}\right)^{\varepsilon-1} - 1\right] dG(\varphi).$$

Similarly, the expected export profits, i.e., the second term of (5), in every country are

$$(N-1)f_X \int_{\varphi_X}^{\infty} \left[\left(\frac{\varphi}{\varphi_X} \right)^{\varepsilon-1} - 1 \right] dG(\varphi) + (N-1)^2 f_{XM} \int_{\varphi_{XM}}^{\infty} \left[\left(\frac{\varphi}{\varphi_{XM}} \right)^{\varepsilon-1} - 1 \right] dG(\varphi).$$

Finally, using the definition of $J(\varphi_c)$ and noting that the sum of the expected profits equals the entry cost f_E under free entry gives us the expression in (8).

A.3 Revenue/Expenditure Shares

We first show the derivation of the revenue share σ_c in (9). From $r_D = p_D q_D$, the firm's domestic revenue is

$$r_D = \begin{cases} \frac{B}{1-\alpha} \varphi^{\varepsilon-1} & \text{if non-importers,} \\ \frac{B(1+(N-1)\tau_M^{1-\varepsilon})}{1-\alpha} \varphi^{\varepsilon-1} & \text{if importers.} \end{cases}$$

Let σ_D (σ_{DM}) denote all non-importers' (all importers') revenue share in aggregate revenue in the domestic market. Aggregating the firm's domestic revenue for the relevant productivity ranges,

$$\sigma_D = M_E \int_{\varphi_D}^{\varphi_{DM}} \frac{B}{(1-\alpha)R} \varphi^{\varepsilon-1} dG(\varphi),$$
$$\sigma_{DM} = M_E \int_{\varphi_{DM}}^{\infty} \frac{B(1+(N-1)\tau_M^{1-\varepsilon})}{(1-\alpha)R} \varphi^{\varepsilon-1} dG(\varphi).$$

Similarly, from $r_X = p_X q_X$, the firm's export revenue is

$$r_X = \begin{cases} \frac{B\tau_X^{1-\varepsilon}}{1-\alpha} \varphi^{\varepsilon-1} & \text{if non-importers,} \\ \frac{B\tau_X^{1-\varepsilon}(1+(N-1)\tau_M^{1-\varepsilon})}{1-\alpha} \varphi^{\varepsilon-1} & \text{if importers.} \end{cases}$$

Let σ_X (σ_{XM}) denote all exporters' (all importer-exporters') revenue share in aggregate revenue in the domestic market. Aggregating the firm's export revenue for the relevant productivity ranges,

$$\sigma_X = (N-1) M_E \int_{\varphi_X}^{\varphi_{XM}} \frac{B\tau_X^{1-\varepsilon}}{(1-\alpha)R} \varphi^{\varepsilon-1} dG(\varphi),$$
$$\sigma_{XM} = (N-1) \int_{\varphi_{XM}}^{\infty} \frac{B\tau_X^{1-\varepsilon}(1+(N-1)\tau_M^{1-\varepsilon})}{(1-\alpha)R} \varphi^{\varepsilon-1} dG(\varphi).$$

Using the productivity cutoff φ_c in (7) and $R = \beta \overline{L}$, we get the revenue shares in (9). Moreover, the fact that $\sum_c \sigma_c = 1$ follows immediately from noting that

$$R = M_E \int_{\varphi_D}^{\infty} r_D dG(\varphi) + (N-1) M_E \int_{\varphi_X}^{\infty} r_X dG(\varphi).$$

Next, we show the derivation of the expenditure share $\tilde{\sigma}_c$. Under country symmetry, the firm chooses the quantities of domestic and foreign inputs $\{z_D, z_M\}$ to solve

$$e = \min_{z_D, z_M} \left\{ z_D + (N-1)\tau_M z_M \quad \text{s.t.} \quad \varphi x \geq q \right\}.$$

Cost minimization yields the demand function for domestic and foreign inputs:

$$z_D = c^{\varepsilon-1} e/\varphi,$$
$$z_M = \tau_M^{-\varepsilon} c^{\varepsilon-1} e/\varphi.$$

Using $e = cq$ where $q = Ap^{-\varepsilon}$, the firm's domestic input expenditure in every country, $e_D = z_D$, is

$$e_D = \begin{cases} \frac{\alpha B}{1-\alpha}\varphi^{\varepsilon-1} & \text{if non-exporters,} \\ \frac{\alpha B \tau_X^{1-\varepsilon}}{1-\alpha}\varphi^{\varepsilon-1} & \text{if exporters.} \end{cases}$$

Let $\tilde{\sigma}_D$ ($\tilde{\sigma}_X$) denote all non-exporters' (all exporters') expenditure share in aggregate expenditure in the domestic market. Aggregating the firm's domestic input expenditure for the relevant productivity ranges,

$$\tilde{\sigma}_D = M_E \int_{\varphi_D}^{\infty} \frac{\alpha B}{(1-\alpha)E}\varphi^{\varepsilon-1} dG(\varphi),$$
$$\tilde{\sigma}_X = M_E \int_{\varphi_X}^{\infty} \frac{\alpha B (N-1)\tau_X^{1-\varepsilon}}{(1-\alpha)E}\varphi^{\varepsilon-1} dG(\varphi).$$

Similarly, the firm's foreign input expenditure in every country, $e_M = \tau_M z_M$, is

$$e_M = \begin{cases} \frac{\alpha B \tau_M^{1-\varepsilon}}{1-\alpha}\varphi^{\varepsilon-1} & \text{if non-exporters,} \\ \frac{\alpha B (\tau_X \tau_M)^{1-\varepsilon}}{1-\alpha}\varphi^{\varepsilon-1} & \text{if exporters.} \end{cases}$$

Let $\tilde{\sigma}_{DM}$ ($\tilde{\sigma}_{XM}$) denote all importers' (all importer-exporters') expenditure share in aggregate expenditure in the domestic market. Aggregating the firm's foreign expenditure for the relevant productivity ranges,

$$\tilde{\sigma}_{DM} = M_E \int_{\varphi_{DM}}^{\infty} \frac{\alpha B (N-1)\tau_M^{1-\varepsilon}}{(1-\alpha)E}\varphi^{\varepsilon-1} dG(\varphi),$$
$$\tilde{\sigma}_{XM} = M_E \int_{\varphi_{XM}}^{\infty} \frac{\alpha B (N-1)^2(\tau_X \tau_M)^{1-\varepsilon}}{(1-\alpha)E}\varphi^{\varepsilon-1} dG(\varphi)$$

Using the productivity cutoff φ_c in (7) and using $E = \alpha R = \alpha\beta\overline{L}$, we get the expenditure shares in (12). Moreover, the fact that $\sum_c \tilde{\sigma}_c = 1$ follows immediately from noting that

$$E = M_E \int_{\varphi_D}^{\infty} e_D dG(\varphi) + (N-1)\int_{\varphi_{DM}}^{\infty} e_M dG(\varphi).$$

A.4 Changes in Productivity Cutoffs

We show the derivation of (14). Taking the log and differentiating (7) with respect to $\ln\tau_M = \ln\tau_X \equiv \ln\tau$,

$$\begin{aligned} d\ln B + (\varepsilon-1)d\ln\varphi_D &= 0, \\ d\ln B - (\varepsilon-1)d\ln\tau + (\varepsilon-1)d\ln\varphi_{DM} &= 0, \\ d\ln B - (\varepsilon-1)d\ln\tau + (\varepsilon-1)d\ln\varphi_X &= 0, \\ d\ln B - 2(\varepsilon-1)d\ln\tau + (\varepsilon-1)d\ln\varphi_{XM} &= 0. \end{aligned} \tag{A.2}$$

Further, differentiating (8) with respect to τ,

$$\begin{aligned} & f_D J'(\varphi_D)\varphi_D d\ln\varphi_D + (N-1) f_{DM} J'(\varphi_{DM})\varphi_{DM} d\ln\varphi_{DM} \\ & \quad + (N-1) f_X J'(\varphi_X)\varphi_X d\ln\varphi_X + (N-1)^2 f_{XM} J'(\varphi_{XM})\varphi_{XM} d\ln\varphi_{XM} = 0. \end{aligned} \tag{A.3}$$

Just like (2) and (5) jointly provide implicit solutions for *levels* of the productivity cutoffs $\varphi_D, \varphi_{DM}, \varphi_X, \varphi_{XM}$ and the market demand B, (A.2) and (A.3) jointly provide implicit solutions for *changes* of the productivity cutoffs $d\ln\varphi_D, d\ln\varphi_{DM}, d\ln\varphi_X, d\ln\varphi_{XM}$ and the market demand $d\ln B$.

To solve for the equilibrium in changes, note from (A.2) that $d\ln\varphi_{DM}, d\ln\varphi_X, d\ln\varphi_{XM}$ can be expressed in terms of $d\ln\varphi_D$ only:

$$\begin{aligned} d\ln\varphi_{DM} &= d\ln\varphi_D + d\ln\tau, \\ d\ln\varphi_X &= d\ln\varphi_D + d\ln\tau, \\ d\ln\varphi_{XM} &= d\ln\varphi_D + 2d\ln\tau. \end{aligned} \tag{A.4}$$

Further, differentiating $J(\varphi_c) = \int_{\varphi_c}^{\infty} \left[\left(\frac{\varphi}{\varphi_c} \right)^{\varepsilon-1} - 1 \right] dG(\varphi)$ with respect to φ_c,

$$J'(\varphi_c) = -\left(\frac{\varepsilon - 1}{\varphi_c} \right) [J(\varphi_c) + 1 - G(\varphi_c)].$$

From the functional forms of $J(\varphi_c)$ and $V(\varphi_c) = \int_{\varphi_c}^{\infty} \varphi^{\varepsilon-1} dG(\varphi)$, we have $J(\varphi_c) + 1 - G(\varphi_c) = \varphi_c^{1-\varepsilon} V(\varphi_c)$. Substituting this equality into the above $J'(\varphi_c)$,

$$J'(\varphi_c) = -(\varepsilon - 1)\varphi_c^{-\varepsilon} V(\varphi_c).$$

Moreover, substituting this and (A.4) into (A.3) and solving for $\ln\varphi_D$,

$$d\ln\varphi_D = -\left(\frac{v_X + v_{XM}}{\sum_c v_c} \right) d\ln\tau - \left(\frac{v_{DM} + v_{XM}}{\sum_c v_c} \right) d\ln\tau, \tag{A.5}$$

where

$$\begin{aligned} v_D &\equiv f_D \varphi_D^{1-\varepsilon} V(\varphi_D), & v_{DM} &\equiv (N-1) f_{DM} \varphi_{DM}^{1-\varepsilon} V(\varphi_{DM}), \\ v_X &\equiv (N-1) f_X \varphi_X^{1-\varepsilon} V(\varphi_X), & v_{XM} &\equiv (N-1)^2 f_{XM} \varphi_{XM}^{1-\varepsilon} V(\varphi_{XM}). \end{aligned}$$

Using the productivity cutoff φ_c (7) for v_c and the revenue share σ_c in (9), the value in the first brackets in (A.5) is expressed as $\frac{\sigma_X + \sigma_{XM}}{\sum_c \sigma_c}$, which equals the foreign output share $1 - \lambda_D$. Similarly, using the expenditure share $\tilde{\sigma}_c$ in (12), the value in the second brackets in (A.5) is expressed as $\frac{\tilde{\sigma}_{DM} + \tilde{\sigma}_{XM}}{\sum_c \tilde{\sigma}_c}$, which equals the foreign input share $1 - \delta_D$. These results establish the first equality in (14). The other equalities in (14) follow from substituting this $d\ln\varphi_D$ into (A.4).

Finally, we show that $-1 + \delta_D + \lambda_D$ in the second equality of (14) can be positive or negative, depending on the number of trading countries N. From v_c, the domestic shares, $\lambda_D = \frac{\sigma_D + \sigma_{DM}}{\sum_c \sigma_c}$, $\delta_D = \frac{\tilde{\sigma}_D + \tilde{\sigma}_X}{\sum_c \tilde{\sigma}_c}$, are

$$\lambda_D = \frac{v_D + v_{DM}}{\sum_c v_c}, \quad \delta_D = \frac{v_D + v_X}{\sum_c v_c}.$$

Then $-1+\delta_D+\lambda_D>0$ if and only if $v_D>v_{XM}$. From φ_c in (7), this inequality can be rewritten as

$$\frac{(\tau_X\tau_M)^{\varepsilon-1}}{(N-1)^2}\frac{V(\varphi_D)}{V(\varphi_{XM})}>1.$$

When $N=2$, this inequality always holds as $(\tau_X\tau_M)^{\varepsilon-1}>1$ and $V(\varphi_D)>V(\varphi_{XM})$ (where the latter comes from noting that $\varphi_{XM}>\varphi_D$ and $V(\varphi_c)$ is decreasing in φ_c). However, if N is sufficiently large, this might not hold in which case $-1+\delta_D+\lambda_D<0$.

A.5 Changes in Mass of Entrants

We show that changes in the mass of entrants are proportional to changes in sectoral labor supply when the extensive margin elasticity is constant ($\gamma_{cs}=\gamma_s$). With country symmetry, trade is balanced sector-by-sector and hence $R_s=L_s$. Then using v_c introduced above for $R_s=R_{Ds}+R_{Xs}$, we get

$$L_s=\frac{M_{Es}}{1-\alpha}\sum_c v_{cs}.$$

Taking the log and differentiating this equality,

$$d\ln L_s=d\ln M_{Es}-(\varepsilon_s-1+\gamma_s)\frac{\sum_c v_{cs}d\ln\varphi_{cs}}{\sum_c v_{cs}}.$$

Using (A.4), this expression can be written as

$$d\ln L_s=d\ln M_{Es}-(\varepsilon_s-1+\gamma_s)\left(d\ln\varphi_{Ds}+\left(\frac{v_{Xs}+v_{XMs}}{\sum_c v_{cs}}\right)d\ln\tau_s+\left(\frac{v_{DMs}+v_{XMs}}{\sum_c v_{cs}}\right)d\ln\tau_s\right),$$

where the second term is zero from (A.5). This establishes the desired result.

References

Antràs P, Chor D. 2022. Global Value Chains. *Handbook of International Economics* Volume 5, Elsevier: North Holland, 297-376.

Antràs P, Fort TC, and Tintelnot F. 2017. The Margins of Global Sourcing: Theory and Evidence from U.S. Firms. *American Economic Review* 107, 2514-2564.

Arkolakis C, Costinot A, Rodríguez-Clare A. 2012. New Trade Models, Same Old Gains? *American Economic Review* 102, 94-130.

Bernard AB, Jensen JB, Redding SJ, Schott SK. 2007. Firms in International Trade. *Journal of Economic Perspectives* 21, 105-130.

Bernard AB, Jensen JB, Redding SJ, Schott SK. 2012. The Empirics of Firm Heterogeneity and International Trade. *Annual Review of Economics* 4, 283-313.

Bernard AB, Jensen JB, Redding SJ, Schott SK. 2018. Global Firms. *Journal of Economic Literature* 56, 565-619.

Blaum J. 2019. Global Firms in Large Devaluations. Mimeo.

Chaney T. 2008. Distorted Gravity: The Intensive and Extensive Margins of International Trade. *American Economic Review* 98, 1707-1721.

Costinot A. 2009. An Elementary Theory of Comparative Advantage. *Econometrica* 77, 1165-1192.

Costinot A, Rodríguez-Clare A. 2014. Trade Theory with Numbers: Quantifying the Consequences of Globalization. *Handbook of International Economics*, Volume 4, Elsevier: North Holland, 197-261.

Eaton J, Kortum S. 2002. Technology, Geography, and Trade. *Econometrica* 70, 1741-1779.

Eaton J, Kortum S, Kramarz F. 2011. An Anatomy of International Trade: Evidence from French Firms. *Econometrica* 79, 1453-1498.

Fieler AC, Eslava M, Xu DY. 2018. Trade, Quality Upgrading, and Input Linkages: A Theory with Evidence from Colombia. *American Economic Review* 108, 109-146.

Halpern L, Koren M, Szeidl A. 2015. Imported Inputs and Productivity. *American Economic Review* 105, 3660-3703.

Helpman E, Melitz MJ, Yeaple SR. 2004. Export Versus FDI with Heterogeneous Firms. *American Economic Review* 94, 300-316.

Kasahara H, Lapham B. 2013. Productivity and the Decision to Import and Export: Theory and Evidence. *Journal of International Economics* 89, 297-316.

Melitz MJ. 2003. The Impact of Trade on Intra-industry Reallocations and Aggregate Industry Productivity. *Econometrica* 71, 1695-1725.

Melitz MJ, Redding SJ. 2014a. Heterogeneous Firms and Trade. *Handbook of International Economics* Volume 4, Elsevier: North Holland, 1-54.

Melitz MJ, Redding SJ. 2014b. Missing Gains from Trade? *American Economic Review* 104, 317-321.

Tintelnot F. 2017. Global Production with Export Platforms. *Quarterly Journal of Economics* 132, 157-209.